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A backward sweep method for power flow solution in distribution networks

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ABSTRACT

A methodology for the analysis of radial or weakly meshed distribution systems supplying voltage dependent loads is here developed. The solution process is iterative and, at each step, loads are simulated by means of impedances. Therefore, at each iteration, it is necessary to solve a network made up only of impedances; for this kind of network, all the voltages and currents can be expressed as linear functions of a single unknown current (in radial systems) or of two unknown currents for each independent mesh (for meshed systems). The methodology has been called "backward" since the unique equation, in case of radial network, and the linear system of equations, in case of meshed network, in which such unknown currents appear can be determined by starting from the ending nodes of the radial system, or of the radialized network (obtained by means of cuts in meshed networks). After a brief presentation of the b/f method, which is currently the most commonly used technique for solving distribution networks, the solution methodology is detailed both for radial and for meshed systems. Then, the way in which PV nodes can be considered is also described.

Finally, the results obtained in the solution of some networks already studied in the literature are presented with other methods, in order to compare their performances.

The applications show the efficiency of the proposed methodology in solving distribution networks with many meshes and PV nodes.

1. Introduction

The method currently adopted for the analysis of radial distribution systems is the backward/forward method (b/f) [1–12], which, in only one iteration for constant current loads, or in more than one iteration for other types of loads (constant power, mixed, etc.) finds the solution.

It is well known that there exist three main variants of the b/f method that differ from each other based on the type of electric quantities that at each iteration, starting from the terminal nodes and going up to the source node (backward sweep), are calculated:

(i) the current summation method, in which the branch currents are evaluated;
(ii) the power summation method, in which the power flows in the branches are evaluated;
(iii) the admittance summation method, in which, node by node, the driving point admittances are evaluated.

In other terms, the three variants of the b/f method simulate the loads within each iteration, with a constant current, a constant power and a constant admittance model. In the forward phase, the three variants are identical since, based on quantities calculated in the backward phase, the bus voltages are calculated starting from the source node and going towards the ending nodes. Voltages are then used to update, based on the dependency of loads on the voltage, the quantities used in the backward sweep in order to proceed to another iteration.

The process stops when a convergence criterion is verified. If the network is meshed, the most commonly adopted solution process is that of radializing the network by means of a certain number of cuts [13–16]. For each couple of nodes, created by each cut, two equal and opposite currents are injected, the value of which is determined by imposing the condition that the voltage difference between the two cut nodes goes to zero. This is the compensation currents method [17]; it uses a reduced Thévenin impedance matrix and a vector of known terms that are the open circuit voltages between the cut nodes. The latter are determined, for the radialized network, at the end of a forward phase. Since the condition defined at the cut nodes is linear (equality of the voltages) in the unknowns (the compensation currents), the system to be solved is also linear and its resolution requires the inversion of the reduced Thévenin impedance matrix. The latter is composed of terms that do not depend on bus voltages; therefore, it is enough to invert it only once and keep the coefficients of the inverse matrix in order to use them in the different iterations.

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The compensation currents method is also used to solve a network with PV nodes [18–21]; in this case, fictitious meshes are added; these are obtained by connecting a null impedance branch between the PV node and the node taken as the reference for voltage at the source node. In such a branch, an ideal voltage generator is inserted, the magnitude of which is equal to the imposed voltage at the PV node. The solution of a network having real and fictitious meshes associated to PV nodes is carried out with the above-described method, executing cuts in all the meshes in order to radialize the network. The cuts in the fictitious meshes are executed so that the two cut nodes are: one, the PV node of the network; the other, the pole of the ideal voltage generator. The construction of the reduced Thévenin impedance matrix is carried out based on all the meshes, both real and fictitious.

A wide review and a comparison study are presented in [22], where various distribution system load flow algorithms, based on the forward/backward sweeps, are reviewed, and their convergence ability is quantitatively evaluated for different loading conditions, R/X ratios, and substation voltage levels; moreover, the effect of static load modeling on the convergence characteristics of algorithms is investigated.

The analysis methodology set-up here is based on the solution, at each iteration, of a radial or radialized network made up of series and shunt impedances and supplied by one point. The series impedances are those of the lines, while the shunt impedances are: the capacitances of the lines (concentrated at the two ends); the capacitors for the reactive power compensation; and the load equivalents. The load impedances are evaluated at the beginning of each iteration, based on the rated values of the power of the loads, on the dependency of the loads on the voltage, and on the loads' bus voltages (such values are fixed at the first iteration, and are calculated in the subsequent iterations).

The simulation of the loads by means of impedances is used in [23] to solve radial or meshed distribution systems; the unknowns of the problem are, for radial systems, all the loads currents; meshed systems are turned into radial by means of cuts and, in this case, the unknowns are all the loading currents and the cut currents of the meshes. The solution of the network goes through the construction of an impedance matrix linking the unknown currents with the source node voltage; all the unknown currents are determined by solving a linear system of equations.

Differently from the methodology developed in [23], the technique here set up, at each iteration allows, in the case of radial systems, to have only one unknown current in the entire system, the value of which can be obtained based on the value of the imposed voltage at the source node of the network. For meshed systems, the unknown currents, the number of which is twice the number of independent meshes, can be gained by solving a linear system of equations obtained by starting from the radialized network and related to the source node of the network and to the nodes from which branches belonging to the meshes spread out.

Differently from the compensation currents method, in which the meshes are considered after having solved the radialized system, in the methodology here set up the unknown currents of the meshed system are solved all together; in this way, it is not necessary to execute the correction of the bus voltages following the interesting and efficient technique proposed by Rajicic et al. in [19] and set up in the aim of limiting the drawbacks of the compensation currents method.

The other quantities of the network, bus voltages and branch currents can be obtained directly from the values of the unknowns and as a linear combination of them. Differently from the b/f method, the calculation of the voltages is not carried out sequentially starting from the source node and going towards the ending nodes; at each iteration, each voltage can be evaluated independently from the others. The methodology is "backward" since the equations with the unknown currents (for radial networks) and the linear system with the unknown currents (for meshed networks) are obtained starting from the ending nodes of the radial system, or from the cut nodes in radialized systems.

The applications show the efficiency of the proposed methodology in solving distribution networks in complex situations, namely in networks with many meshes and fixed voltage nodes. These features are particularly favorable in optimization problems solved by means of the analysis of many possible solutions as, for example, the reactive power flows compensation ([24]) or the service restoration [25].

2. General analysis methodology

Within the proposed methodology, at each iteration, the loads at the nodes are modeled by means of impedances calculated based on the bus power and voltage. So, for each iteration, the main problem is how to solve efficiently a network made up of series and shunt impedances; to this aim, a methodology to solve radial and meshed networks (presented in the following sections) has been set up. Once the bus voltages have been determined, they are compared with those that have been used to evaluate the load impedances. If the error is below a prefixed margin, the iterative process stops, otherwise another iteration is started.

2.1. Radial networks solution

Consider the network in Fig. 1 made up of a single feeder supplying N loads; node 0 is the source node with constant voltage equal to $\boldsymbol{V}_0$ (hereafter, bold letters indicate phasors or complex quantities, and roman letters real quantities).

