A Theoretical Study of the Effect of Inertia Force of the Vanes on Performance of Hydraulic Balanced Vane Pump by Using Advanced Hypertrochoid Curve at the Inner Surface of Its Stator

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Abstract: In the latest our work, the possibility of application of the advanced hypertrochoid curve in the inner surface of the stator of fixed displacement hydraulic balanced vane pump, theoretically, was studied. By considering the properties of this curve, sealing action between pressure and suction sides of the pump can be improved. One of the important characteristics of a new profile must be improving of the inertial reaction of the vanes in each position where these vanes have a radial movement because it causes a smooth sliding motion of the vanes, and hence, a higher performance of the pump while attaining a longer life of it. Then in the present paper, the effect of the inertial force of the vanes on the performance of the pump with this curve will be studied. The regular motion of the vanes on the inner surface of the stator with smooth profile suppresses the lateral reactional force applied to the vanes and results to diminish local wear of the vanes tip and noise.

Keywords: Hydraulic balanced vane pump, Stator, Advanced hypertrochoid curve, Inertia force

INTRODUCTION

A hydraulic balanced vane pump, now widely used in many hydraulic power systems because of its compactness, lightweight and low cost, is specially suitable for a hydraulic power source such as a power steering system of a vehicle, in which low pressure pulsation and low noise are required. Also it is suited for low viscosity, non-lubricating liquids up to ۲۲۰۰cst/۱۰۰۰۰SSU such as LPG, ammonia, solvents, alcohol, fuel oils, gasoline and refrigerants. When designing a hydraulic balanced vane pump, selecting of a suitable curve for using at the inner surface of its stator constitutes a key factor in improving a sufficient sealing action between pressure and suction sides of the pump, and so decreasing the amount of leakage of working fluid from the clearance between the rotor and the stator and also between the vanes tip and the inner surface of the stator in the sliding contact between the two and, accordingly, increasing the volumetric efficiency and its output flow because of the good continuity between circular arc and selected curve at contact points in the end of sealing zone and smooth sliding motion of the vanes on the inner surface of the stator.

STRUCTURE OF BALANCED VANE PUMP

As shown in Figure ۱, Hydraulic balanced vane pumps comprise a casing, a shaft, a tubular stator (cam ring), a rotor having a plurality of vane grooves fixed on the shaft, two pressure side plates and vanes adapted to slide in said vane grooves. When the vane pump operates, the vanes reciprocate within the respective vane grooves while sliding on the inner surface of the stator in accordance with the rotation of the rotor.

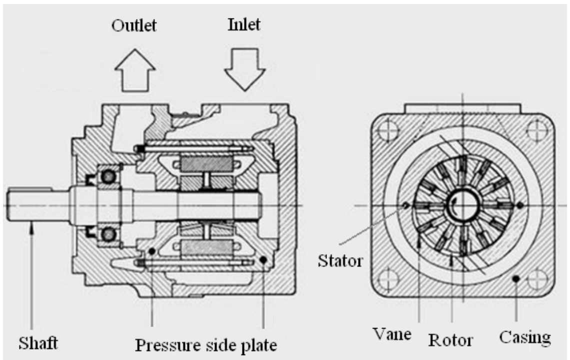

Figure ۱ - Cross- sectional view of vane pump

It is noted that the initial action of the vanes will be caused by centrifugal force and once the pump has developed pressure, the vanes will be exposed to pressure and will be held against the inner surface of the stator at all times. Rotation of the rotor, by virtue of the increasing area between the rotor and the stator surfaces, will cause inlet vacuum and entrance of oil into the pockets between the vanes. By increasing of rotation angle of rotor, the radius of the stator and then the vane pocket area will gradually be decrease and a volume of oil will be delivered under pressure to the outlet port [۱].

