Asymmetric Information, Heterogeneity in Risk Perceptions and Insurance: An Explanation to a Puzzle

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Abstract

Competitive models of asymmetric information predict a positive relationship between coverage and risk. In contrast, most recent empirical studies find either negative or zero correlation. This paper, by introducing heterogeneity in risk perceptions into an asymmetric information competitive model, provides an explanation to this puzzle. If optimism discourages precautionary effort, there exist separating equilibria exhibiting the observed empirical patterns. It is also shown that zero correlation is consistent with information barriers to trade in insurance markets. The predictions of our model allow us to empirically distinguish it from standard asymmetric information models.

Key Words: Asymmetric Information, Insurance, Risk Perceptions

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Introduction

Starting with the seminal Rothschild-Stiglitz paper (1976), most theoretical models of competitive insurance markets under asymmetric information predict a positive relationship between coverage and the accident probability of the buyer of the contract. This prediction is shared by models of pure adverse selection (e.g. Rothschild and Stiglitz (1976)), pure moral hazard (e.g. Arnott and Stiglitz (1988)) as well as models of adverse selection plus moral hazard (e.g. Chassagnon and Chiappori (1997) and Chiappori et al. (2004)). In fact, Chiappori et al. (2004) show that the positive correlation property holds true in a very general framework involving multiple levels of losses, multidimensional adverse selection, moral hazard and even non-expected utility.

In contrast, most empirical studies find either negative or zero correlation.\(^1\) De Meza and Webb (2001) provide casual evidence for a negative relationship in the credit card insurance market.\(^2\) Cawley and Philipson (1999) study of life insurance contracts shows a negative relationship which, however, is not statistically significant. They also report that insurance premiums fall with coverage. Chiappori and Salanie (2000) and Dionne, Gourieroux and Vanasse (2001) also find a not statistically significant negative relation for the automobile insurance market.\(^3\) More recently, Finkelstein and McGarry (2004) analyse the long-term care insurance market and find no evidence of a positive relationship between insurance coverage and care utilisation (actual risk). However, they find a positive relationship between perceived risk and coverage.

Several recent papers provide different explanations for the negative relation between coverage and the accident rate. De Meza and Webb (2001) consider a model where agents are heterogeneous with respect to their risk aversion and face a moral hazard problem. Also, insurance companies pay a fixed administrative cost per claim. In this model, there exist a separating and a partial pooling equilibrium predicting a negative relationship but due to the fixed per claim cost the less risk-averse agents go

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1 Of course, there are exceptions. In one of the earliest studies, Puelz and Snow (1994) find a positive relationship in the automobile insurance market in Georgia. Similar findings are reported by Finkelstein and Poterba (2002, 2004) for the UK annuities market. Individuals who purchase annuities tend to live longer than those who do not buy. However, Dionne, Gourieroux and Vanasse (2001) argue that the result in Puelz and Snow is likely to be a spurious effect of their linear specification.

2 4.8% of U.K. credit cards are reported lost or stolen each year. The corresponding figure for insured cards is 2.7%.

3 In the Chiappori and Salanie (2000) study those opting for less coverage purchase the legal minimum of third-party coverage. Dionne et al. (2001) look at contracts with two different levels of deductibles.
uninsured. The fact that the administrative costs are now incurred only by the insured agents changes the computation of the premiums which allows Chiappori et al. (2004) to derive the positive correlation result.  

Therefore, in the presence of fixed administrative costs, competitive models of insurance markets under asymmetric information can potentially explain the observed negative or zero correlation between coverage and risk. 

Jullien, Salanie and Salanie (2001) provide a different explanation for the negative correlation between risk and coverage. As far as the insurees are concerned, their model is similar to de Meza-Webb (2001) but in their case the insurer has monopoly power. In order to reveal their type and obtain insurance at a lower per unit price, the less risk-averse insurees accept low coverage. Moreover, not only are the more risk-averse agents willing to pay a higher per unit price to purchase more coverage but also take more precautions and so have a lower accident probability. The positive correlation property breaks because the insurer exploits his monopoly power and extracts more surplus from the more risk-averse insurees. However, more coverage is associated with a higher per unit price.

Finally, Villeneuve (2000) also considers a model with a monopolistic insurer but he reverses the adverse selection hypothesis. He assumes that insurer knows better the insuree’s accident probability than the insuree himself, and obtains separating equilibria displaying a negative relationship between risk and coverage. Insurees try to extract information about their type from the contracts the insurer offers them. In order to convince the high-risk of his type, the monopolistic insurer must offer him a contract that he would not propose to the low-risk type. Profit maximisation then requires that the high-risk type be offered less coverage.

