A soft soil model that accounts for creep

P.A. Vermeer & H.P. Neher
Institute of Geotechnical Engineering, University of Stuttgart, Germany

Keywords: aging, creep, consolidation, soft soil

ABSTRACT: The well-known logarithmic creep law for secondary compression is transformed into a differential form in order to include transient loading conditions. This 1-D creep model for oedometer-type strain conditions is then extended towards general 3-D states of stress and strain by incorporating concepts of Modified Cam-Clay and viscoplasticity. Considering lab test data it is shown that phenomena such as undrained creep, overconsolidation and aging are well captured by the model.

1 INTRODUCTION

As soft soils we consider near-normally consolidated clays, clayey silts and peat. The special features of these materials is their high degree of compressibility. This is best demonstrated by oedometer test data as reported for instance by Janbu in his Rankine lecture (1985). Considering tangent stiffness moduli at a reference oedometer pressure of 100 kPa, he reports for normally consolidated clays E_{oed} = 1 to 4 MPa, depending on the particular type of clay considered. The differences between these values and stiffnesses for NC-sands are considerable as here we have values in the range of 10 to 50 MPa, at least for non-cemented laboratory samples. Hence, in oedometer testing normally consolidated clays behave ten times softer than normally consolidated sands. This illustrates the extreme compressibility of soft soils.

Another feature of the soft soils is the linear stress-dependency of soil stiffness. According to the Hardening-Soil model we have $E_{oed} = E_{oed}^{ref} (\sigma / p^{ref})^m$, at least for $c = 0$, and a linear relationship is obtained for $m = 1$. Indeed, on using an exponent equal to one, the above stiffness law reduces to $E_{oed} = \sigma / \lambda^*$, where $\lambda^* = p^{ref} / E_{oed}^{ref}$. For this special case of $m = 1$, the Hardening-Soil model yields $\dot{\varepsilon} = \lambda^* \dot{\sigma}/\sigma$, which can be integrated to obtain the well-known logarithmic compression law $\varepsilon = \lambda^* \ln\sigma$ for primary oedometer loading. For many practical soft-soil studies, the modified compression index λ^* will be known and we can compute the oedometer modulus from the relationship $E_{oed}^{ref} = p^{ref} / \lambda^*$.

From the above considerations it would seem that the HS-model is perfectly suitable for soft soils. Indeed, most soft soil problems can be analysed using this model, but the HS-model is not suitable when considering creep, i.e. secondary compression. All soils exhibit some creep, and primary compression is thus always followed by a certain amount of secondary compression. Assuming the secondary compression (for instance during a period of 10 or 30 years) to be a small percentage of the primary compression, it is clear that creep is important for problems involving large primary compression. This is for instance the case when constructing road or river embankments on soft soils. Indeed, large primary settlements of dams and embankments are usually followed by substantial creep settlements in later years.

In some cases dams or buildings may also be founded on initially overconsolidated soil layers that yield relatively small primary settlements. Then, as a consequence of the loading, a state of normal consolidation may be reached and significant creep may follow. This is a treacherous situation as considerable secondary compression is not preceeded by the warning sign of large primary compression.

Apart from foundation-related problems, creep plays an important role in steep slopes. Many natural slopes have a relatively small factor of safety and they show continuous displacements due to creep. Gradual geometric changes and associated degradation of strength may then lead to slope slides. The various different problems that relate to creep have motivated us to develop a stress-strain relationship that takes creep into account.

Buisman (1936) was probably the first to propose a creep law for clay after observing that soft-soil settlements could not be fully explained by classical consolidation theory. The work on 1D-secondary compression was continued by researchers including, for example, Bjerrum (1967), Garlanger (1972) and Mesri (1977) and Leroueil (1977). A more mathematical lines of research in the area were followed by, for example, Sekiguchi (1977), Adachi and Oka (1982) and Borja et al. (1985). This line of mathematical 3D-creep modelling was influenced by the more experimental line of 1D-creep modelling, but conflicts exist.

From the authors' viewpoint, 3D-creep should be a straight forward extension of 1D-creep, but this is hampered by the fact that present 1D-models have not been formulated as differential equations. For the presentation of the Soft-Soil-Creep model we will first complete the line of 1D-modelling by conversion to a differential form, from which an extension is made to a 3D-model. In addition, attention is focused on the model parameters. Finally the model will be validated by considering both model predictions and data from triaxial tests. Here attention is focused on constant strain-rule shear tests and undrained creep tests.

2 BASICS OF ONE-DIMENSIONAL CREEP

When reviewing previous literature on secondary compression in oedometer tests, one is struck by the fact that it concentrates on behaviour related to step loading even though natural loading processes tend to be continuous or transient in nature. Buisman (1936) who was probably the first to consider such a classical creep test. He proposed the following to describe creep behaviour under constant effective tress,

$$\varepsilon = \varepsilon_c + C_B \log \frac{t}{t_c} \qquad \text{for} \qquad t > t_c \tag{1}$$

where ε_c is the strain up to the end of consolidation, t the time measured from the beginning of loading, t_c the time to the end of primary consolidation and C_B is a material constant. As is often the case in soil mechanics, compression is assumed to be positive. For further consideration, it is convenient to rewrite this equation as

$$\varepsilon = \varepsilon_c + C_B \log \frac{t_c + t'}{t_c} \qquad \text{for} \qquad t' > 0 \tag{2}$$

with $t' = t - t_c$ being the effective creep time.

Based on the work by Bjerrum on creep, as published for instance in 1967, Garlanger (1972), proposed a creep equation of the form

$$e = e_c - C_\alpha \log \frac{\tau_c + t'}{\tau_c} \qquad \text{with} \qquad C_\alpha = C_B\,(1 + e_0) \qquad \text{for} \qquad t' > 0 \tag{3}$$

Differences between Garlanger's and Buisman's forms are modest. The engineering strain ε is replaced by void ratio e and the consolidation time t_c is replaced by a parameter τ_c. Equations (2) and (3) are entirely identical when choosing $\tau_c = t_c$. For the case that $\tau_c \neq t_c$ differences between both formulations will vanish when the effective creep time t' increases.