HYPERTROCHOID CURVE CHARACTERISTICS

As shown in Equation (۱), the hypertrochoid curve is described by the following complex equations [۲]:

$$Z = X + jY = \sum_{k=۱}^{n+۱} A_k \exp j(\alpha_k K + \beta_k) \qquad (۱)$$

where $۰ \langle K \langle K^*$

Wherein j is imaginary unit, and exp j is imaginary exponential function; A_k, α_k and β_k are real number defining parameters of a particular form of the hypertrochoid; K is a real parameter varying between zero and a particular values K*, where the affix once covers the hypertrochoid; n is an integer defining the order of the hypertrochoid (if n=۱, Basic hypertrochoid [۵-۷] and second order and if n=۲, advanced hypertrochoid). This curve has an order of symmetry, S_s, with respect to its centre point with centre point angle θ_c ($\theta_c \neq ۰$). In a plane curve, an order of symmetry, S_s, with respect to a point represents the quality of the curve by which after one revolution with an amplitude $۲\pi/S_S$ radians about that point, the curve will be brought in coincidence with itself. By contrast, the closed hypertrochoid curve has order of symmetry S_H with respect to a centre point with centre point angle θ_H that differs from the order of symmetry S_s and is expressed by a rational number according to the Equation (۲):

$$S_H = \frac{\theta_c + \theta_H}{\theta_H} \cdot S_s = \frac{a}{b} \quad \text{wherein} \quad \theta_H = \left(\frac{۲\pi}{S_s}\right) - \theta_c \qquad (۲)$$

By substituting n=۲ in Equation (۱), the advanced hypertrochoid curve, as shown in Equation (۳), will be achieved [۸].

$$Z = \sum_{k=۱}^{۳} A_k \exp j(\alpha_k K + \beta_k) = A_۱ \exp j(\alpha_۱ K + \beta_۱) + A_۲ \exp j(\alpha_۲ K + \beta_۲) + A_۳ \exp j(\alpha_۳ K + \beta_۳) \qquad (۳)$$

Wherein:

$$Z = X + jY = R \exp J\theta = R(\cos\theta + J\sin\theta) \qquad (۴)$$

By considering the characteristics of the hypertrochoid curve, we can determine all parameters of the advanced hypertrochoid curves.

Calculating of the Parameters α_k and β_k

$$\alpha_۲ = \frac{b}{a} = \alpha_m, \ \alpha_۱ = \alpha_m(-1) = \frac{la-b}{a}, \ \alpha_۳ = \alpha_m(+1) = \frac{la+b}{a}, \ \beta_۲ = \frac{\theta_C}{۲} \ \text{and} \ \beta_۱ = \beta_۳ = \frac{۲\pi}{s_S} + \frac{\theta_C}{۲} \qquad (۵)$$

Calculating of the Parameter A_k

$$n = ۲ \Rightarrow R_e^* = \sum_{k=۱}^{۳} A_k = A_۱ + A_۲ + A_۳$$

Because $R_e \le R_e^* \Rightarrow R_e = A_۱ + A_۲ + A_۳$

If assume $m = ۲$:

$$R_i^* = A_m - \sum_{\substack{k=۱ \\ k \neq m}}^{۳} A_k = A_۲ - (A_۱ + A_۳) \qquad (۶)$$

Because $R_i \ge R_i^* \Rightarrow R_i = A_۲ - (A_۱ + A_۳)$

Because of assuming $R_e = R_r + H$ and $R_i = R_r$ then:

$A_۲ = R_r + \frac{H}{۲}$ and $A_۱ = A_۳ = \frac{H}{۴}$

Wherein R_r is radius of the rotor, H is maximum vane stroke.