Under the assumption that loads are impedances, the network can be represented by the cascade of series and shunt impedances shown in Fig. 2; the series impedances, $\boldsymbol{Z}_{\boldsymbol{ser},\boldsymbol{i}}$, include the line impedances, while the load impedances, the line capacitances and the capacitors for reactive power compensation are included in the shunt impedances, $\boldsymbol{Z}_{\boldsymbol{sh},i}$. Calling $\boldsymbol{I}_{\boldsymbol{Zsh},N}$ the unknown current in the last shunt impedance, $\boldsymbol{Z}_{\boldsymbol{sh},N}$, starting from the terminal nodes and going up to the source node, all the bus voltages ($\boldsymbol{V}_{\boldsymbol{i}}$), all the load currents ($\boldsymbol{I}_{\boldsymbol{L},\boldsymbol{i}}$) and all the branch currents ($\boldsymbol{I}_{\boldsymbol{b},\boldsymbol{k}}$) can be calculated. These quantities are proportional to the current $\boldsymbol{I}_{\boldsymbol{Zsh},N}$ following relations such as:

$$\boldsymbol{V}_{\boldsymbol{i}} = \boldsymbol{H}_{\boldsymbol{V}}(i,N) \cdot \boldsymbol{I}_{\boldsymbol{Zsh},N} \tag{1}$$

$$\boldsymbol{I}_{\boldsymbol{L},\boldsymbol{i}} = \boldsymbol{H}_{\boldsymbol{L}}(i,N) \cdot \boldsymbol{I}_{\boldsymbol{Zsh},N} \tag{2}$$

$$\boldsymbol{I}_{\boldsymbol{b},\boldsymbol{k}} = \boldsymbol{H}_{\boldsymbol{I}_{\boldsymbol{b}}}(k,N) \cdot \boldsymbol{I}_{\boldsymbol{Zsh},N} \tag{3}$$

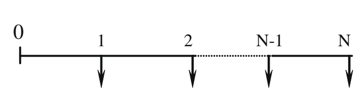

Fig. 1. Main feeder.

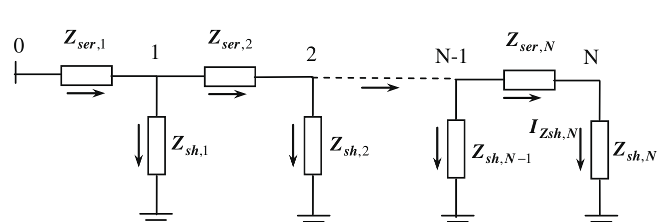

Fig. 2. Scheme of the main feeder shown in Fig. 1 with series and shunt impedances.

where $\boldsymbol{H_V}(i, N)$ is the transfer function of the voltage between the ith node and the Nth node, $\boldsymbol{H_L}(i, N)$ is the transfer function of the load current between the ith node and the Nth node, and $H_{I_b}(k, N)$ is the transfer function of the branch current between branch k and node N. $\boldsymbol{H}$ functions only depend on the series and shunt impedances in the path between node i (branch k) and node N; such impedances include the series and shunt impedances of the lines, those of the loads and the reactances of the capacitor banks (for reactive flow compensation). In the iterative process, only the capacitors' and lines' impedances keep a constant value: the load impedances change at each iteration due to the variation of the bus voltages.

Going towards the source node, for this latter a relation similar to (1) can be found:

$$\boldsymbol{V}_0 = \boldsymbol{H_V}(0, N) \cdot \boldsymbol{I_{Zsh,N}} \tag{4}$$

in which the voltage $\boldsymbol{V}_0$ is the known source node voltage. Then, from (4), the value of the only unknown, the current $\boldsymbol{I_{Zsh,N}}$, can be found and thus, based on (1)–(3), the values of all the bus voltages and all the branches' and loads' currents can be evaluated.

If the network has a branching node, node D in Fig. 3, from which two laterals spread out, in the system two ending nodes can be identified: N_1 and N_2.

Representing the system by means of the load and branch impedances and indicating with $\boldsymbol{I_{Zsh,N_1}}$ and $\boldsymbol{I_{Zsh,N_2}}$ the unknown currents in the shunt impedances at the two ending nodes, Fig. 4, the voltage at branching node D can be expressed by the two following relations:

$$\boldsymbol{V}'_D = \boldsymbol{H_V}(D, N_1) \cdot \boldsymbol{I_{Zsh,N_1}}$$

$$\boldsymbol{V}''_D = \boldsymbol{H_V}(D, N_2) \cdot \boldsymbol{I_{Zsh,N_2}}$$

The first is obtained when node D is reached starting from node N_1, the second in the case that node D is reached starting from node N_2. Since

$$\boldsymbol{V}'_D = \boldsymbol{V}''_D = \boldsymbol{V}_D$$

a coefficient of proportionality, k^*, can be determined; it links the unknown current $\boldsymbol{I_{Zsh,N_2}}$ with $\boldsymbol{I_{Zsh,N_1}}$:

$$\boldsymbol{I_{Zsh,N_2}} = [\boldsymbol{H_V}(D, N_1)/\boldsymbol{H_V}(D, N_2)] \cdot \boldsymbol{I_{Zsh,N_1}} = k^* \cdot \boldsymbol{I_{Zsh,N_1}} \tag{5}$$

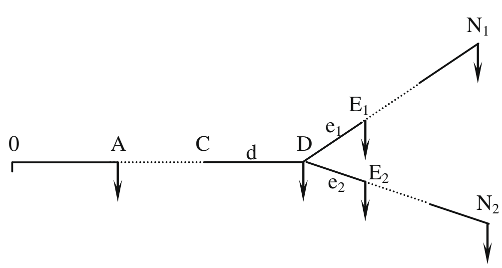

Fig. 3. Main feeder and two laterals (D is a branching node).

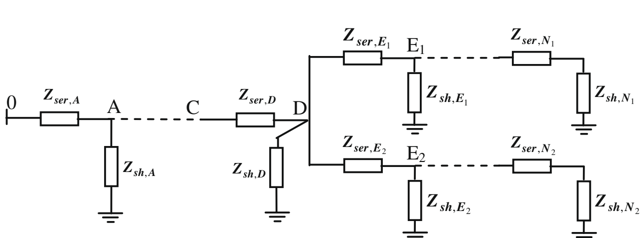

Fig. 4. Scheme of the system shown in Fig. 3 with series and shunt impedances.

Substituting (5) in the expressions of voltages and currents evaluated starting from the terminal node N_2, new relations are determined in which only the current $\boldsymbol{I_{Zsh,N_1}}$ appears as unknown. In particular, the current in the branch e_2, downstream from node D, can be expressed as a function of $\boldsymbol{I_{Zsh,N_1}}$:

$$\boldsymbol{I_{b,e_2}} = \boldsymbol{H_{I_b}}(e_2, N_2) \cdot \boldsymbol{I_{Zsh,N_2}} = \boldsymbol{H_{I_b}}(e_2, N_2) \cdot \boldsymbol{k}^* \cdot \boldsymbol{I_{Zsh,N_1}} \tag{6}$$

The balance of currents at node D allows the evaluation of the current in branch d, upstream from node D, that only depends on current $\boldsymbol{I_{Zsh,N_1}}$:

$$\boldsymbol{I_{b,d}} = [\boldsymbol{H_{I_b}}(d, N_1) + \boldsymbol{H_{I_b}}(j, N_2) \cdot k^*] \cdot \boldsymbol{I_{Zsh,N_1}} \tag{7}$$

Then, the bus voltage at node C, upstream from node D, can be calculated:

$$\begin{aligned}\boldsymbol{V}_C &= \boldsymbol{V}'_{\boldsymbol{D}} + \boldsymbol{Z_{ser,D}} \boldsymbol{I_{b,d}} \\ &= [\boldsymbol{H_V}(D, N_1) + \boldsymbol{Z_{ser,D}}(\boldsymbol{H_{I_b}}(d, N_1) + \boldsymbol{H_{I_b}}(e_2, N_2) \cdot \boldsymbol{k}^*)] \cdot \boldsymbol{I_{Zsh,N_1}} = \\ &= \boldsymbol{H_V}(C, N_1) \cdot \boldsymbol{I_{Zsh,N_1}}\end{aligned} \tag{8}$$

Also, the voltage at node C only depends on current $\boldsymbol{I_{Zsh,N_1}}$. In this case, the transfer function $\boldsymbol{H_V}(\boldsymbol{C}, \boldsymbol{N_1})$ between node C and the ending node N_1 takes into account the branching that appears along the path C–N_1.