For practical consulting, oedometer tests are usually interpreted by assuming t_c = 24 h. Indeed, the standard oedometer test is a *Multiple Stage Loading Test* with loading periods of precisely one day. Due to the special assumption that this loading period coincides to the consolidation time t_c, it follows that such tests have no effective creep time. Hence one obtains $t' = 0$ and the log-term drops out of equation (3). It would thus seem that there is no creep in this standard oedometer test, but this suggestion is entirely false. Even highly impermeable oedometer samples need less than one hour for primary consolidation. Then all excess pore pressures are zero and one observes pure creep for the other 23 hours of the day. Therefore we will not make any assumptions about the precise values of τ_c and t_c.

Another slightly different possibility to describe secondary compression is the form adopted by Butterfield (1979)

$$\varepsilon^H = \varepsilon_c^H + C \ln \frac{\tau_c + t'}{\tau_c} \tag{4}$$

where ε^H is the logarithmic strain defined as

$$\varepsilon^H = -\ln \frac{V}{V_o} = -\ln \frac{1+e}{1+e_o} \tag{5}$$

with the subscript 'o' denoting the initial values. The superscript 'H' is used for denoting logarithmic strain. We use this particular symbol, as the logarithmic strain measure was originally used by Hencky. For small strains it is possible to show that

$$C = \frac{C_\alpha}{(1+e_o) \cdot \ln 10} = \frac{C_B}{\ln 10} \tag{6}$$

because then logarithmic strain is approximately equal to the engineering strain. Both Butterfield (1979) and Den Haan (1994) showed that for cases involving large strain, the logarithmic small strain supersedes the traditional engineering strain.

3 ON THE VARIABLES τ_C AND ε_C

In this section attention will first be focussed on the variable τ_c. Here a procedure is to be described for an experimental determination of this variable. In order to do so we depart from equation (4). By differentiating this equation with respect to time and dropping the superscript 'H' to simplify notation, one finds

$$\dot{\varepsilon} = \frac{C}{\tau_c + t'} \quad \text{or inversely} \quad \frac{1}{\dot{\varepsilon}} = \frac{\tau_c + t'}{C} \tag{7}$$

which allows one to make use of the construction developed by Janbu (1969) for evaluating the parameters C and τ_c from experimental data. Both the traditional way, being indicated in Figure 1a, as well as the Janbu method of Figure 1b can be used to determine the parameter C from an oedometer test with constant load. The use of the Janbu method is attractive, because both τ_c and C follow directly when fitting a straight line through the data. In Janbu's representation of Figure 1b,

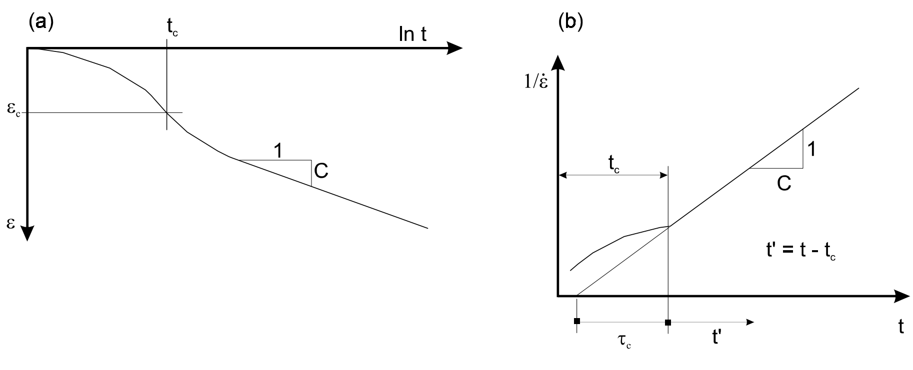

Figure 1. Consolidation and creep behaviour in standard oedometer test.

τ_c is the intercept with the (non-logarithmic) time axis of the straight creep line. The deviation from a linear relation for $t < t_c$ is due to consolidation.

Considering the classical literature it is possible to describe the end-of-consolidation strain ε_c, by an equation of the form

$$\varepsilon_c \;=\; \varepsilon_c^e \;+\; \varepsilon_c^c \;=\; A\,\ln\frac{\sigma'}{\sigma'_0} \;+\; B\,\ln\frac{\sigma_{pc}}{\sigma_{p0}} \qquad (8)$$

Note that ε is a logarithmic strain, rather than a classical small strain although we conveniently omit the superscript $\mathcal{H}'$. In the above equation $\sigma_0{}'$ represents the initial effective pressure before loading and $\sigma'$is the final effective loading pressure. The values σ_{p0} and σ_{pc} representing the preconsolidation pressure corresponding to before-loading and end-of-consolidation states respectively. In most literature on oedometer testing, one adopt e instead of ε, and log instead of ln, and the swelling index C_r instead of A, and the compression index C_c instead of B. The above constants A and B relate for small strains to C_r and C_c as

$$A \;=\; \frac{C_r}{(1+\;\;e_o)\;\cdot\ln 10} \qquad\qquad B \;=\; \frac{(C_c\;\;C_r)}{(1+\;\;e_o)\;\cdot\ln 10} \qquad (9)$$

Combining equations (4) and (8) it follows that

$$\varepsilon \;=\; \varepsilon^e \;+\; \varepsilon^c \;=\; A\,\ln\frac{\sigma'}{\sigma'_0} \;+\; B\,\ln\frac{\sigma_{pc}}{\sigma_{p0}} \;+\; C\,\ln\frac{\tau_c + t'}{\tau_c} \qquad (10)$$

where ε is the total logarithmic strain due to an increase in effective stress from $\sigma_0{}'$ to σ' and a time period of t_c+t'. In Figure 2 the terms of equation (10) are depicted in a ε-lnσ diagram.