This paper, by introducing heterogeneity in risk perceptions in a competitive model of asymmetric information, provides an alternative explanation to the negative or zero correlation between coverage and risk. It can also explain the empirical

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4 See Koufopoulos (2004) for a formal argument.
5 Negative correlation equilibria may also arise in cases where there is more than one level of loss and one of the two contracts chosen in equilibrium offers more comprehensive coverage. Comprehensive insurance allows claims to be made for contingencies not covered by the less comprehensive contract (e.g. two different levels of deductibles). Therefore, the more comprehensive contract entails higher expected administrative costs that could result in the breaking of the positive correlation property.
6 However, their prediction is not consistent with negative or zero correlation in insurance markets where all agents opt for strictly positive coverage and there are just two events (loss/no loss), (e.g. the Cawley and Philipson (1999) findings).
7 This explanation does not rely on the presence of fixed costs and holds true even if there is just one level of loss and all insurees buy some insurance.
findings of Cawley and Philipson (1999) and Finkelstein and McGarry (2004). Numerous empirical studies both by psychologists and economists indicate that the majority of people tend to be unrealistically optimistic, in the sense that overestimate their ability and the outcome of their actions and underestimate the probability of various risks.\(^8\) For example, Svenson (1981) finds that 90 percent of the automobile drivers in Sweden consider themselves “above average”. Similar results are reported by Rutter, Quine and Alberry (1998) for motorcyclists in Britain. On average, motorcyclists both perceive themselves to be less at risk than other motorcyclists and underestimate their absolute accident probability.

In several other studies people appear to overestimate their survival probabilities (Gan, Hurd and McFadden (2003)) and underestimate their susceptibility to health problems (Weinstein (1987)) and the health risk of smoking (Sutton (1998), Hammar and Johansson-Stenman (2004)).\(^9\) As these studies indicate, people do hold different beliefs about the same risk and, on average, they underestimate it.\(^10\)\(^11\) Furthermore, some authors present evidence indicating that optimism discourages precautionary effort. Finkelstein and McGarry (2004), Lee (1989), Lundborg and Lindgren (2002) and Viscusi (1990) find that those who perceive a higher risk tend to take more precautions.\(^12\)

In line with the empirical evidence, this paper drops the assumption that all insurees have an accurate estimate of their accident probability.\(^13\) It assumes that some agents, the optimists (henceforth Os), underestimate it both in absolute terms and relative to the less optimistic ones (henceforth Rs). They also tend to be less willing to take precautions. However, both the Os and the Rs are certain that their perceived probabilities coincide with the true ones and so their choices are determined


\(^9\) On the other hand, Viscusi (1990) finds that more individuals overestimate the risk of lung-cancer associated with smoking than underestimate it and, on average, they overestimate it. However, the overestimation of smoking risk reported by Viscusi (1990) may be due to the fact that he asked individuals to provide an estimate of the average smoker’s risk not their own risk. Weinstein (1998) reviews a sizable literature demonstrating that smokers perceive the risks to themselves from smoking to be substantially lower than the smoking risks faced by other smokers (see also Slovic (2000)).

\(^10\) Schulman et al. (1993) study twins and provide evidence that differences in the degree of optimism are intrinsic rather than an acquired characteristic.

\(^11\) See Landier and Thesmar (2003) for a discussion of theories developed to explain optimism.

\(^12\) Also, Coelho and de Meza (2005) provide experimental evidence that unrealistic optimism is prevalent and optimism and performance are negatively correlated.

\(^13\) A large number of papers have investigated the implications of overconfidence and unrealistic optimism in securities markets and firm financing. See Barberis and Thaler (2002) for a recent survey of the behavioural finance literature.
by these (mistaken for the Os) beliefs. That is, insurees aim at maximising their expected utility given their beliefs.

In this framework there exist separating equilibria exhibiting negative or zero correlation between coverage and risk. In the first case, the Os not only take fewer precautions (high-risk type) but also purchase less coverage than the Rs. Competition among insurance companies then implies that the Os also pay a higher per unit premium. Because they underestimate their accident probability, the Os purchase low coverage at a high per unit price, although contracts offering more insurance at a lower per unit price are available. There also exists a separating equilibrium that exhibits zero correlation between coverage and risk and involves the Rs being quantity-constrained. In order to reveal their type, the Rs accept lower coverage than they would have chosen under full information about types.