By substituting the above parameters in the advanced hypertrochoid curve and using the definition of $e^{j\theta}$ (or $\exp j\theta$), the above equation will be became as follow:

$$\begin{cases} X = \frac{H}{۴}\cos\left(\frac{la-b}{a}K + \frac{۲\pi}{s_S} + \frac{\theta_C}{۲}\right) + \left(R_r + \frac{H}{۲}\right)\cos\left(\frac{b}{a}K + \frac{\theta_C}{۲}\right) + \frac{H}{۴}\cos\left(\frac{la+b}{a}K + \frac{۲\pi}{s_S} + \frac{\theta_C}{۲}\right) \\ Y = \frac{H}{۴}\sin\left(\frac{la-b}{a}K + \frac{۲\pi}{s_S} + \frac{\theta_C}{۲}\right) + \left(R_r + \frac{H}{۲}\right)\sin\left(\frac{b}{a}K + \frac{\theta_C}{۲}\right) + \frac{H}{۴}\sin\left(\frac{la+b}{a}K + \frac{۲\pi}{s_S} + \frac{\theta_C}{۲}\right) \end{cases} \qquad (۷)$$

DESCRIPTION OF THE PREFERRED EMBODIMENT

For using the advanced hypertrochoid curve in the inner surface of the stator of a balanced vane pump, as shown in Table ۱, we assume the recommended parameters for a typical vane pump.

Table ۱ – Recommended parameters for a vane pump

Order of Symmetry, S_s	Radius of the Rotor, R_r	Max. Vane Stroke, H
۲	۳۱.۷۵ mm	۸.۰۵ mm

The analysis of the advanced hypertrochoid curve characteristics with the above given parameters was carried out by FORTRAN Power station software and then the results was imported to TECHPLOT software for drawing the curve. The advanced hypertrochoid curve that drawn in accordance with the before described considerations is shown in Figure ۲. The main disadvantage of this curve is the difficulty in providing a sufficient seal between the pressure side and the adjoining suction side between the stator and the rotor that caused by the fact that the hypertrochoid can not conform the profile of the rotor over a finite centre point angle (θ_c).

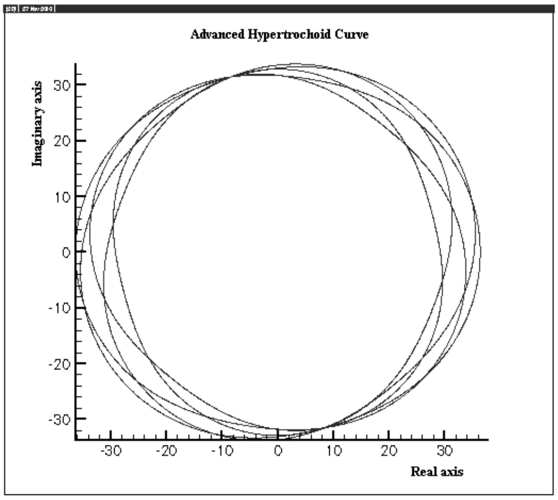

Figure ۲- Advanced hypertrochoid curve (n=۲)

According to some techniques that mentioned in some U.S. Patents [۲, ۳ and ۴], as shown in Figure ۳, a special case of an advanced hypertrochoid (with θ_c=۵۰°) suitable for using at the inner surface of the stator was obtained [۸].

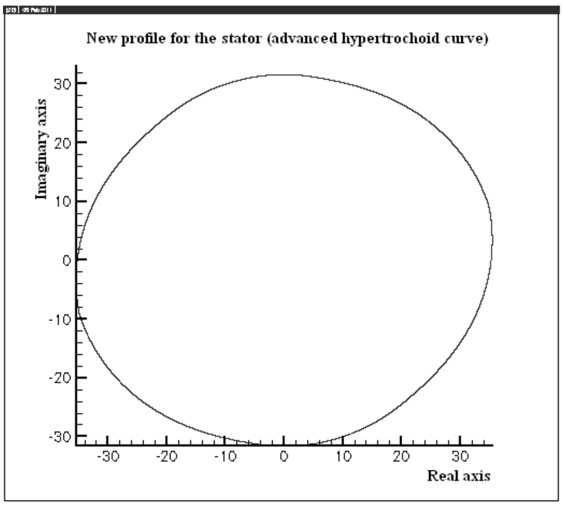

Figure ۳- Advanced hypertrochoid curve with circular arc in the sealing zone

Figure ۴ illustrates a graphical picture of the ring of the stator by using the advanced hypertrochoid curve at the inner surface of it. Also the schematic of cartridge (the rotor, the shaft and the stator without vanes) of the vane pump with new profile is shown in Figure ۵. In this figure, the sealing zone (clearance between rotor and stator hypertrochoid with θ_c=۵۰°) and working zone (pump chamber) are illustrated.