Up to this point, the more general problem of creep under transient loading conditions has not yet been addressed, as it should be recalled that restrictions have been made to creep under constant load. For generalising the model, a differential form of the creep model is needed. No doubt, such a general equation may not contain $t'$and neither τ_c as the consolidation time is not clearly defined for transient loading conditions.

4 DIFFERENTIAL LAW FOR 1D-CREEP

The previous equations emphasize the relation between accumulated creep and time, for a given constant effective stress. For solving transient or continuous loading problems, it is necessary to formulate a constitutive law in differential form, as will be described in this section. In a first step we will derive an equation for τ_c. Indeed, despite the use of logarithmic strain and ln instead of log, the formula (10) is classical without adding new knowledge. We may also deviate a bit as we write

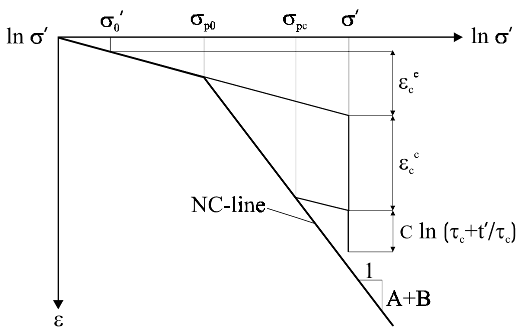

Figure 2. Idealised stress-strain curve from oedometer test with division of strain increments into an elastic and a creep component. For $t' + t_c$ = 1 day one arrives precisely on the NC-line.

σ_{pc} when some other authors write σ'. Moreover, the question on the physical meaning of τ_c is still open. In fact, we have not been able to find precise information on τ_c, apart from Janbu's method of experimental determination.

In order to find an analytical expression for the quantity τ_c, we adopt the basic idea that all inelastic strains are time dependent. Hence total strain is the sum of an elastic part ε^e and a time-dependent creep part ε^c. For non-failure situations as met in oedometer loading conditions, we do not assume an instantaneous plastic strain component, as used in traditional elastoplastic modelling. In addition to this basic concept, we adopt Bjerrum's idea that the preconsolidation stress depends entirely on the amount of creep strain being accumulated in the course of time. In addition to equation (10) we therefore introduce the expression

$$\varepsilon = \varepsilon^e + \varepsilon^c \;=\; A \ln \frac{\sigma'}{\sigma'_0} \;+\; B \ln \frac{\sigma_p}{\sigma_{p0}} \qquad \rightarrow \qquad \sigma_p = \sigma_{p0} \exp\left(\frac{\varepsilon^c}{B} \right) \tag{11}$$

The longer a soil sample is left to creep the larger σ_p grows. The time-dependency of the preconsolidation pressure σ_p is now found by combining equations (10) and (11) to obtain

$$\varepsilon^c - \;\varepsilon_c{}^c \;=\; B \;\ln \frac{\sigma_p}{\sigma_{pc}} \;=\; C \;\ln \frac{\tau_c + \; t'}{\tau_c} \tag{12}$$

This equation can now be used for a better understanding of τ_c, at least when adding knowledge from standard oedometer loading. In conventional oedometer testing the load is stepwise increased and each load step is maintained for a constant period of $t_c + t' = \tau$, where τ is precisely one day. In this way of stepwise loading the so-called normal consolidation line (NC-line) with $\sigma_p = \sigma'$ is obtained. On entering $\sigma_p = \sigma'$ and $t' = \tau - t_c$ into equation (12) it is found that

$$B \;\ln \frac{\sigma'}{\sigma_{pc}} \;=\; C \;\ln \frac{\tau_c + \;\tau \;-\; t_c}{\tau_c} \qquad \text{for} \qquad \text{OCR} \;=\; 1 \tag{13}$$

It is now assumed that $(\tau_c - t_c) \ll \tau$. This quantity can thus be disregarded with respect to τ and it follows that

$$\frac{\tau}{\tau_c} \;=\; \left(\frac{\sigma'}{\sigma_{pc}} \right)^{\frac{B}{C}} \qquad \text{or} \qquad \tau_c \;=\; \tau \;\left(\frac{\sigma_{pc}}{\sigma'} \right)^{\frac{B}{C}} \tag{14}$$

Hence τ_c depends both on the effective stress σ' and the end-of-consolidation preconsolidation stress σ_{pc}. In order to verify the assumption $(\tau_c - t_c) \ll \tau$, it should be realised that usual oedometer samples consolidate for relatively short periods of less than one hour. Considering load steps on the normal consolidation line, we have OCR = 1 both in the beginning and at the end of the load step. During such a load step σ_p increases from σ_{p0} up to σ_{pc} during the short period of (primary) consolidation. Hereafter σ_p increases further from σ_{pc} up to σ' during a relatively long creep period. Hence, at the end of the day the sample is again in a state of normal consolidation, but directly after the short consolidation period the sample is under-consolidated with $\sigma_p < \sigma'$. For the usually very high ratios of $B/C \geq 15$, we thus find very small τ_c-values from equation (14). Hence not only t_c but also τ_c tend to be small with respect to τ. It thus follows that the assumption $(\tau_c - t_c) \ll \tau$ is certainly correct.