These results have several interesting implications. First, they explain both the negative correlation between coverage and risk and the fact that insurance premiums display quantity discounts reported by Cawley and Philipson (1999).

Second, they are consistent with the puzzling empirical findings of Finkelstein and McGarry (2004): the positive correlation between perceived risk and coverage and the simultaneous lack of a positive relation between coverage and actual risk.

Third, they suggest that a lack of a positive correlation between coverage and risk can be consistent with informational barriers to trade in insurance markets.

Finally, they allow us to empirically distinguish our approach from standard asymmetric information models. To this end, we rely on a very general result derived by Chiappori et al. (2004). If an agent, who correctly estimates his risk, chooses one contract over another offering more coverage, then it must be true that his accident probability under the contract chosen is strictly lower than the per unit premium of the additional coverage offered by the other contract. This is a revealed preference argument. Its validity is independent of the market structure, proportional loadings/taxes or whether some agents go uninsured.14 However, because some agents underestimate their risk, this prediction fails in both equilibria presented in this paper. Therefore, its rejection by the data is consistent with our model but not with asymmetric information models in which insurees accurately estimate their accident

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14 Given risk aversion and state-independent utility, this revealed preference argument may fail only if some agents underestimate their risk. For example, it holds true in the models of de Meza and Webb (2001) and Jullien, Salanie and Salanie (2001). It even holds in the Villeneuve (2000) model provided insurance companies do not observe all the insuree’s characteristics that affect his accident probability.
probability. That is, our model can be empirically distinguished from the latter models even if these models also predict negative or zero correlation between coverage and risk.\(^{15}\)

This paper is organised as follows. In Section 1, we present a model where agents differ with respect to their risk perceptions and face a moral hazard problem. Section 2 provides a diagrammatic proof for the existence of the two separating equilibria described above. Section 3 deals with the empirical implications of our results. Finally, section 4 concludes.

1. The Model

For simplicity, we employ a model similar to de Meza and Webb (2001).\(^{16}\) In particular, there are two states of nature: good and bad. In the good state there is no loss whereas in the bad state the individual (insuree) suffers a gross loss of \(D\). Before the realisation of the state of nature all individuals have the same wealth level, \(W\). Also, all individuals are risk averse and have the same utility function but differ with respect to their perception of the probability of suffering the loss. There are two types of individuals, the Rs and the Os. The Rs have an accurate estimate of their true loss probability whereas the Os underestimate it.\(^{17}\) However, both the Os and the Rs believe that they correctly estimate their accident probabilities and their choices are driven by these (mistaken for the Os) beliefs.\(^{18}\) That is, all agents maximise their expected utility given their beliefs.

Furthermore, all agents can affect the true loss probability by undertaking preventive activities. Given the level of precautionary effort, the true loss probability is the same for both types. We consider the case where agents either take precautions or not (two effort levels). If an individual takes precautions \((C_i = C)\), he incurs a

\(^{15}\) The introduction of proportional loadings/taxes into competitive asymmetric information models could give rise to separating equilibria predicting negative or zero correlation between coverage and risk even if all agents buy some insurance. However, the revealed preference argument would still hold true.

\(^{16}\) Our model differs from de Meza and Webb (2001) in two respects: (i) In our model, individuals differ with respect to their degree of optimism whereas in de Meza and Webb they differ with respect to their degree of risk aversion, (ii) In our case, administrative costs are zero. In our model, the results are only driven by heterogeneous risk perceptions (optimism).

\(^{17}\) For expositional simplicity, we assume that the more optimistic agents are optimists whereas the less optimistic are realists. However, all the results go through if two types are respectively optimists and pessimists.

\(^{18}\) Unlike insurees in Villeneuve model (2000), insurees in our model do not try to extract any information about their accident probabilities from the contracts offered because they are certain that their perceived probabilities coincide with the true ones.
utility cost of $C$ and his true probability of avoiding the loss $p(C_i)$ is $p_C$. If he takes no precautions ($C_i = 0$), his utility cost is 0 but his true probability of avoiding the loss $p(C_i)$ is $p_0$, where $p_C > p_0$.

Now, let $p^i_j$ ($i = O, R, j = C, 0$) be the perceived probability of avoiding the loss. Given our assumption about the risk perceptions of the Os and the Rs, we have $p^R_j = p_j$, $p^O_j > p_j$ ($j = C, 0$), where $p_j$ is the true probability of avoiding the loss.