Following the described methodology, namely going towards the source node, the voltage of the latter can be expressed as a function of the unknown current $\boldsymbol{I_{Zsh,N_1}}$. Imposing that in the source node, the voltage gets the prefixed value, the unknown current can be calculated and, from this, through the transfer functions, all the other network quantities.

The just-described process for a network with two ends can be applied to a radial system with many ending nodes. Having chosen one of the unknown currents circulating in the terminal shunt impedances, $\boldsymbol{I_{Zsh,N^*}}$, all the others can be related to it by means of the proportionality coefficient $\boldsymbol{k}^*$ (Eq. (5)) which is calculated, at a branching node, based on the transfer functions of the voltage between the same branching node and the different ending nodes downstream from it.

In this way, for the entire system, there is only one unknown, the current $\boldsymbol{I_{Zsh,N*}}$, the value of which is determined by imposing that the voltage calculated at the source node has the prefixed value:

$$\boldsymbol{V}_0 = \boldsymbol{H}_V(0, N^*) \cdot \boldsymbol{I_{Zsh,N^*}} \tag{9}$$

All the other quantities of the system, voltages and currents, can be determined directly from $\boldsymbol{I_{Zsh,N*}}$ since these are directly proportional to it through the relevant transfer functions $\boldsymbol{H}$.

Note that the implementation of the methodology just illustrated is extremely simple, since it is not necessary to calculate the value of the single transfer functions, but it is sufficient to evaluate all the network quantities (bus voltages, branch currents and load currents) depending on an arbitrary value assigned to the currents circulating on the shunt impedances on the terminal buses. In Appendix I it is illustrated, for a network with a few buses, the sequence of calculations that must be carried out to solve the system.

2.2. Meshed networks solution

Consider a meshed distribution system supplied by a single node. If m is the number of independent meshes, the network can be turned into a radial network by means of m cuts; obviously, at cut nodes, the injection of unknown tie-currents must be considered. For the couple of nodes, $\boldsymbol{TI}_k$ and $\boldsymbol{TII}_k$, created by the cut of mesh k, Fig. 5, the following relations are valid:

$$\boldsymbol{I_{TII_k}} = -\boldsymbol{I_{TI_k}} \tag{10}$$

$$\boldsymbol{V_{TI_k}} = \boldsymbol{V_{TII_k}} \tag{11}$$

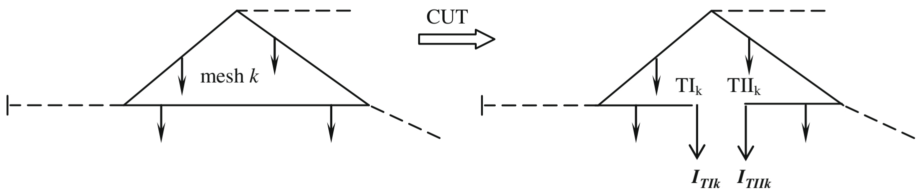

Fig. 5. Cut scheme of the mesh k ($\boldsymbol{T_{Ik}}$ and $\boldsymbol{TII_k}$ are cut nodes) and current injection ($\boldsymbol{I_{TIk}}$ and $\boldsymbol{I_{TIIk}}$) in the cut nodes.

Under the hypothesis that loads are impedances and the network is 'purely meshed', namely that all the branches of the network belong to meshes or to paths from the source node to the nodes of the mesh (in other terms, that there are no purely radial parts), the network can be solved following a procedure similar to the one described in the preceding paragraph.

The network is therefore represented as a set of series and shunt impedances and can be solved using a procedure in which the unknowns are the currents in the terminal shunt impedances of the network and the tie-currents injected in the cut nodes of the meshes, Fig. 6.

Under the hypothesis of the absence of purely radial parts, the terminal nodes of the network are all the cut nodes of the meshes.

It is convenient to execute the cuts at the load nodes and divide the load into two equal parts between the two cut nodes. In this way, two benefits can be obtained: (i) the overall number of nodes of the network increases only by m, and (ii) the unknown currents in the two impedances, $\boldsymbol{I}_{\boldsymbol{Zsh},\boldsymbol{TI}_k}$ and $\boldsymbol{I}_{\boldsymbol{Zsh},\boldsymbol{TII}_k}$, are the same. Indeed, calling $\boldsymbol{Z_{sh}}$ the shunt impedance of the load in the node chosen for the execution of the cut in the mesh k, the shunt impedances in the two cut nodes, $\boldsymbol{Z}_{\boldsymbol{sh},\boldsymbol{TI}_k}$ and $\boldsymbol{Z}_{\boldsymbol{sh},\boldsymbol{TII}_k}$, are equal and equal to $2\boldsymbol{Z_{sh}}$ and (11) gives:

$$\boldsymbol{Z}_{\boldsymbol{sh},\boldsymbol{TI}_k}\boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_k} = \boldsymbol{Z}_{\boldsymbol{sh},\boldsymbol{TII}_k}\boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TII}_k} \tag{12}$$

from which obviously derives:

$$\boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_k} = \boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TII}_k} \tag{13}$$

Finally, in the radialized system, the $2m$ shunt currents, $\boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_k}$ and $\boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TII}_k}$, are unknowns, and the $2m$ tie-currents injected in the cut nodes, $\boldsymbol{I}_{\boldsymbol{TI}_k}$ and $\boldsymbol{I}_{\boldsymbol{TII}_k}$, are also unknowns. Considering (10) and (13), the total number of unknown currents is reduced from $4m$ to $2m$. In what follows, we will refer, as far as the cut currents in the meshes are concerned, to the set of currents $\boldsymbol{I}_{\boldsymbol{TI}_k}$, and, as far as the currents in the shunt impedances at the ending nodes are concerned, to the set $\boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_k}$.

The $2m$ relations allowing the determination of $2m$ unknown currents can be determined, imposing that:

- for the source node, the voltage is set to a prefixed value;
- for the branching nodes, namely for the nodes from which two or more ramification branches spread out downstream (in the radialized system, both the terms downstream and upstream are valid), the voltages calculated through the quantities related to any couple of ramification branches are the same.