Having derived the simple expression (14) for τ_c, it is now possible to formulate the differential creep equation. To this end equation (10) is differentiated to obtain

$$\dot{\varepsilon} \;=\; \dot{\varepsilon}^e \;+\; \dot{\varepsilon}^c \;=\; A \;\frac{\dot{\sigma}'}{\sigma'} \;+\; \frac{C}{\tau_c + \; t'} \tag{15}$$

where $\tau_c + t'$ can be eliminated by means of equation (12) to obtain

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^c = A\,\frac{\dot{\sigma}'}{\sigma'} + \frac{C}{\tau_c}\left(\frac{\sigma_{pc}}{\sigma_p}\right)^{\frac{B}{C}} \quad \text{with} \quad \sigma_p = \sigma_{p0}\,\exp\!\left(\frac{\varepsilon^c}{B}\right) \tag{16}$$

Equation (14) can now be introduced to eleminate τ_c and σ_{pc} and to obtain

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^c = A\,\frac{\dot{\sigma}'}{\sigma'} + \frac{C}{\tau}\left(\frac{\sigma'}{\sigma_p}\right)^{\frac{B}{C}} \quad \text{where} \quad \sigma_p = \sigma_{p0}\,\exp\!\left(\frac{\varepsilon^c}{B}\right) \tag{17}$$

5 THREE-DIMENSIONAL-MODEL

On extending the 1D-model to general states of stress and strain, the well-known stress invariants for pressure $p = \sigma_{oct}$ and deviatoric stress $q = 3\tau_{oct}/\sqrt{2}$ are adopted, with σ_{oct} and τ_{oct} being the octahedral normal and shear stresses respectively. These invariants are used to define a new stress measure named p^{eq}:

$$p^{eq} = p' + \frac{q^2}{M^2\,p'} \tag{18}$$

$$\text{with} \quad p' = \frac{1}{3}\,(\sigma_{1'} + \sigma_{2'} + \sigma_{3'}) \quad \text{and} \quad q = \frac{1}{\sqrt{2}}\,\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$$

In Figure 3 it is shown that the stress measure p^{eq} is constant on ellipses in p-q-plane. In fact we have the ellipses from the *Modified Cam-Clay-Model* as introduced by Roscoe and Burland (1968). The soil parameter M represents the slope of the so-called *'critical state line'* as also indicated in Figure 3. We use the above equation for the deviatoric stress q and

$$M = \frac{6\sin\varphi_{cv}}{3\sin\varphi_{cv}} \tag{19}$$

where φ_{cv} is the critical-void friction angle, also referred to as critical-state friction angle. On using the above definition for q, the equivalent pressure p^{eq} is constant along ellipsoids in principal stress space. To extend the 1D-theory to a general 3D-theory, attention is now focused on normally consolidated states of stress and strain as met in oedometer testing. In such situations it yields $\sigma_2' = \sigma_3' = K0{,}N^C\sigma_1'$, and it follows from equation (18) that

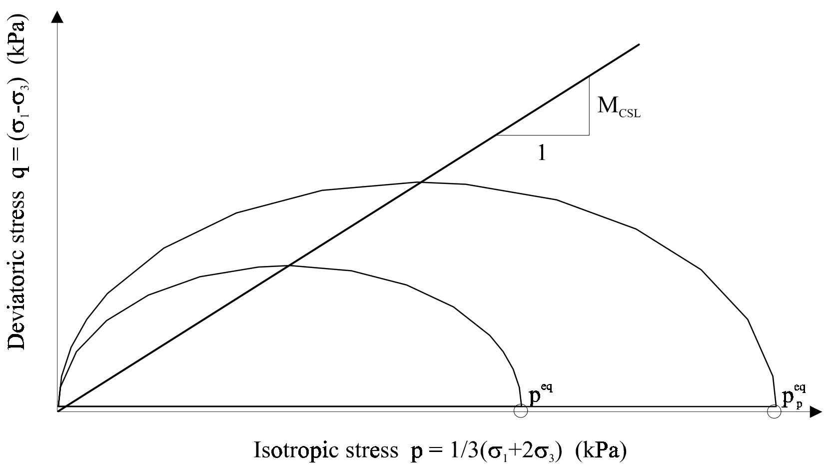

Figure 3. Diagram of p^{eq}-ellipse in a p-q-plane.

$$p^{eq} = \sigma' \left[\frac{1 + 2K_0^{NC}}{3} + \frac{3\left(1 - K_0^{NC}\right)^2}{M^2 \left(1 + 2K_0^{NC}\right)} \right]$$

$$p_p^{eq} = \sigma_p \left[\frac{1 + 2K_0^{NC}}{3} + \frac{3\left(1 - K_0^{NC}\right)^2}{M^2\left(1 + 2K_0^{NC}\right)} \right] \tag{20}$$

where $\sigma' = \sigma_1{}'$, and p_p^{eq} is a generalised preconsolidation pressure, being simply proportional to the one-dimensional one. For known values of K_0^{NC}, p^{eq} can thus be computed from σ' and p_p^{eq} can thus be computed from σ_p. Omitting the elastic strain in the 1D-equation (17), introducing the above expressions for p^{eq} and p_p^{eq} and writing ε_v instead of ε it is found that

$$\dot{\varepsilon}_v^c = \frac{C}{\tau} \left(\frac{p^{eq}}{p_p^{eq}} \right)^{\frac{B}{C}} \quad \text{where} \quad p_p^{eq} = p_{p0}^{eq} \exp\left(\frac{\varepsilon_v^c}{B} \right) \tag{21}$$

For one-dimensional oedometer conditions, this equation reduces to equation (17), so that one has a true extension of the 1D-creep model. It should be noted that the subscript '*0*' is once again used in the equations to denote initial conditions and that $\varepsilon_v^c = 0$ for time $t = 0$.