In this environment, the (perceived) expected utility of an agent $i$ is given by:

$$EU(C_i, y_i, \lambda_i, W) = p^i_j U(W - y) + (1 - p^i_j) U(W - D + (\lambda - 1)y) - C_i$$

(1)

where

- $W$: insuree’s initial wealth
- $D$: gross loss
- $y$: insurance premium
- $(\lambda - 1)y$: net payout in the event of loss, $\lambda > 1$
- $\lambda y$: coverage (gross payout in the event of loss)

Hence, the increase in the (perceived) expected utility from taking precautions is:

$$\Delta_i = (p^i_C - p^i_0)[U(W - y) - U(W - D + (\lambda - 1)y)] - C_i, \quad i = O, R$$

(2)

where $U$ is increasing and strictly concave and $W - y$, $W - D + (\lambda - 1)y$ are the wealth levels in the good and the bad state respectively.

Insurance companies know the true accident probability (given the precautionary effort level) and the perceived accident probabilities of the Os and Rs but they can observe neither the type nor the actions of each insuree. They also know the cost for the insuree corresponding to each precautionary effort level, the utility function of the insurees and the proportion of the Os and Rs in the population. The insurance contract $(y, \lambda y)$ specifies the premium $y$ and the coverage $\lambda y$. As a result, since insurance companies have an accurate estimate of the true accident probability, the expected profit of an insurer offering such a contract is:

$$\pi_j = p_j y - (1 - p_j)(\lambda - 1)y, \quad j = C, 0$$

(3)
**Equilibrium**

Here, we use the equilibrium concept employed by Rothschild and Stiglitz (1976). That is, we look for equilibria in a free-entry game with many insurers. A pair of contracts \( z_O = (y_O, \lambda_O y_O) \) and \( z_R = (y_R, \lambda_R y_R) \) is an equilibrium if the following conditions are satisfied: i) No contract in the equilibrium pair \( (z_O, z_R) \) makes negative expected profits. ii) No other set of contracts introduced alongside those already in the market would increase an insurer’s expected profits.

Depending on parameter values both separating and pooling equilibria can arise. Below, we provide a diagrammatical description of the two most interesting separating equilibria.\(^{19}\)

**2. Negative and Zero-Correlation Equilibria**

Let \( H = W - y \) and \( L = W - D + (\lambda - 1)y \) denote the income of an insuree who has chosen the contract \((y, \lambda y)\) in the good and bad state respectively. Let also \( \bar{H} = W \) and \( \bar{L} = W - D \) denote the endowment of an insuree after the realisation of the state of nature.

**2.1. Effort Incentive Constraints**

Let us first consider the moral hazard problem an insuree of type \( i \) faces. A given contract \((y, \lambda y)\) will induce an agent of type \( i \) to take precautions if

\[
(p_i' - p_0')[U(H) - U(L)] \geq C \quad \Leftrightarrow \quad \Delta_i \geq 0, \quad i = O, R
\]

(4)

Let \( P_i P_i' \) be the locus of combinations \((L, H)\) such that \( \Delta_i = 0 \). Since \( C, \ U' > 0 \), the \( P_i P_i' \) locus lies entirely below the \( 45^\circ \) line in the \((L, H)\) space. This locus divides the \((L, H)\) space into two regions: On and below the \( P_i P_i' \) locus the insurees take

\(^{19}\) Details about the other equilibria are available from the author upon request.
precautions (this is the set of effort incentive compatible contracts) and above it they do not. The slope of $P_iP'_i$ in the $(L, H)$ space is given by:

$$\left. \frac{dL}{dH} \right|_{P'_i} = \frac{U'(H)}{U'(L)} > 0 \quad \text{since} \quad U' > 0$$

That is, $P_iP'_i$ is upward-sloping. Also, since both types have the same utility function, the shape of $P_iP'_i$ is independent of the type of the insuree.\(^{20}\)

However, the location of $P_iP'_i$ does depend upon the insuree’s type. Although the Os overestimate their probability of avoiding the loss at any given precautionary effort level, they may either overestimate or underestimate the increase in that probability from choosing a higher preventive effort level. In principle, both cases are possible. However, if, given that no precautions are taken, the optimists’ perceived accident probability is very low, then the latter seems to be more reasonable. Also, this case is consistent with the empirical findings of Finkelstein and McGarry (2004), Lee (1989), Lundborg and Lindgren (2002) and Viscusi (1990). All these studies find that those who perceive a higher risk tend to take more precautions.\(^{21}\) In this paper, the analysis is carried out under the assumption that the latter case is relevant.\(^{22}\) In particular, the following assumption is made:

**Assumption 1:**

$$p_c^R - p_o^R > p_c^O - p_o^O$$

That is, the Rs’ set of effort incentive compatible contracts is strictly greater than that of the Os. It should be noted that Assumption 1 is required for but does not necessarily imply a negative relationship between coverage and risk. It may well be the case that Assumption 1 holds and a separating equilibrium arises exhibiting a positive relationship. We also assume that

**Assumption 2:**

$$p_c^i - p_o^i \left[ U(H) - U(L) \right] > C, \quad i = O, R$$

\(^{20}\) The curvature of these loci does not affect our analysis.

\(^{21}\) Coelho and de Meza (2005) also provide experimental evidence that optimism and performance are negatively correlated.

\(^{22}\) Details about the equilibria arising when optimism encourages precautionary effort are available from the author upon request.
Assumption 2 implies that both $P_R P'_R$ and $P_O P'_O$ pass above the endowment point, and so the effective set of effort incentive compatible contracts is not empty for either type. If Assumption 2 does not hold for either type, the corresponding type never takes precautions. Although, Assumption 1 is necessary for the negative correlation prediction, Assumption 2 does not need to hold for the Os. In fact, this result obtains more easily if the Os never take precautions. In contrast, the zero-correlation result requires Assumption 2 but not Assumption 1.\(^{23}\) It obtains even if the Os overestimate the decrease in their accident probability from taking precautions.

2.2. Indifference Curves

The indifference curves, labelled $I_i$, are kinked where they cross the corresponding $P_i P'_i$ locus. Above $P_i P'_i$, insurees of the the i-type do not take precautions, their perceived probability of avoiding the loss is $p_0^i$, and so the slope of $I_i$ is:

$$\frac{dL}{dH}_{I_i, p=p_0} = -\frac{p_0^i}{1 - p_0^i} \frac{U'(H)}{U'(L)} \quad i = O, R$$  \(6\)

On and below $P_i P'_i$ insurees of the i-type do take precautions, their perceived probability of avoiding the loss rises to $p_c^i$ and so the slope of $I_i$ becomes:

$$\frac{dL}{dH}_{I_i, p=p_c} = -\frac{p_c^i}{1 - p_c^i} \frac{U'(H)}{U'(L)} \quad i = O, R$$  \(7\)

Hence, just above $P_i P'_i$ the i-type indifference curves become flatter.

Furthermore, because the Os underestimate their accident probability, at any given identical preventive effort level and $(L, H)$ pair, the Os indifference curve is steeper in the $(L, H)$ space. Intuitively, the Os are less willing to exchange consumption in

\(^{23}\) The zero-correlation result obtains even if the direction of the inequality in Assumption 2 is reversed. However, this assumption would imply that both types never take precautions and so this case is not very interesting.
the good state for consumption in the bad state because their perceived probability of the bad state occurring is lower than that of the Rs.

### 2.3. Insurers’ Zero-profit Lines (Offer Curves)

Using the definitions \( H = W - y \) and \( L = W - D + (\lambda - 1)y \), and the fact that insurance companies have an accurate estimate of the true accident probabilities, given the precautionary effort level, the insurers’ expected profit function becomes:

\[
\pi_j = p_j(W - H) - (1 - p_j)(L - W + D)
\]

(8)

The zero-profit lines are given by:

\[
L = \frac{1}{1 - p_j} \left( W - \frac{p_j}{1 - p_j}H - D \right), \quad j = C,0
\]

(9)

Conditional on the preventive effort level chosen by the two types of insurees, there are three zero-profit lines with slopes:

\[
\left. \frac{dL}{dH} \right|_{x=0} = \frac{p_0}{1 - p_0} \quad \text{(EN’ line, effective from N’ to } P_0P'_0) \]

(10)

\[
\left. \frac{dL}{dH} \right|_{x=0} = \frac{p_c}{1 - p_c} \quad \text{(EJ’ line, effective from E to } P_{R_0}P'_{R_0}) \]

(11)

\[
\left. \frac{dL}{dH} \right|_{x=0} = \frac{q}{1 - q} \quad \text{(EM’ line, effective between } P_{R_0}P'_{R_0} \text{ and } P_0P'_0) \]

(12)

where \( q = \mu p_c + (1 - \mu)p_0 \), and \( \mu \) is the proportion of the Rs in the population of insurees. Also, EM’ is the pooling zero-profit line when the Rs take precautions whereas the Os do not.