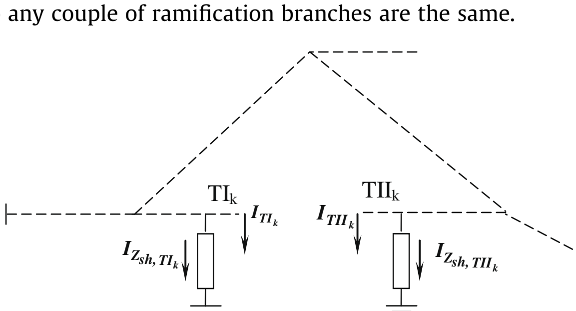

Fig. 6. Unknown currents related to the mesh k.

The process allowing the determination of the $2m$ equations in $2m$ unknown currents is similar to the one described for radial systems: starting from the terminal nodes, the currents (in the branches and loads) and the voltages at the nodes can be evaluated as the superposition of the homonymous quantities calculated by forcing the system first with the terminal shunt currents, then with the tie-currents injected into the tie-nodes.

Therefore, the bus voltages and the currents of the loads or of the branches are expressed by linear combinations of the unknown currents (all or part of them) following relations, similar to (1)–(3):

$$\boldsymbol{V_i} = \sum[\boldsymbol{H_V}(i, TI_j)\cdot \boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_j} + \boldsymbol{H}^*_{\boldsymbol{V}}(i, TI_j)\cdot \boldsymbol{I}_{\boldsymbol{TI}_j}] \tag{14}$$

$$\boldsymbol{I_{L,i}} = \sum[\boldsymbol{H_L}(i, TI_j)\cdot \boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_j} + \boldsymbol{H}^*_{\boldsymbol{L}}(i, TI_j)\cdot \boldsymbol{I}_{\boldsymbol{TI}_j}] \tag{15}$$

$$\boldsymbol{I_{b,k}} = \sum[\boldsymbol{H}_{\boldsymbol{I_b}}(k, TI_j)\cdot \boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_j} + \boldsymbol{H}^*_{\boldsymbol{I_b}}(k, TI_j)\cdot \boldsymbol{I}_{\boldsymbol{TI}_j}] \tag{16}$$

where for the branch current, $\boldsymbol{I_{b,k}}$, the summation is extended to the set of terminal nodes T_j (cut nodes) that in the radialized network are supplied through branch k. For load voltage and current, summation is extended to the terminal nodes supplied by the branch with the sending bus node i.

If D is the sending bus of p branches, namely if D is a branching node, Fig. 7, p relations similar to (14) can be found for the voltage at node D; calling r and s a couple of branches which have D as the sending bus, the voltage in D, calculated through the quantities of branch r, is given by:

$$\boldsymbol{V}_{D,r} = \sum_r[\boldsymbol{H_V}(D, TI_j)\cdot \boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_j} + \boldsymbol{H}^*_{\boldsymbol{V}}(D, TI_j)\cdot \boldsymbol{I}_{TI_j}] \tag{17}$$

where summation is extended to the set of terminal nodes that is supplied through branch r. In the same way, the voltage in D, calculated through the quantities of branch s, is given by:

$$\boldsymbol{V}_{D,s} = \sum_s[\boldsymbol{H_V}(D, TI_j)\cdot \boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_j} + \boldsymbol{H}^*_{\boldsymbol{V}}(D, TI_j)\cdot \boldsymbol{I}_{\boldsymbol{TI}_j}] \tag{18}$$

where summation is extended to the set of terminal nodes that is supplied through branch s. As it must be:

$$\boldsymbol{V}_{D,r} = \boldsymbol{V}_{D,s} \tag{19}$$

from which the following relation can be obtained:

$$\sum_s[\boldsymbol{H_V}(D, TI_j)\cdot \boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_j} + \boldsymbol{H}^*_{\boldsymbol{V}}(D, TI_j)\cdot \boldsymbol{I}_{\boldsymbol{TI}_j}] - \sum_s[\boldsymbol{H_V}(D, TI_j)\cdot \boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_j} + \boldsymbol{H}^*_{\boldsymbol{V}}(D, TI_j)\cdot \boldsymbol{I}_{\boldsymbol{TI}_j}] = 0 \tag{20}$$

For a node with p branches, among all the possible equations similar to (20), $p-1$ equations are independent. For a network with m meshes, it is possible to find $2m-1$ independent equations of the same type as (20). Indeed, making the network radial with m cuts, $2m$ terminal branches can be determined. In the paths connecting each of these $2m$ branches with the source node, it is possible to determine $2m$ expressions of the voltages of the same type as (17) and (18). From these, equating at the branching nodes all the possible couples, $2m-1$ independent equations (like 20) can

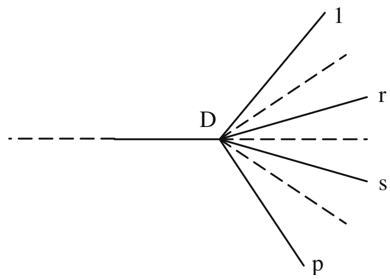

Fig. 7. Branching node.

be gained. In order to solve the problem another equation is needed; the latter is the one expressing the voltage at the source node:

$$\boldsymbol{V}_0 = \sum [\boldsymbol{H}_{\boldsymbol{V}}(0, TI_j) \cdot \boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_j} + \boldsymbol{H}^*_{\boldsymbol{V}}(0, TI_j) \cdot \boldsymbol{I}_{\boldsymbol{TI}_j}] \tag{21}$$

where obviously, summation is extended to all the terminal nodes of the radialized network.

Consider now the presence of radial parts in the above-mentioned meshed system, namely of radial structures that have the origin node belonging to one of the meshes or to the path connecting the source node to one of the nodes belonging to a mesh, Fig. 8.

The general methodology set-up can also be applied to any such partial network. The generic partial radial network, having node O_M as the origin node and a certain number of terminal nodes, is represented by a set of series and shunt impedances. For a radial structure, we are reminded that having chosen one of the unknown currents as a reference, all the other terminal unknown currents can be expressed as proportional to it. Fixed then as the reference, for each of the radial parts, one of its terminal nodes N^* and the relevant unknown shunt current, $\boldsymbol{I}_{\boldsymbol{Z_{sh}},N^*}$, the voltage in the origin node of the partial radial network is given by:

$$\boldsymbol{V}_{O_M} = \boldsymbol{H}_{\boldsymbol{V}}(O_M, N^*) \cdot \boldsymbol{I}_{\boldsymbol{Z_{sh}},N^*} \tag{22}$$

where $\boldsymbol{H}_{\boldsymbol{V}}(O_M, N^*)$ also takes into account possible ramifications in the partial radial network. On the other hand, the node O_M belongs to the meshed system, therefore its voltage is also given by:

$$\boldsymbol{V}_{O_M} = \sum [\boldsymbol{H}_{\boldsymbol{V}}(O_M, TI_j) \cdot \boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_j} + \boldsymbol{H}^*_{\boldsymbol{V}}(O_M, TI_j) \cdot \boldsymbol{I}_{\boldsymbol{TI}_j}] \tag{23}$$

where summation is extended to the set of terminal nodes that are supplied, in the radialized network without radial parts, through the branch, or one of the branches, having node O_M as the sending bus. Equating (22) and (23), the relation between the unknown current of the partial radial network and the unknown currents of the purely meshed network appearing in (23) can be deduced:

$$\begin{aligned}\boldsymbol{I}_{\boldsymbol{Z_{sh}},N^*} = {} & \left[\sum \boldsymbol{H}_{\boldsymbol{V}}(O_M, TI_j)/\boldsymbol{H}_{\boldsymbol{V}}(O_M, \boldsymbol{N}^*)\right] \cdot \boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_j} \\ & + \left[\sum \boldsymbol{H}^*_{\boldsymbol{V}}(O_M, TI_j)/\boldsymbol{H}_{\boldsymbol{V}}(O_M, \boldsymbol{N}^*)\right] \cdot \boldsymbol{I}_{\boldsymbol{TI}_j}\end{aligned} \tag{24}$$

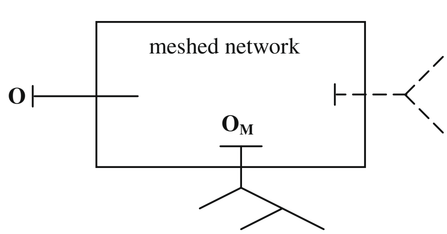

Fig. 8. Partial radial networks spreading from the meshed system.