Instead of the parameters *A*, *B* and *C* of the 1D-model, we will now change to the material parameters κ^*, λ^* and μ^*, who fit into the framework of critical state soil mechanics. Conversion between constants follows the rules

$$\kappa^* \approx \frac{3\,(1 - \nu_{ur})}{(1 + \nu_{ur})} A\,, \qquad B = \lambda^* - \kappa^*\,, \qquad \mu^* = C \tag{22}$$

On using these new parameters, equation (21) changes to become

$$\dot{\varepsilon}_v^c = \frac{\mu^*}{\tau} \left(\frac{p^{eq}}{p_p^{eq}} \right)^{\frac{\lambda^* - \kappa^*}{\mu^*}}, \qquad p_p^{eq} = p_{p0}^{eq} \exp\left(\frac{\varepsilon_v^c}{\lambda^* - \kappa^*} \right) \tag{23}$$

As yet the 3D-creep model is incomplete, as we have only considered a volumetric creep strain ε_v^c, whilst soft soils also exhibit deviatoric creep strains. For introducing general creep strains, we adopt the view that creep strain is simply a time-dependent plastic strain. It is thus logic to assume a flow rule for the rate of creep strain, as usually done in plasticity theory. For formulating such a flow rule, it is convenient to adopt the vector notation

$$\underline{\sigma} = (\sigma_1,\ \sigma_2,\ \sigma_3)^T \qquad \text{and} \qquad \underline{\varepsilon} = (\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3)^T$$

where *T* is used to denote a transpose. Similar to the 1D-model we have both elastic and creep strains in the 3D-model. Using Hook's law for the elastic part, and a flow rule for the creep part, one obtains

$$\dot{\underline{\varepsilon}} = \dot{\underline{\varepsilon}}^e + \dot{\underline{\varepsilon}}^c = \underline{\underline{D}}^{-1}\, \dot{\underline{\sigma}}' + \lambda \frac{\partial g^c}{\partial \underline{\sigma}'} \tag{24}$$

where the elasticity matrix and the plastic potential function are defined as

$$\underline{\underline{D}}^{-1} = \frac{1}{E_{ur}} \begin{bmatrix} 1 & -\nu_{ur} & -\nu_{ur} \\ -\nu_{ur} & 1 & -\nu_{ur} \\ -\nu_{ur} & -\nu_{ur} & 1 \end{bmatrix} \quad \text{and} \quad g^c = p^{eq}$$

Hence we use the so-called equivalent pressure p^{eq} as a plastic potential function for deriving the individual creep strain-rates components. The superscripts '*ur*' are introduced to emphasize that both the elasticity modulus and Poisson's ratio will determine unloading-reloading behaviour. Now it follows from the above equations that

$$\dot{\varepsilon}_v^c = \dot{\varepsilon}_1^c + \dot{\varepsilon}_2^c + \dot{\varepsilon}_3^c = \lambda \left(\frac{\partial p^{eq}}{\partial \sigma_1'} + \frac{\partial p^{eq}}{\partial \sigma_2'} + \frac{\partial p^{eq}}{\partial \sigma_3'} \right) = \lambda \frac{\partial p^{eq}}{\partial_p'} \tag{25}$$

Hence we define $\alpha = \partial p^{eq} / \partial_p'$. Together with equations (23) and (24) this leads to

$$\dot{\underline{\varepsilon}} = \underline{\underline{D}}^{-1} \dot{\underline{\sigma}}' + \frac{\dot{\varepsilon}_v^c}{\alpha} \frac{\partial p^{eq}}{\partial \underline{\sigma}'} = \underline{\underline{D}}^{-1} \dot{\underline{\sigma}}' + \frac{1}{\alpha} \frac{\mu^*}{\tau} \left(\frac{p^{eq}}{p_p^{eq}} \right)^{\frac{\lambda^* - \kappa^*}{\mu^*}} \frac{\partial p^{eq}}{\partial \underline{\sigma}'} \tag{26}$$

$$\text{where} \quad p_p^{eq} = p_{p0}^{eq} \exp\left(\frac{\varepsilon_v^c}{\lambda^* - \kappa^*} \right) \quad \text{or inversely} \quad \varepsilon_v^c = \left(\lambda^* - \kappa^* \right) \ln \frac{p_p^{eq}}{p_{p0}^{eq}}$$

6 FORMULATION OF ELASTIC 3D-STRAINS

Considering creep strains, it has been shown that the 1D-model can be extended to obtain the 3D-model, but as yet this has not been done for the elastic strains. To get a proper 3D-model for the elastic strains as well, the elastic modulus E_{ur} has to been defined as a stress-dependent tangent stiffness according to

$$E_{ur} = 3\,(1 - 2\nu_{ur})\; K_{ur} = 3\,(1 - 2\nu_{ur})\; \frac{p'}{\kappa^*} \tag{27}$$

Hence, E_{ur} is not a new input parameter, but simply a variable quantity that relates to the input parameter κ^*. On the other hand ν_{ur} is an additional true material constant. Hence similar to E_{ur}, the bulk modulus K_{ur} is stress dependent according to the rule $K_{ur} = p'/\kappa^*$. Now it can be derived for the volumetric elastic strain that

$$\dot{\varepsilon}_v^e = \frac{\dot{p}'}{K_{ur}} = \kappa^* \frac{\dot{p}'}{p'} \quad \text{or by integration} \quad \varepsilon_v^e = \kappa^* \ln \frac{p'}{p_0'} \tag{28}$$