Also, at \( H = \overline{H} = W \), Eq. (9) becomes:
Eq. (13) is independent of the value of \( p_j \). This implies that all three zero-profit lines have the same origin (the endowment point, E).

We can now provide a diagrammatic proof of the existence of the two separating equilibria. The negative correlation result is shown in Proposition 1 whereas Proposition 2 provides an example of a separating equilibrium exhibiting zero correlation between coverage and risk.

**PROPOSITION 1:** If the Os’ indifference curve tangent to \( EN' \), \( I_O^* \), passes above the intersection of \( EJ' \) and \( P_RP'_R \), above the endowment point \( E \), and meets \( P_OP'_O \) above \( EJ' \), then there exists a unique separating equilibrium \((z_R, z_O)\) where the Rs take precautions whereas the Os do not. Both types choose strictly positive coverage but the Rs buy more than the Os (see Fig. 1).24

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24 This is true if i) the degree of optimism is sufficiently high so that the Os’ perceived probability of avoiding the loss is larger than the Rs’ true probability given that the Rs take precautions whereas the Os do not \( (p_O^O > p_C^R) \), ii) the Os significantly underestimate the effect of precautions on the accident probability (the distance between \( P_RP'_R \) and \( P_OP'_O \) is large). iii) the difference between the true probabilities of avoiding the loss \( (p_C - p_O) \) is sufficiently small relative to the loss size.
Proof: Suppose that types are publicly observable but the effort level is not contractible. Then, given their perceptions about accident probabilities and the effect of precautionary effort on these probabilities, in a competitive equilibrium the Os take contract \( z_O \) and the Rs take contract \( z_R \). Also, notice that the Os strictly prefer \( z_O \) to \( z_R \) and the Rs strictly prefer \( z_R \) to \( z_O \). Thus, the revelation constraints of both types are satisfied (that is, both types choose the contracts they would have chosen under full information about types). Therefore, there is no profitable deviation and so \((z_R, z_O)\) is the unique equilibrium.\(^{25}\) Q.E.D.

Intuitively, given the contracts offered, because the Os considerably underestimate the reduction in their accident probability from taking precautions, they choose to take no precautions. Also, although insurance is offered at actuarially fair terms, because they underestimate their accident probability, the Os underinsure choosing a contract with low coverage while contracts with higher coverage are available at a lower per unit premium.

That is, this separating equilibrium exhibits two interesting features. The Os not only purchase less coverage than the Rs but also take fewer precautions and so their accident probability is higher. Competition among insurance companies then implies that the Os also pay a higher per unit premium. Therefore, this equilibrium is consistent with both the negative correlation between coverage and risk (point estimate) and the fact that per unit premiums fall with the quantity of insurance purchased as reported by Cawley and Philipson (1999).

**PROPOSITION 2:** Suppose \( EM' \) does not cut \( I_R \) through the intersection point of \( EJ' \) and \( I'_O \) (the Os’ indifference curve tangent to \( EJ' \) below \( P_OP'_O \) and to the left of E). Then there exists a unique separating equilibrium where both types purchase strictly positive coverage and take precautions but the Rs buy more insurance than the

\(^{25}\) Here, we assume that the Os do not update their beliefs after observing the contract they are offered although the terms they are offered are, according to their beliefs, actuarially unfair. Alternatively, one could introduce an arbitrarily small fraction of agents whose true accident probability and effect of precautions on this probability coincide with those perceived by the Os. Now, the beliefs of the Os would not necessarily be shaken since they know that the terms they are offered are determined by the mass of optimists and they have a strong prior that they are one of the high ability agents (many empirical studies report that the overwhelming majority of people believe they are “above average” e.g. Svenson (1981)). In this case, in equilibrium, the contract taken by the Os would be arbitrarily close to \( z_O \) whereas the contract taken by the Rs would be exactly \( z_R \).
Os. That is, this equilibrium exhibits zero correlation between coverage and the accident probability (see Fig. 2).