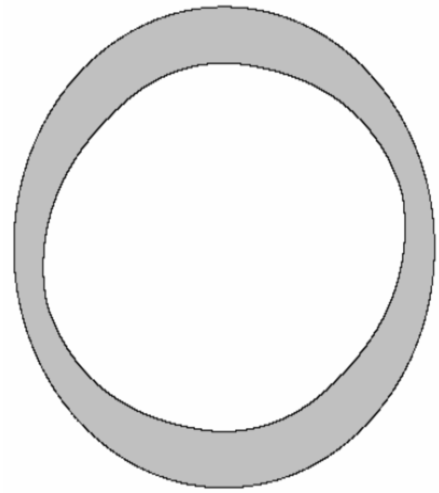

Figure ۴- Stator with new profile

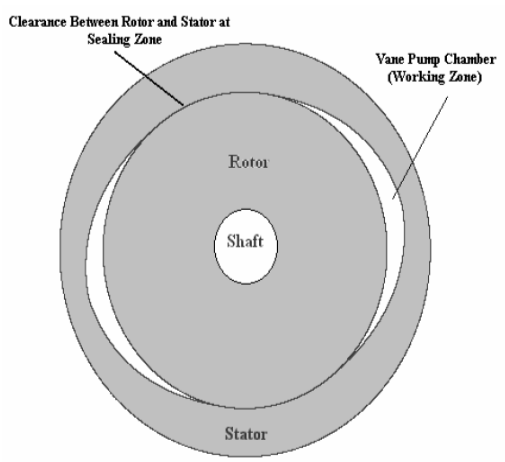

Figure ۵- Schematic of the cartridge

POLAR COORDINATE

Polar coordinate is considered where the particle is located by the radial distance *R* from a fixed pole and by an angular measurement ? to the radial line. Figure ۶ shows the polar coordinate *R* and ? which locate a vane tip travelling (sliding motion) on a curved path (the inner surface of the stator). An arbitrary fixed line, such as the x- axis is used as a reference for measurement of ? . The length of vector $\vec{R}$ that represents displacement of the vane is $R=\sqrt{X^{۲}+Y^{۲}}$ and its direction angle is $\phi=\tan^{-۱}(Y/X)$.

DISPLACEMENT, VELOCITY AND ACCELERATION OF THE VANE

Calculating of Displacement of the Vane

By considering the polar coordinates, when the advanced hypertrochoid curve is used in the inner surface of the stator, as shown in Equation ٨, the length of $\vec{R}$, or R, will be:

$$R=|Z|=\sqrt{X^{2}+Y^{2}}=\sqrt{\left(\sum_{k=1}^{3}A_k\,\mathrm{Cos}\left(\alpha_k K+\beta_k\right)\right)^{2}+\left(\sum_{k=1}^{3}A_k\,\mathrm{Sin}\left(\alpha_k K+\beta_k\right)\right)^{2}} \quad (٨)$$

After substituting all parameters of the curve in above equation, displacement of the vane can be determined:

$$R=\sqrt{\left[A_1^{2}+A_2^{2}+A_3^{2}+2A_1A_2\,\mathrm{Cos}\left(\alpha' K+\beta'\right)+2A_1A_3\,\mathrm{Cos}\left(\alpha'' K+\beta''\right)+2A_2A_3\,\mathrm{Cos}\left(\alpha''' K+\beta'''\right)\right]} \quad (٩)$$

Wherein $\alpha'=\alpha_1-\alpha_2$, $\alpha''=\alpha_1-\alpha_3$, $\alpha'''=\alpha_2-\alpha_3$, $\beta'=\beta_1-\beta_2$, $\beta''=\beta_1-\beta_3$ and $\beta'''=\beta_2-\beta_3$.