Since the current circulating in branch k of the partial radial network, having O_M as the sending bus, is proportional to $\boldsymbol{I}_{\boldsymbol{Z_{sh}},N^*}$:

$$\boldsymbol{I}_{\boldsymbol{b},\boldsymbol{k}} = \boldsymbol{H}_{\boldsymbol{I_b}}(k, N^*) \cdot \boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{N}^*} \tag{25}$$

substituting (24) for the preceding expression, the current in branch k can be determined as a function of the unknown currents of the purely meshed network:

$$\begin{aligned}\boldsymbol{I}_{\boldsymbol{b},\boldsymbol{k}} = {} & \boldsymbol{H}_{\boldsymbol{I_b}}(k, N^*) \cdot \left[\sum \boldsymbol{H}_{\boldsymbol{V}}(O_M, TI_j)/\boldsymbol{H}_{\boldsymbol{V}}(O_M, N^*)\right] \cdot \boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_j} \\ & + \boldsymbol{H}_{\boldsymbol{I_b}}(k, N^*) \cdot \left[\sum \boldsymbol{H}^*_{\boldsymbol{V}}(O_M, TI_j)/\boldsymbol{H}_{\boldsymbol{V}}(O_M, \boldsymbol{N}^*)\right] \cdot \boldsymbol{I}_{\boldsymbol{TI}_j}\end{aligned} \tag{26}$$

The balance of the currents at node O_M can thus be carried out without the introduction of the further unknown represented by the terminal shunt current of the partial radial network, $\boldsymbol{I}_{\boldsymbol{Z_{sh,*}},N^*}$. As a conclusion, the presence of radial parts does not modify the total number of unknown currents: instead, the coefficients – on which the voltages and the currents in the purely meshed network linearly depend – are influenced by the unknown currents of the radial parts.

The system of equations, made up of $2m - 1$ relations like (20) and of the relation (21), in $2m$ unknowns $\boldsymbol{I}_{\boldsymbol{Z_{sh}},\boldsymbol{TI}_j}$ and $\boldsymbol{I}_{\boldsymbol{TI}_j}$, is linear: once the unknowns are determined, it is possible to evaluate directly all the voltages and currents of the meshed network by equations (14)–(16). For radial parts, the unknown current $\boldsymbol{I}_{\boldsymbol{Z_{sh,*}},N^*}$ must first be calculated by means of (24) and then all the electric quantities that are proportional to it can be evaluated.

In Appendix II it is illustrated, for a network with a few buses and only one mesh, the sequence of calculations that must be carried out to solve the system.

3. PV nodes

The simulation of PV nodes is carried out considering, in such nodes, the presence of shunt reactances that can provide reactive power to keep the bus voltage magnitude at a specified value. The currents injected into the PV nodes from such reactances are unknowns and can be evaluated by the Thévenin theorem based on the initial and final conditions of the network. The initial conditions are related to the absence of PV nodes, while the final conditions refer to the presence of reactances in the PV nodes and to the voltage values imposed at the same nodes.

Indicating with $[\boldsymbol{V}^f_{PV}]$ and with $[\boldsymbol{V}^i_{PV}]$ the array of the PV node voltages in the final unknown and initial conditions and with $[\boldsymbol{I}^f_{PV}]$ the vector of the currents injected into the nodes, the Thévenin theorem gives:

$$[\boldsymbol{V}^f_{PV}] - [\boldsymbol{V}^i_{PV}] = [\boldsymbol{Z}] \cdot [\boldsymbol{I}^f_{PV}] \tag{27}$$

where $[\boldsymbol{Z}]$ is the reduced bus impedance matrix obtained by neglecting the load impedances and considering only the PV nodes and some branching nodes in the network. Indeed, if the network is radial, only the branching nodes from which PV nodes are seen on totally different paths must be considered; if the network is meshed, the purely radial parts must be omitted (unless they include PV nodes) and only all the branching nodes of the meshes are to be considered.

The final conditions of the network are the following:

– for the jth PV node, indicating with B_j the unknown susceptance (positive if capacitive) of the shunt apparatus, the aim of which is to have a voltage with a specified magnitude V^{sp}_j, the following relation holds:

$$\boldsymbol{I}^f_j = -jB_j \boldsymbol{V}^f_j \tag{28}$$

where $\boldsymbol{V}^f_j$ is the jth bus voltage phasor and the minus sign takes into account that the currents injected into the network must be considered positive; the in-phase and in-quadrature components of V_j^f must meet the following condition:

$$\boldsymbol{V}_{j,p}^{f^2} + \boldsymbol{V}_{j,q}^{f^2} = \boldsymbol{V}_j^{sp^2} \tag{29}$$

Defining a matrix $[\boldsymbol{B}]$ of order N_{PV}

$$[\boldsymbol{B}] = \begin{bmatrix} B_1 & 0 & \dots & 0 \\ 0 & B_2 & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & B_{N_{PV}} \end{bmatrix} \tag{30}$$

and indicating with $[V_{PV,p}^f]$, $[V_{PV,q}^f]$, the vectors of the in-phase and in-quadrature components of the voltages at the PV nodes and with $[V_{PV}^{sp}]$, the vector of specified values for the magnitudes of the voltages at the same nodes, the following relations can be written:

$$[\boldsymbol{V}_{PV}^f] - [\boldsymbol{V}_{PV}^i] = -\boldsymbol{j}[\boldsymbol{Z}] \cdot [\boldsymbol{B}] \cdot [\boldsymbol{V}_{PV}^f] \tag{31}$$

$$[V_{PV,p}^f] \cdot [V_{PV,p}^f]^t + [V_{PV,q}^f] \cdot [V_{PV,q}^f]^t = [V_{PV}^{sp}] \cdot [V_{PV}^{sp}]^t \tag{32}$$

where apex t indicates the transposition operator. From each complex equation of the system (31), two equations can be obtained, in terms of the components in phase and in quadrature of the voltages at the PV nodes; for the generic PV node the two equations are:

$$V_{i,p}^f - V_{i,p}^i = -\sum_{j=1,NPV} B_j(R_{ij}V_{j,q}^i + X_{ij}V_{j,p}^i) \tag{33}$$

$$V_{i,q}^f - V_{i,q}^i = -\sum_{j=1,NPV} B_j(X_{ij}V_{j,q}^i - R_{ij}V_{j,p}^i) \tag{34}$$

where R_{ij} and X_{ij} are the real and the imaginary part of the term $\boldsymbol{Z}_{ij}$ belonging the matrix $[\boldsymbol{Z}]$.