Hence in the 3D-model the elastic strain is controlled by the mean stress p', rather than by principal stress σ' as in the 1D-model. However mean stress can be converted into principal stress. For one-dimensional compression on the normal consolidation line, we have both $3p' = (1+2\,K_0^{NC})\sigma'$ and $3p_0' = (1+2\,K_0^{NC})\sigma_0'$ and it follows that $p'/p_0 = \sigma'/\sigma_0$. As a consequence we derive the simple rule $\varepsilon_v^e = \kappa^* \ln(\sigma'/\sigma_0')$, whereas the 1D-model involves $\varepsilon_v^e = A \ln(\sigma'/\sigma_0')$. It would thus seem that κ^* coincides with A. Unfortunately this line of thinking can not be extended towards overconsolidated states of stress and strain. For such situations, it can be derived that

$$\frac{\dot{p}'}{p'} = \frac{1 + \nu_{ur}}{1 - \nu_{ur}} \; \frac{1}{1 + 2K_0} \; \frac{\dot{\sigma}'}{\sigma'} \tag{29}$$

and it follows that

$$\dot{\varepsilon}_v^e = \kappa^* \frac{\dot{p}'}{p'} = \frac{1 + \nu_{ur}}{1 - \nu_{ur}} \frac{\kappa^*}{1 + 2K_0} \frac{\dot{\sigma}'}{\sigma'} \tag{30}$$

where K_0 depends to a great extend on the degree of overconsolidation. For many situations, it is reasonable to assume $K_0 \approx 1$ and together with $\nu_{ur} \approx 0.2$ one obtains $2\varepsilon_v^e \approx \kappa^* \ln(\sigma'/\sigma_0')$. Good agreement with the 1D-model is thus found by taking $\kappa^* \approx 2A$.

7 REVIEW OF THE MODEL PARAMETERS

As soon as the failure yield criterion $f(\underline{\sigma}', c, \varphi) = 0$ is met, instantaneous plastic strain rates develop according to the flow rule $\underline{\dot{\varepsilon}}^p = \lambda\, \partial g/\partial\underline{\sigma}'$ with $g = g(\sigma', \psi)$. This gives as additional soil parameters the effective cohesion, c, the Mohr-Coloumb friction angle, φ, and the dilatancy angle ψ. For fine grained, cohesive soils, the dilatancy angle tends to be small, it may often be assumed that ψ is equal to zero. In conclusion, the Soft-Soil-Creep model requires the following material constants.

Failure parameters as in the Mohr-Coulomb model:

c	:	Cohesion	[kN/m^2]
φ	:	Friction angle	[°]
ψ	:	Dilatancy angle	[°]

Parameters of the Soft-Soil-Creep model:

κ^*	:	Modified swelling index	[-]
λ^*	:	Modified compression index	[-]
μ^*	:	Modified creep index	[-]
ν_{ur}	:	Poisson's ratio for unloading-reloading	[-]
M	:	Slope of the so-called *'critical state line'*	[-]

7.1 *Modified swelling index, modified compression index and modified creep index*

These parameters can be obtained both from an isotropic compression test and an oedometer test. When plotting the logarithm of stress as a function of strain, the plot can be approximated by two straight lines (see Figure 2). The slope of the normal consolidation line gives the modified compression index λ^*, and the slope of the unloading (or swelling) line can be used to compute the modified swelling index κ^*, as explained in section 6. Note that there is a difference between the modified indices κ^* and λ^* and the original Cam-Clay parameters κ and λ. The latter parameters are defined in terms of the void ratio e instead of the volumetric strain ε_v. The parameter μ^* can be obtained by measuring the volumetric strain on the long term and plotting it against the logarithm of time (see Figure 1).

Relationship to Cam-Clay parameters:

$$\lambda^* = \frac{\lambda}{1 + e} \qquad \kappa^* = \frac{\kappa}{1 + e} \tag{31}$$

Relationship to internationally normalized parameters:

$$\lambda^* = \frac{C_c}{2.3\,(1 + e)} \qquad \kappa^* \approx \frac{3}{2.3} \frac{1 - \nu_{ur}}{1 + \nu_{ur}} \frac{C_r}{1 + e} \qquad \mu^* = \frac{C_\alpha}{2.3\,(1 + e)} \tag{32}$$

As already indicated in section 6, there is no exact relation between the isotropic compression indices κ and κ^* and the one-dimensional swelling index C_r, because the ratio of horizontal and vertical stress changes during one-dimensional unloading. For the approximation it is assumed that the average stress state during unloading is an isotropic stress state, i.e. horizontal and vertical stress equal.

For a rough estimate of the model parameters, one might use the correlation $\lambda^* \approx I_p$ (%) / 500, the fact that λ^*/μ^* is in the range between 15 to 25 and the general observation λ^*/κ^* is in the range between 5 to 10.

For characterising a particular layer of soft soil, it is also necessary to know the initial preconsolidation pressure σ_{p0}. This pressure may e.g. be computed from a given value of the overconsolidation ratio (OCR). Subsequently σ_{p0} can be used to compute the initial value of the generalised preconsolidation pressure p_p^{eq}.

7.2 *Poisson's ratio*

In the case of the Soft-Soil-Creep model, Poisson's ratio is purely an elasticity constant rather than a pseudo-elasticity constant as used in the Mohr-Coulomb model. Its value will usually be in the range between 0.1 and 0.2. For loading of normally consolidated materials, Poisson's ratio plays a minor role, but it becomes important in unloading problems. For example, for unloading in a one-dimensional compression test (oedometer), the relatively small Poisson's ratio will result in a small decrease of the lateral stress compared with the decrease in vertical stress. As a result, the ratio of horizontal and vertical stress increases, which is a well-known phenomenon for overconsolidated materials. Hence, Poisson's ratio should be based on the ratio of difference in horizontal stress to difference in vertical stress in oedometer unloading and reloading.

$$\frac{\nu_{ur}}{1 - \nu_{ur}} = \frac{\Delta\ \sigma_{xx}}{\Delta\ \sigma_{yy}} \qquad (33)$$

8 VALIDATION OF THE 3D-MODEL

This section briefly compares the simulated response of undrained triaxial creep behaviour of Haney clay with test data provided by Vaid and Campanella (1977), using the material parameters summarized in Table 1. A extensive validation is provided in Stolle et al.(1997). All triaxial tests considered were completed by initially consolidating the samples under an effective isotropic confining pressure of 525 kPa for 36 hours and then allowing them to stand for 12 hours under undrained conditions before starting the shearing part of the test.