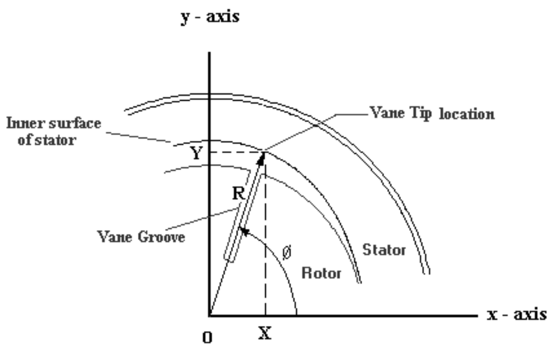

Figure ۶ - Polar coordinates – the geometric presentation of complex number

Calculating of Velocity of the Vane

For calculating of velocity of the vane, the displacement diagram must be drawn. This graph will show displacement of the vane which plotted as a function of time. Degrees of rotor rotation are plotted along the horizontal axis, and the length of the diagram represents one revolution of the rotor. Since the rotor speed (rpm) is constant, equal angular divisions also represent equal time increments. Then by successive differentiation of displacement of the vane, as shown in Equation (٥), the velocity of the vane will be achieved:

$$V=\frac{dR}{dt}=\frac{dR}{dK}\cdot\frac{dK}{d\varphi}\cdot\omega=-\frac{R\cdot\eta}{\xi}\cdot\omega \quad (١٠)$$

Wherein:

$\omega=d\phi/dt$ is angular velocity of rotor (rad/sec),

$$\eta=A_1A_2\alpha'\,\mathrm{Sin}\left(\alpha' K+\beta'\right)+A_1A_3\,\alpha''\,\mathrm{Sin}\left(\alpha'' K+\beta''\right)+A_2A_3\alpha'''\,\mathrm{Sin}\left(\alpha''' K+\beta'''\right),$$

$$\xi=A_1^{2}\alpha_1+A_2^{2}\alpha_2+A_3^{2}\alpha_3+A_1A_2\alpha_1'\,\mathrm{Cos}\left(\alpha' K+\beta'\right)+A_1A_3\,\alpha_1''\,\mathrm{Cos}\left(\alpha'' K+\beta''\right)+A_2A_3\,\alpha_1'''\,\mathrm{Cos}\left(\alpha''' K+\beta'''\right),$$

$\alpha_1'=\alpha_1+\alpha_2$, $\alpha_1''=\alpha_1+\alpha_3$ and $\alpha_1'''=\alpha_2+\alpha_3$,

$$\varphi=\tan^{-1}\left(\frac{Y}{X}\right)=\tan^{-1}\left(\frac{\sum_{k=1}^{3}A_k\,\mathrm{Sin}\left(\alpha_k K+\beta_k\right)}{\sum_{k=1}^{3}A_k\,\mathrm{Cos}\left(\alpha_k K+\beta_k\right)}\right)\Rightarrow$$

$$\frac{dK}{d\varphi}=\frac{R^{2}}{\xi} \quad \text{and}$$

$$\frac{dR}{dK}=-\frac{\eta}{R}$$

Calculating of A cceleration of the Vane

For calculating of acceleration of the vane, the second derivative of displacement of the vane must be determined then as shown in Equation (١١), the acceleration will be:

$$A=\frac{d^{2}R}{dt^{2}}=\frac{dV}{dt}=\frac{dV}{dK}\cdot\frac{dK}{d\varphi}\cdot\omega=-\frac{\left[\frac{dR}{dK}\cdot\eta+R\cdot\varphi\right]\xi+R\cdot\eta\cdot\psi}{\xi^{2}}\cdot\frac{dK}{d\varphi}\cdot\omega^{2} \quad (١١)$$

Wherein:

$$\varphi=A_1A_2\alpha'^{2}\,\mathrm{Cos}\left(\alpha' K+\beta'\right)+A_1A_3\,\alpha''^{2}\,\mathrm{Cos}\left(\alpha'' K+\beta''\right)+A_2A_3\,\alpha'''^{2}\,\mathrm{Cos}\left(\alpha''' K+\beta'''\right)$$