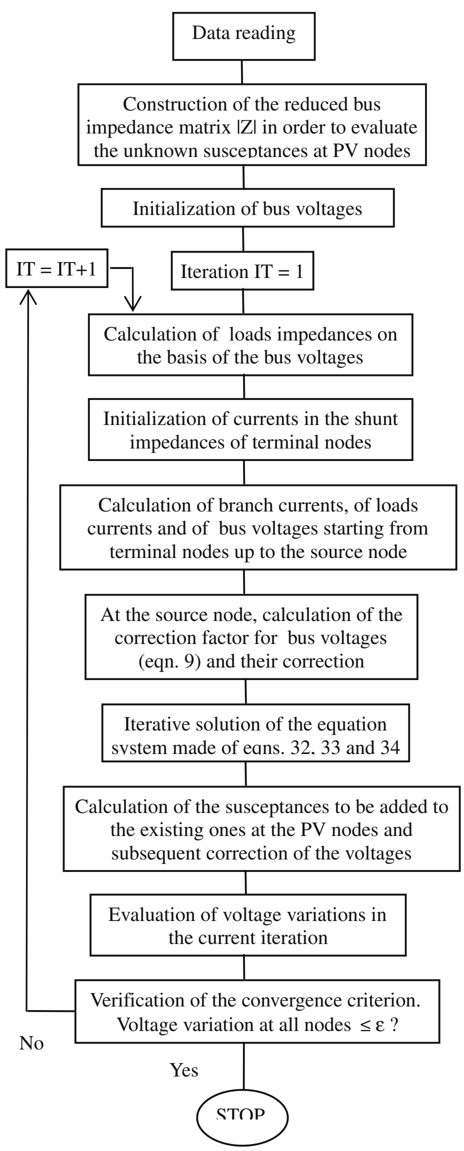

Fig. 9. Flow-chart of the procedure for radial systems

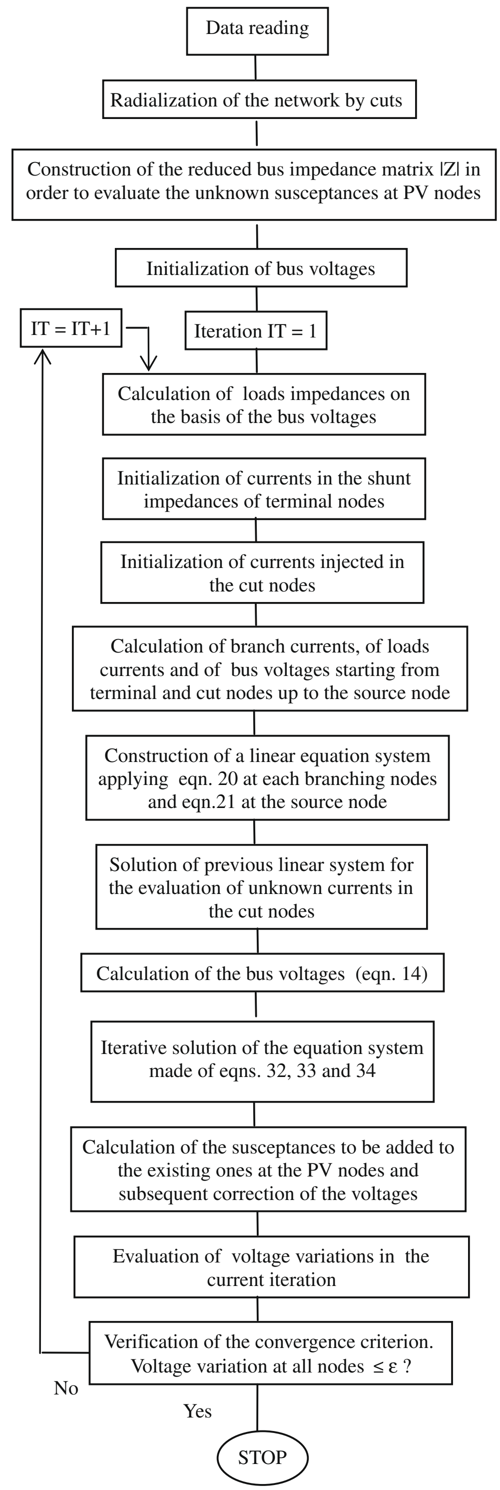

Fig. 10. Flow-chart of the procedure for meshed systems.

The system composed by NPV (32), NPV (33) and NPV (34) is non-linear and for its solution, with respect to the 3 NPV unknown quantities ($V^f_{PV,p}$, $V^f_{PV,q}$, B_j), an easy iterative method can be applied (in the implementation of the methodology, the Gauss-Seidel method has been employed).

4. Implementation of the methodology

To solve a radial or meshed distribution network in the absence of PV nodes, with the methodology set-up, at each iteration it is required to solve an equation with a single unknown current ($\boldsymbol{I}_{L,N^*}$) for a radial system, or a system of $2m$ equations with $2m$ unknown currents ($\boldsymbol{I}_{Z_{sh},\Pi_j}$ and $\boldsymbol{I}_{\Pi_j}, j = 1, 2, \ldots, m$), for a network with m independent meshes. The problem is the evaluation, at each iteration, of the coefficient of (9), or of the coefficients of the unknowns in the system of Eqs. (20) and (21).

If PV nodes are present in the network, the susceptances of such nodes are evaluated at the end of each iteration when the PV nodes' voltages are available. Since the mathematical models (32)–(34) related to PV nodes are non-linear, the susceptances are evaluated by means of an iterative technique. Moreover, at the end of the first iteration, the values of the susceptance calculated for the different PV nodes are given to these nodes; then, in the following iterations, their variations are calculated and these are added to the existing susceptances. To accelerate the convergence, based on an efficient technique proposed in [9], once the values of susceptances to be added in the PV nodes have been calculated, before proceeding to a new iteration, the bus voltages' values are corrected to take into account the susceptances just calculated.

For radial and meshed systems, the main steps of the procedure are shown in the flow-charts reported in Figs. 9 and 10 respectively.

Table 1
Results attained in the literature and with the proposed method

[...]	Nodes	Meshes	PV nodes	f_c	ε	$IT_{[...]}$	IT_{est}	It_{int}/IT_{est}
19	11	3	2	1.0	10^{-6}	11	7	10.7
18	14	7	4	0.5	10^{-4}	7	4	6.3
18	14	7	4	1.0	10^{-4}	9	6	5.5
18	14	7	4	1.4	10^{-4}	14	8	5.1
18	14	7	4	1.5	10^{-4}	15	9	5.0
18	14	7	4	1.8	10^{-4}	19	10	7.1
19	30	12	5	0.5	10^{-4}	10	5	6.0
19	30	12	5	0.7	10^{-4}	11	6	6.0
19	30	12	5	0.9	10^{-4}	14	6	6.7
19	30	12	5	1.0	10^{-4}	14	6	6.7
19	30	12	5	1.2	10^{-4}	18	7	6.4
15	69	5	4	1	10^{-4}	6	5	12.4

5. Applications

The main aim of the applications is to compare the developed methodology with other methodologies already proposed in the literature.