The end-of-consolidation preconsolidation pressure just before shearing, p_{pc}^{eq}, was found to be 373 kPa. This value was determined by simulating the consolidation part of the test. The preconsolidation pressure p_{pc}^{eq} of 373 kPa is less than 525 kPa, which would have been required for an OCR_o of 1. It is clear that the preconsolidation pressure not only depends on the applied maximum consolidation stress, but also on time as discussed in previous section. In Figure 4 we can see the results of Vaid and Campanella's tests (1977) for different strain rates and the computed curves, that were calculated with the present creep model. It is observed that the model describe the tests very well.

Table 1. Material properties for Haney clay.

$\kappa^* = 0.016$	$\lambda^* = 0.105$	$\mu^* = 0.004$
v = 0.25		
$\varphi_{mc} = 32^o$	$\psi = 0^o$	$c = 0$ kPa
$\varphi_{cs} = 32.1^o$		

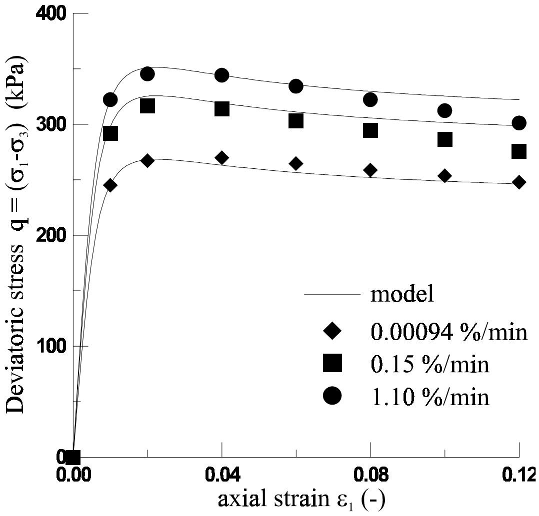

Figure 4. Results of the undrained triaxial tests (CU-tests) with different rates of strain. The faster the test the higher the undrained shear strength.

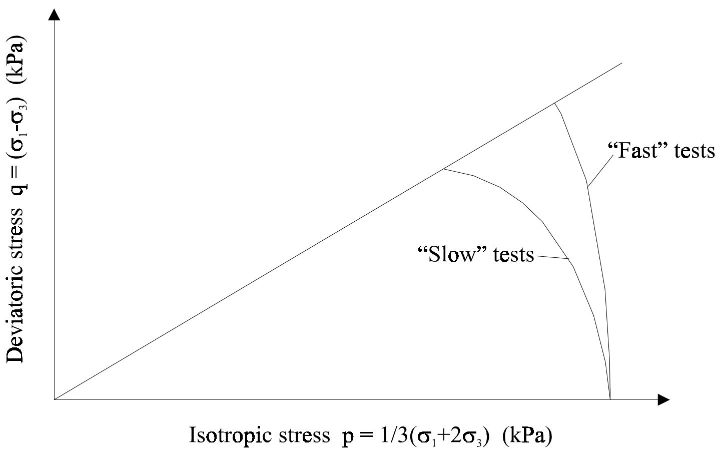

Figure 5. Strain rate dependency of the effective stress path in undrained triaxial tests.

8.1 *Constant strain rate shear tests*

Undrained triaxial compression tests, as considered in Figure 4, are performed under constant rates of vertical strain $\dot{\varepsilon}_1$ and constant horizontal pressure σ_3, so the vertical stress σ_1 is allowed to increase. In the range of classical triaxial compression tests the shear stress *q*, for different strain rates increases with the increase of the strain rate, though there is a limit for very slow and very fast tests between the shear stress *q* can move. This behaviour is shown in Figure 5.

During the undrained test the condition $\dot{\varepsilon}_v = \dot{\varepsilon}_v^e + \dot{\varepsilon}_v^c = 0$ or $\dot{\varepsilon}_v^e = -\dot{\varepsilon}_v^c$ is valid. Therefore the total volumetric change must be zero, but it exists a creep behaviour, though the creep compaction is compensated by an elastic swelling. The slower a test is perform, the lager the creep compaction is and finally the elastic swelling. The expression $\dot{p}' = K_{ur}\ \dot{\varepsilon}_v^e$, where K_{ur} is the elastic bulk modulus, shows that elastic swelling implies a decrease of the effective mean stress. The slower the shear-rate is the more the stress path will curve against the origin of the *p*-*q*-plane.

For extremely fast tests there is no time for creep and it yields $\dot{\varepsilon}_v^c = 0$ and consequently also $\dot{\varepsilon}_v^e = 0$. Hence in this extreme case there is no elastic change and consequently neither a change of the mean stress. This implies a straight vertical path for the effective stress in *p*-*q*-plane.

On inspecting all numerical results, it appears that the undrained shear strength, c_u, may be approximated by equation (34),

$$\frac{c_u}{c_u^*} \approx 1.02 + 0.09 \log \dot{\varepsilon} \tag{34}$$

where c_u^* is the undrained shear strength in an undrained triaxial test with a strain rate of 1% per hour. This agrees well with the experimental data summarized by Kulhawy and Mayne (1990).

8.2 *Undrained triaxial creep tests*

In undrained triaxial creep tests the vertical and horizontal stresses are kept constant, after a initial shear stress q has been applied. The creep behaviour in these tests depends on the stress level. Under a relatively small applied shear stresses (about 30% of the shear strength as determined in conventional tests) the creep movements are small and the creep strain rate, $\dot{\varepsilon}$, decreases, while the excess pore-water pressure increases. After a certain time, an equilibrium condition is reached, in which the creep strain rate is zero and the excess pore-water pressure is constant. Under a shear stress on higher level (but less than 70% of the shear strength of the conventional tests), creep movements seem to continue at constant strain rates. Under shear stresses of still higher intensity, an acceleration of the creep rate takes place followed by complete failure of the specimen ('creep rupture').