, and

$$\psi=A_1A_2\alpha'\alpha_1'\,\mathrm{Sin}\left(\alpha' K+\beta'\right)+A_1A_3\,\alpha''\alpha_1''\,\mathrm{Sin}\left(\alpha'' K+\beta''\right)+A_2A_3\,\alpha'''\,\alpha_1'''\,\mathrm{Sin}\left(\alpha''' K+\beta'''\right)$$

DESCRIPTION OF THE VANE MOTION

The contact points between circular arcs (sealing arcs) and hypertrochoid curve lobes (displacement curves) at the different angular positions (or different real parameter of K) are shown in Figure ٧. If at these contact points, the inner surface of the stator has discontinuity, the vanes sliding on the said surface are made to move irregularly when they pass such discontinuous points. The irregular motion of the vanes causes chattering of the vanes resulting in local wear of the vanes, a poor seal between the vanes tip and the inner surface of the stator in discontinuous points, generation of noise and other troubles [٢ - ۴]. In this figure, the good continuity between said curves is shown by magnification the contact point area. So this continuity causes the vanes sliding on the inner surface of the stator regularly when they pass such contact points.

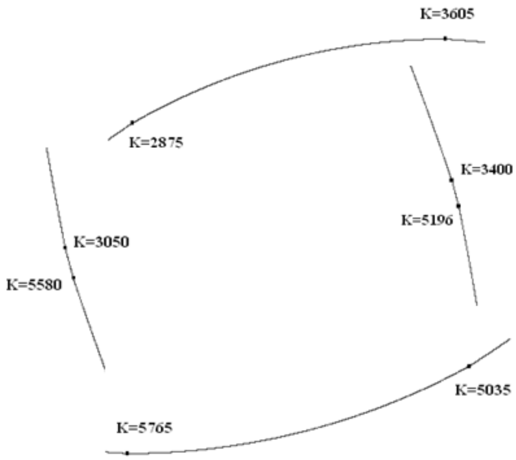

Figure ٧- Good continuity in Contact points at different real parameter K

DIAGRAM OF DISPLACEMENT, VELOCITY AND ACCELERATION OF THE VANES

In accordance with the inner surface of the improved stator shown in Figure ٣, displacement of the vane versus degrees of rotor rotation is shown in Figure ٨. The horizontal axis represents one revolution of the rotor and displacement of the vane is plotted along the vertical axis. Since the rotor speed (rpm) is constant, equal angular divisions also represent equal time increments, so the diagram is in reality a displacement versus time graph therefore by successive differentiation, the velocity (Figure ٩) and the acceleration versus degrees of rotor rotation (Figure ١٠) graphs can be obtained.

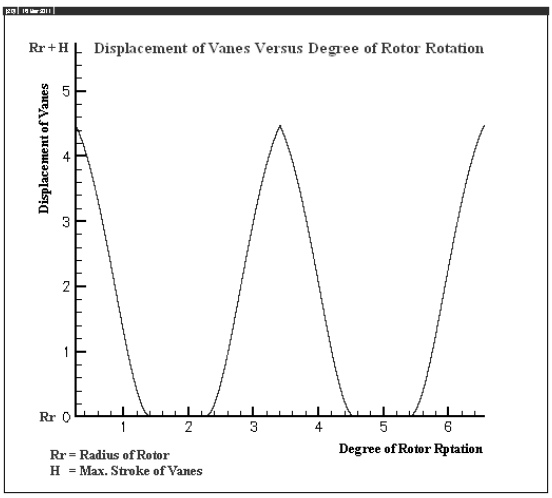

Figure ٨- Vane displacement versus degree of rotor rotation curve

ILLUSTRATION OF INERTIA FORCE OF THE VANES

The acceleration of the vane is important in high speed rotor because it affects inertia forces which result in vibration, noise, high stresses and wear. As illustrated in Figures ٩ and ١٠, the differential $dR/d\theta$ and the second differential $d^2R/d\theta^2$ at the contact points are equal to zero and the acceleration diagram has no discontinuity point, and, accordingly, the derivative of acceleration with respect to time, jerk, or the time rate of change of the inertia force of the vanes is suppressed and hence, the impact characteristics of the vane, oscillation and noise of the pump is decreased [٩].