Concerning radial or weakly meshed systems, the performances of the proposed methodology are analogue to those of other similar methods based, at each iteration, on loads simulation by means of impedances; in [22] for different methodologies present in literature and applied to electrical systems with different number of buses, these performances are detailed, varying loading factor and the R/X lines ratio. The conclusions reported in [22] underline that, in the case of radial or weakly meshed systems, the loads simulation by means of impedances does not improve the speed of convergence. To complete results and conclusions reported in [22], the applications here executed regard the analysis of complex situations, i.e. electrical distribution systems with a high number of meshes and PV nodes.

In particular, networks reported in [16–20] have been analysed in all the hypothesized loading conditions. The comparison is based on the number of iterations required to reach the solution for a given convergence factor. In Table 1, the following quantities are reported: reference number of the paper in which the results are presented, number of nodes of the considered networks, number of meshes, number of PV nodes, loading factor f_c, convergence factor ε, number of iterations $IT_{[...]}$ reported in the paper, number of iterations obtained with the proposed methodology IT_{est}, and ratio between the number of iterations of the internal and that of the external cycle It_{int}/IT_{est}.

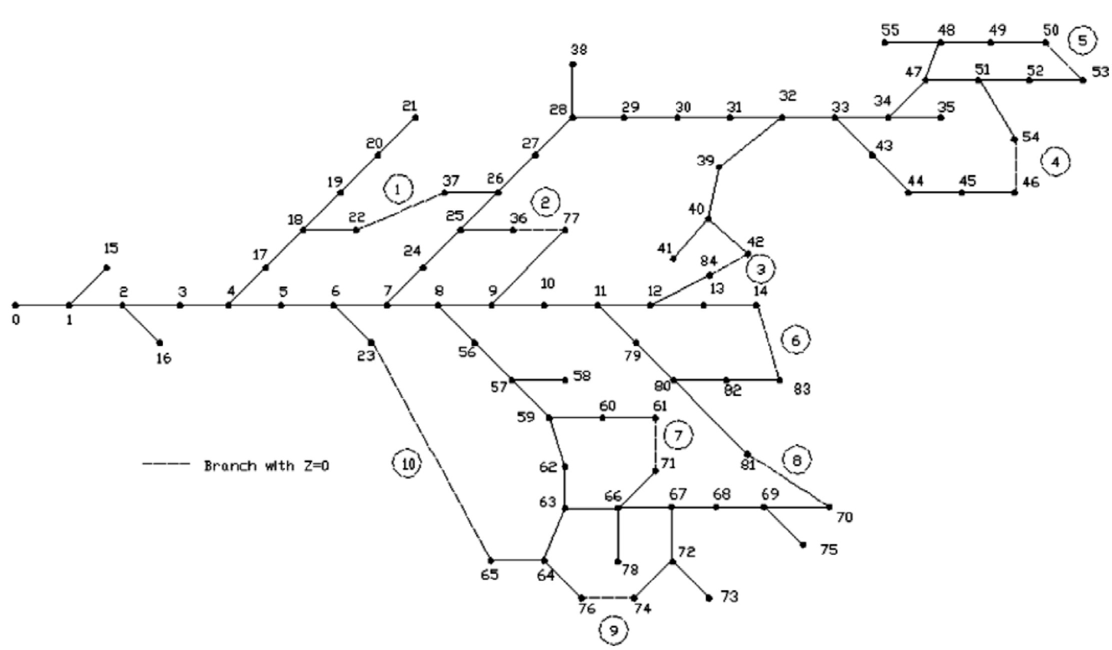

Fig. 11. Test system with 85 nodes and 10 meshes.

Table 2
Results.

Meshes		1			5			10		
PV nodes	f_c	IT_{est}	It_{int}/IT_{est}	CPU (p.u.)	IT_{est}	It_{int}/IT_{est}	CPU (p.u.)	IT_{est}	It_{int}/IT_{est}	CPU (p.u.)
1	1	7	10.6	1	5	7.8	1.38	5	4.6	3.07
2	1	5	6.4	0.73	6	6.7	1.66	4	7.3	2.48
3	1	7	7.6	1.05	6	7.8	1.66	4	9.3	2.49
4	1	6	13.7	0.99	5	19	1.60	4	14.3	2.56
1	2	8	18.5	1.22	6	11.8	1.70	6	6	3.59
2	2	8	33.4	1.43	6	8.2	1.67	6	6.3	3.66
3	2	7	13.1	1.10	6	9.5	1.70	6	9.2	3.72
4	2	6	19.7	1.03	6	13.3	1.82	6	13.8	3.75

In order to obtain some information on CPU times as well, the developed methodology has been applied to solve the system with 85 buses and 10 meshes shown in Fig. 11; data on lines and loads are reported in [3]. The network has been solved considering combinations among the following conditions:

– number of PV nodes: 1, 2, 3 and 4;
– number of meshes: 1, 5 and 10;
– rated load (loading factor $f_c = 1$) and loads increased to 100% ($f_c = 2$).

In Table 2 are reported, for each of the considered conditions, the number of iterations required to reach the final solution, IT_{est}, the ratio between the overall number of iterations of the internal cycle, IT_{int}, and that of the external cycle, and the total CPU time, expressed in p.u. compared to the CPU time spent to solve the system with 1 PV node, 1 mesh and unitary load factor. The external cycle convergence factor, ε_{est}, was fixed to 10^{-4}, while that of the internal cycle was varied based on the cumulative number of iterations of the external cycle, $\varepsilon_{int} = 10\exp[-(1 + IT_{est})]$, with the constraint that: $\varepsilon_{in} \geqslant \varepsilon_{est}$. The voltage magnitude at the PV nodes was fixed, for all the nodes, to 1 p.u. when the load is at the rated value, and to 0.95 p.u. when the load doubles.

The results indicate that the non-linearity of the model, introduced by the presence of PV nodes, does not influence the overall number of iterations required to reach the final solution as the number of PV nodes and meshes varies. The number of iterations increases when the load of the system increases; on the other hand, such an increase is limited, since it is one or two iterations more.

The CPU times are quite sensitive to the number of meshes and do not depend on the number of PV nodes; obviously, they increase with the loading factor.

In all the considered cases, the reactive power injected at the PV nodes has also been evaluated and it has been compared to the value obtained by solving the system with the Newton–Raphson method. In all the cases, the absolute value of the difference between the two values has the same order of magnitude as the convergence factor.

6. Conclusions

The methodology here developed to solve radial or meshed distribution networks is based on an iterative technique in which, at each iteration, loads are simulated as impedances. If the network is radial, all the quantities of the system, voltages and currents are proportional to only one unknown current through transfer functions of the network that depend only on the series and shunt impedances of the system. The only unknown, that is the current in the shunt impedance of a terminal node, can be obtained by the condition that the voltage at the source node must have a prefixed value. The meshed network is transformed to a radial structure by means of cuts and the introduction, in the radialized network, of currents injected at the cut nodes. In this case, the number of unknowns is twice the number of independent meshes and they can be determined by solving a linear system of equations, the coefficients of which must be calculated at each iteration depending on changes of the load impedances.