As for the constant strain rate tests, all triaxial creep tests had been completed by initially consolidating the samples under an effective confining pressure of 525 kPa and for a period such that p_{pc}^{eq} = 373 kPa. Then undrained creep tests were performed at constant shear stresses of q = 278.3, 300.3, 323.4 kPa. Figure 6 illustrates that these tests one also matched by the model.

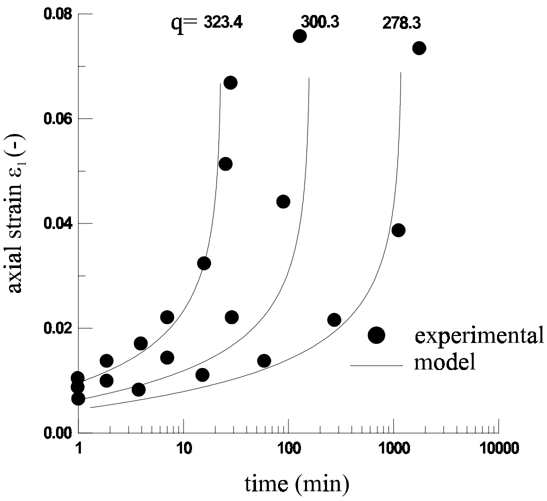

Figure 6. Results of triaxial creep tests. Samples were first consolidated under the same isotropic stress. Then undrained samples were loaded up to different deviatoric stresses. Creep under constant deviatoric stress is observed, being well predicted by the Soft-Soil-Creep model.

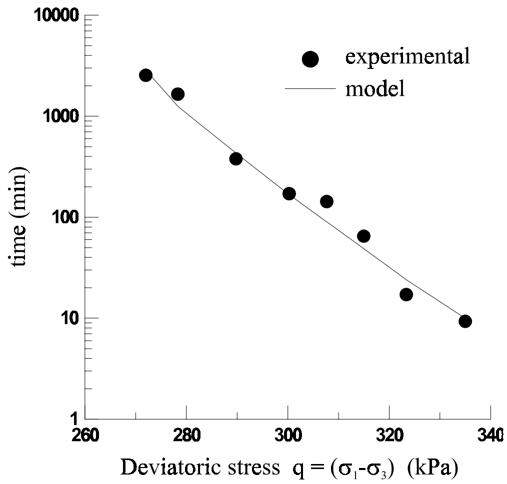

Figure 7. Results of triaxial creep tests. All tests have different constant deviatoric stress. The creep rupture time is the creep time up to a creep rate $\dot{\varepsilon} = \infty$, as indicated by the assumptots in figure 6.

REFERENCES

Adachi, T. & F. Oka 1982. Constitutive equation for normally consolidated clays based on elasto-viscoplasticity. *Soils and Foundations* 22: 57-70.

Bjerrum, L. 1967. Engineering geology of Norwegian normally-consolidated marine clays as related to settlements of buildings. Seventh Rankine Lecture. *Geotechnique* 17: 81-118.

Borja, R.I. & E. Kavaznjian 1985. A constitutive model for the **σ-ε**-t behaviour of wet clays. Geotechnique 35: 283-298.

Borja, R.I. & S.R. Lee 1990. Cam-clay plasticity, part 1: implicit integration of elasto-plastic constitutive relations. *Computer Methods in Applied Mechanics and Engineering* 78: 48-72.

Buisman, K. 1936. Results of long duration settlement tests. *Proceedings 1st International Conference on Soil Mechanics and Foundation Engineering*, Cambridge, Mass. Vol. 1: 103-107.

Butterfield, R. 1979. A natural compression law for soils (an advance on e-log p'). *Geotechnique* 29:469-480.

Den Haan, E.J. 1994. *Vertical Compression of Soils.* Thesis, Delft University.

Garlanger, J.E. 1972. The consolidation of soils exhibiting creep under constant effective stress. *Geotechnique* 22: 71-78.

Janbu, N. 1969. The resistance concept applied to soils. *Proceedings of the 7th ICSMFE, Mexico City* 1:191-196.

Kulhawy, F. H. & Mayne, P. W. 1990. *Manual on Estimating Soil Properties for Foundation Design.* Cornell University, Ithaca, New York

Prevost, J.-H. 1976. Undrained Stress-Strain-Time Behaviour of Clays. *Journal of the Geotechnical Engineering Division* GT12: 1245-1259.

Sekiguchi, H. 1977. Rheological characteristics of clays. *Proceedings of the 9th ICSMFE, Tokyo* 1:289-292.

Stolle, D.F.E. 1991. An interpretation of initial stress and strain methods, and numerical stability. *International Journal for Numerical and Analytical Methods in Geomechanics* 15: 399-416.

Stolle D.F.E., P.G. Bonnier & P.A. Vermeer. 1997. A soft soil model and experiences with two integration schemes. *Numerical Models in Geomechanics*. Numog 1997: 123-128.

Vaid, Y. & R.G. Campanella 1977. Time-dependent behaviour of undisturbed clay. *ASCE Journal of the Geotechnical Engineering Division*, 103(GT7): 693-709.

Vermeer, P.A. & H. van Langen 1989. Soil collapse computations with finite elements. *Ingenieur-Archiv* 59: 221-236.

Vermeer, P.A., Stolle D.F.E. & P.G. Bonnier . 1997. From the classical theory of secondary compression to modern creep. Computer Methods and Advances in Geomechanics Volume 4, Wuhan 1997: 2469-2478.