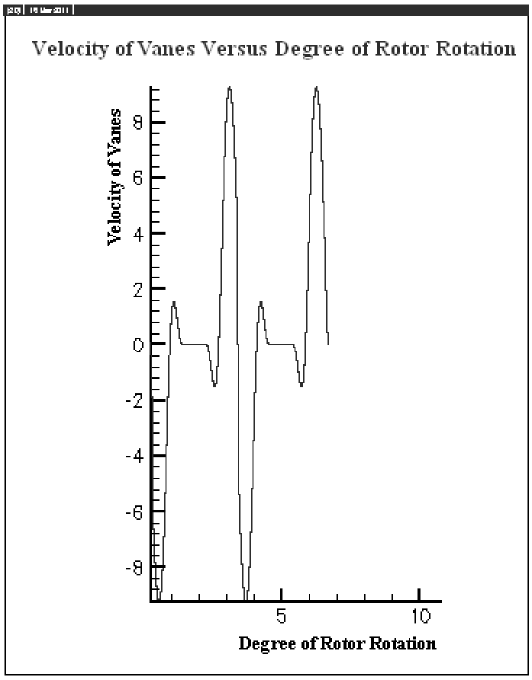

Figure ٩- Vane velocity versus degree of rotor rotation curve

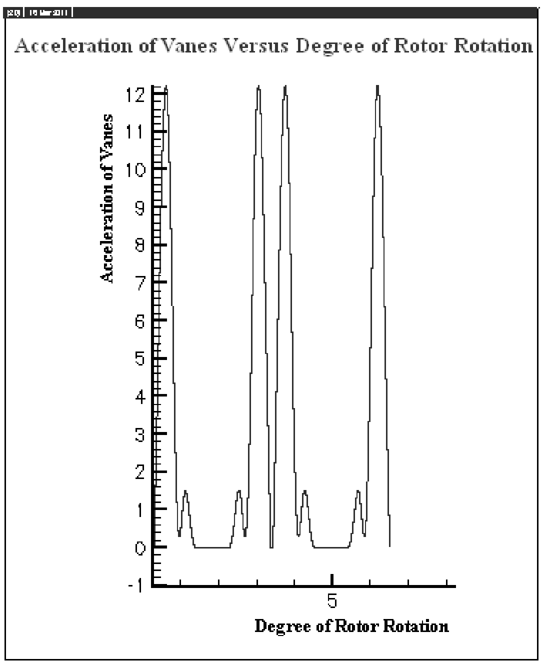

Figure ١٠- Vane acceleration versus degree of rotor rotation curve

CONCLUSIONS

In the latest our work, it was shown that the advanced hypertrochoid curve can be used in the inner surface of the stator of fixed displacement hydraulic balanced vane pump, Hydraulic vane motor and vane type compressor. It will be caused improving a sufficient sealing action between pressure and suction sides of the pump (motor or compressor), and so decreasing the amount of leakage of working fluid from the clearance between the rotor and the stator and between the vanes tip and the inner surface of the stator in the sliding contact between the two and, accordingly, increasing the volumetric efficiency and its output flow because of the good continuity between circular arc and hypertrochoid curve at contact points in the end of sealing zone and smooth sliding motion of the vanes on the inner surface of the stator. Also because there aren't discontinuity points at this profile, the motion of the vanes on the inner surface of the stator will be regular, so the jerk and hence, the lateral reactional force applied to the vanes is suppressed and results to diminish local wear of the vanes tip and the impact characteristics of the vane, high stresses, oscillation and noise of the pump will be decreased and a higher performance and longer life of the parts of the pump will be achieved.

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