The methodology is backward since, to evaluate the coefficients of the unknowns, one proceeds from the terminal nodes of the radial or radialized system to the source node. Unlike the b/f method, in which the calculation of the voltages must be carried out sequentially, starting from the source node and going towards the terminal nodes (forward sweep), in the methodology proposed here, bus voltages can be evaluated one at a time. In the backward sweep, they are directly linked to unknowns through transfer functions. For PV nodes, a non-linear model has been set up allowing results to be gained with a precision comparable to those obtainable with models usually adopted for transmission systems. The evaluation of the unknowns in the PV nodes is carried out at the end of each iteration by an iterative process; in particular, the susceptance of the PV node is given to the node; therefore, in the subsequent iterations, only its increment is evaluated. In the presence of PV nodes, the internal cycle does not significantly worsen the performances of the methodology; indeed, both the number of iterations and the CPU time are insensitive to the number of PV nodes in the network.

The applications have shown that the efficiency of the methodology strongly improves starting from simple situations (radial or weakly meshed systems) till to more complex ones (systems with a high number of meshes and PV nodes); for this type of electrical systems the performances of the proposed methodology are better than those obtained with different approaches.

Other benefits of the methodology are: the possibility to take account of any dependency of the loads on the voltage, reduced computational requirements, possible extension to transmission systems with a limited number of meshes, and high precision of results.

Appendix I. Example of solution of a radial system

Fig. A-1

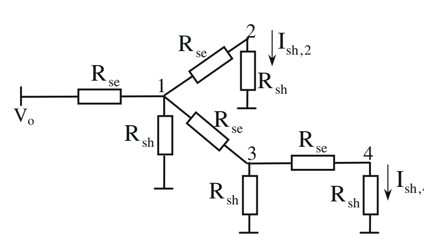

Fig. A-1.

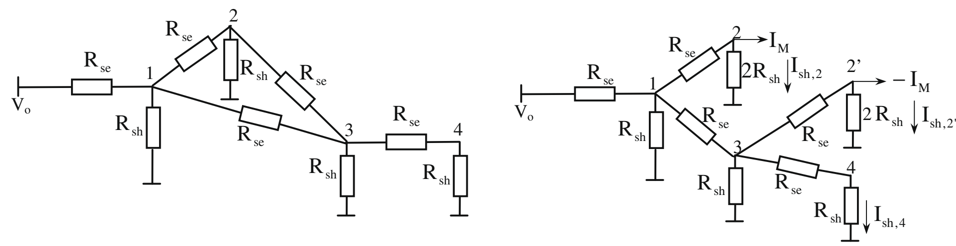

Fig. A-2.

V_0 = 30 V
R_{se} = 1 Ω
R_{sh} = 20 Ω
$I_{sh,4}$ = $I_{sh,2}$ = 1 A values arbitrarily assigned
From terminal bus 2 to branching bus 1:
V_2 = 20 · 1 = 20 V
$I_{1,2}$ = 1 A
V_1 = 20 + 1 · 1 = 21 V
From terminal bus 4 to branching bus 1:
V_4 = 20 · 1 = 20 V
$I_{3,4}$ = 1 A
V_3 = 20 + 1 · 1 = 21 V
$I_{sh,3}$ = 21/20 = 1.05 A
$I_{1,3}$ = 1 + 1.05 = 2.05 A
V'_1 = 21 + 1·2.05 = 23.05 V
The correction factor of the current $I_{sh,2}$ is:
k = 23.05/21 = 1.09762
Correction of the quantities (already calculated) proportional to $I_{sh,2}$
$I^{cor}_{sh,2} = 1 \cdot 1.09762 = 1.09762$ A
$V^{cor}_2 = 20 \cdot 1.09762 = 21.95$ V
$I^{cor}_{1,2}$ = 1.09762 A
$V^{cor}_1 = 21 \cdot 1.09762 = 23.05$ V (=V'_1)
From branching node 1 to source node:
$I_{sh,1}$ = 23.05/20 = 1.1525 A
$I_{0,1} = I^{cor}_{1,2} + I_{1,3} + I_{sh,1}$ = 4.30 A
$V'_0 = 23.05 + 1 \cdot 4.30 = 27.35$ V
The global correction factor is:
K = 30/27.35 = 1.097
Solution of the bus voltages:
V_1 = 1.097 · 23.05 = 25.28 V
V_2 = 1.097 · 21.95 = 24.08 V
V_3 = 1.097 · 21 = 23.03 V
V_4 = 1.097 · 20 = 21.94 V

Appendix II. Example of solution of a meshed system

Fig. A-2
V_0 = 30 V
R_{se} = 1 Ω
R_{sh} = 20 Ω
$I_{sh,4}$ = $I_{sh,2}$ = $I_{sh,2}$ = I_M = 1 A values arbitrarily assigned
$I_{sh,2}$ = $I_{sh,2}$ = I_{sh} and I_M are the unknowns of the system
From terminal bus 2 to branching bus 1 (in parentheses the current whose unitary value determines the value of the coefficient is indicated):
V_2 = 40 (I_{sh})
$I_{1,2}$ = 1(I_{sh}) + 1(I_M)
V_1 = 41(I_{sh}) + 1(I_M)
From terminal bus 4 to branching bus 3:
V_4 = 20 ($I_{sh,4}$)
$I_{1,2}$ = 1($I_{sh,4}$)
V_1 = 21($I_{sh,4}$)
From terminal bus 2′ to branching bus 3:
$V_{2'}$ = 40 (I_{sh})
$I_{3,2'}$ = 1(I_{sh}) – 1(I_M)
$V'_3 = 41(I_{sh}) - 1(I_M)$
From $V_3 = V'_3$ it is possible to derive the equivalence of $I_{sh,4}$ and the unknowns I_{sh} and I_M:
$I_{sh,4}$ = 41/21(I_{sh}) – 1/21(I_M) = 1.95(I_{sh}) – 0.048(I_M)
Correction of the current $I_{3,4}$:
$I_{3,4}$ = 1($I_{sh,4}$) = 1.95(I_{sh}) – 0.048(I_M)
From bus 3 to branching node 1:
$I_{sh,3}$ = 2.05(I_{sh}) – 0.05(I_M)
$I_{1,3} = I'_{3,2} + I_{3,4} + I_{sh,3} = 5(I_{sh}) - 1.098(I_M)$
$V'_1 = 46(I_{sh}) - 2.098(I_M)$
From $V_1 = V'_1$ it is possible to obtain the first equation of the system:
5(I_{sh}) – 3.098(I_M) = 0
From bus 1 to source node:
$I_{sh,1}$ = 2.05(I_{sh}) + 0.05(I_M)
$I_{0,1}$ = $I_{1,2}$ + $I_{1,3}$ + $I_{sh,1}$ = 8.05(I_{sh}) – 0.048(I_M)
V_0 = 49.05(I_{sh}) + 0.952(I_M).
Being V_0 = 30 V it is possible to obtain the second equation of the system:
49.05(I_{sh}) + 0.952(I_M) = 30
5(I_{sh}) – 3.098(I_M) = 0
The solution is:
I_{sh} = 0.593
I_M = 0.957
The bus voltages are:
V_1 = 41 · 0.593 + 0.957 = 25.27 V
V_2 = 40 · 0.593 = 23.72 V
V_3 = 41 · 0.593 – 0.957 = 23.35 V
V_4 = 20 · $I_{sh,4}$ = 20 · (1.95 · 0.593 – 0.048 · 0.957) = 22.21 V

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