Proof: Consider the following deviations. Clearly, offers above \( EJ' \) are loss-making. The same is true for offers above \( PR'P_R \). Between \( PR'P_R \) and \( PO'P_O \) and below \( EJ' \) there is no offer that attracts Rs and does not attract the Os, although there are some offers that attract only the Os. Thus, any offer in this region is unprofitable. Given the equilibrium contracts, below \( EJ' \) and below \( PO'P_O \) there is no offer that is attractive to either type. Hence, the pair \((z_R, z_O)\) is the unique separating equilibrium. Furthermore, the fact that \( EM' \) does not cut \( I_R \) below \( PR'P_R \) rules out any pooling equilibrium. Therefore, the pair \((z_R, z_O)\) is the unique equilibrium. \( Q.E.D. \)

Although they buy more coverage than the Os, the Rs are quantity-constrained. Under full information about types, the Rs would have purchased the contract at the intersection of \( PR'P_R \) and \( EJ' \), instead of \( Z_R \), which involves more insurance. However, since types are hidden, this contract is not offered because it violates the Os’ revelation and effort incentive constraints and so is loss-making for the insurance

![Fig. 2. Zero Correlation Equilibrium](image-url)

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26 This is true if i) the Os moderately underestimate the benefit of precautions (the distance between \( PR'P_R \) and \( PO'P_O \) is not very large) and ii) the degree of optimism is not very high (the Os perceived accident probability is not much smaller than the true one).
companies. In order to reveal their type, the Rs accept lower coverage than they would have chosen if types were observable.

Strictly speaking, the no-correlation prediction is unlikely to be observed in practice. However, if one interprets it as a failure to reject the no-correlation null, then it is consistent with the findings of Cawley and Philipson (1999), Chiappori and Salanie (2000) and Dionne et al. (2001) about the relationship between coverage and the accident rate. Also, this equilibrium can simultaneously explain the positive correlation between perceived risk and coverage and the zero correlation between coverage and the accident rate (actual risk) reported by Finkelstein and McGarry (2004). Because the Os underestimate their risk they purchase less insurance than the Rs although, given the equilibrium contracts, both types have the same accident probability.

Finally, it is worth mentioning that an intervention policy involving a fixed tax per contract sold, with the proceeds returned as a lump-sum subsidy to the whole population can yield a strict Pareto improvement on the laissez-faire equilibrium described in Proposition 2.27 Intuitively, the imposition of the tax results in the Os going uninsured and mitigates the negative externality their presence creates. In the new equilibrium, the Rs purchase more insurance but subsidise the Os. If the proportion of the Os is small, the per capita subsidy is high and so its effect both on the Os’ utility and revelation constraint is large. This, in turn, allows the Rs to purchase a significantly higher amount of insurance. As a result, the welfare gains of the higher coverage more than offset the welfare loss due to the net tax (tax minus subsidy) the Rs pay.28

Surprisingly, although the Os are underinsured in the laissez-faire equilibrium, an intervention scheme which results in the Os going uninsured, leads to a strict Pareto improvement. In contrast, a policy that would result in the Os purchasing more coverage would tighten rather than relax their revelation constraint. As a result, in

27 This result holds true regardless of whether objective or subjective probabilities are used as the welfare criterion. Details are available from the author upon request.
28 These welfare results differ from those in Sandroni and Squintani (2004). Sandroni and Squintani consider a pure adverse selection model where a fraction of the high-risk individuals believe that they are low risks. Because the perceived accident probability of the optimistic high risks coincides with the true accident probability of the low risks (who accurately estimate their accident probability), there exist no intervention policy involving taxes, subsidies and/or compulsory insurance that can yield a Pareto improvement on the laissez-faire equilibrium.
order to reveal their type, the Rs would have to purchase even less insurance and so their welfare would worsen.

3. Implications for Empirical Testing

The results of Propositions 1 and 2 also allow us to empirically distinguish our approach from standard asymmetric information models. To this end, we employ a simplified version of the Chiappori et al. (2004) general framework. There are two states of nature: good and bad. In the good state the agent incurs no loss whereas in the bad state he incurs a loss of $D_\theta$. The parameter $\theta$ represents all the characteristics of the agent (potential insuree) that are his private information (risk, risk aversion, loss, etc). Agents may also privately choose their loss probability $1 - p$. This choice implies a prevention cost that is assumed to be a negative function of the loss probability. A contract consists of coverage and premium: $z = (\lambda y, y), \lambda > 1$. The ex post risk of an insuree is a function of the contract he chooses. The average ex post risk of insurees choosing contract $z$ is $1 - p(z)$. In this general setup, the following assumptions are made:

**A1:** For all contracts offered and all agent types overinsurance is ruled out by assuming $\lambda y \leq D_\theta$.

**A2:** Agents’ preferences over final wealth are state-independent.

**A3:** Agents are risk averse in the sense that they are averse to mean-preserving spreads on wealth.

**A4:** Given the contract chosen, agents accurately estimate their accident probability.

We can now show the following result:

**Lemma 1:** Suppose an agent $\theta$ chooses the contract $z_1 = (\lambda_1 y_1, y_1)$ over the contract $z_2 = (\lambda_2 y_2, y_2)$ that offers higher coverage ($\lambda_2 y_2 > \lambda_1 y_1 \geq 0$). Then if the agent’s ex post risk under $z_1$ is $1 - p(z_1)$, it must be true that

\[
1 - p(z_1) < \frac{y_2 - y_1}{\lambda_2 y_2 - \lambda_1 y_1}
\]
**Proof:** See Appendix A.

Intuitively, given risk aversion, if an agent chooses \( z_1 \) (low-coverage) over \( z_2 \) (high-coverage), then it must be true that his accident probability under \( z_1 \) is strictly lower than the per unit premium of the additional coverage offered by \( z_2 \). Otherwise, he would be strictly better off taking \( z_2 \) while keeping \( 1 - p(z_1) \). This is a revealed preference argument. Its validity is independent of the market structure, proportional loadings/taxes, or whether some agents go uninsured. For example, it holds in the models of Julien, Salanie and Salanie (2001) and de Meza and Webb (2001) where the positive correlation property breaks. It even holds in the Villeneuve (2000) model provided insurance companies do not observe all the insuree’s characteristics that affect his accident probability. In contrast, in our framework, because some agents (the Os) underestimate their true accident probability, this prediction fails.

**Corollary 1:** In the separating equilibria of Propositions 1 and 2 it is respectively true that

\[
(y_R - y_O)/(\lambda_R y_R - \lambda_O y_O) < 1 - p(z_O) = 1 - p_0 \quad \text{and} \quad (15)
\]

\[
(y_R - y_O)/(\lambda_R y_R - \lambda_O y_O) = 1 - p(z_O) = 1 - p_c \quad (16)
\]

**Proof:** See Appendix B.

In words, in both equilibria the per unit price of the additional insurance offered by the high-coverage contract is not higher than the Os’ true accident probability under the low-coverage contract. Nevertheless, due to the underestimation of their accident probability, the Os purchase the low-coverage contract although the high-coverage contract is also available.

Therefore, a rejection of this revealed preference argument by the data is consistent with our model but not with standard asymmetric information models. That is, if the revealed preference condition is not met, our model can be empirically distinguished

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29 Equivalently, given risk aversion, if an agent chooses \( z_1 \) (low-coverage) over \( z_2 \) (high-coverage), then it must be true that his expected income under \( z_1 \) is strictly greater than under \( z_2 \).
from asymmetric information models in which insurees accurately estimate their accident probabilities even if the latter models also predict negative or zero correlation between coverage and risk.\(^{30}\)

In fact, Chiappori et al. (2004) perform this test using data from the French car insurance market and their findings corroborate the prediction of Lemma 1. They also find positive correlation between coverage and risk which, however, is not statistically significant. Although, in this case, our model cannot be distinguished empirically from standard asymmetric information models, it can generate predictions consistent with these empirical findings. The violation of the revealed preference prediction is a sufficient but not necessary condition for the presence of optimism.

If the degree of optimism is not very high, there exist equilibria exhibiting positive correlation between coverage and risk that are consistent with the prediction of Lemma 1. Furthermore, if we introduce fixed costs per claim and extend the model to include more than one levels of loss, there can exist equilibria exhibiting zero (or even negative) correlation between coverage and risk that are consistent with the revealed preference argument. Because they underestimate their risk, the optimists choose a less comprehensive contract (e.g. a contract with a higher level of deductible) that entails lower expected administrative costs. Also, both the optimists and the realists take precautions and have identical loss distributions. Competition then implies that the holders of the less comprehensive contract (the optimists) have greater expected income (the decrease in the premium exceeds the decrease in the expected reimbursement).

4. Conclusions

Most recent empirical studies on the relationship between coverage and risk find either negative or no correlation. Cawley and Philipson (1999) also report that, in the US life insurance market, insurance premiums exhibit quantity discounts. Furthermore, Finkelstein and McGarry (2004) analyse the US long-term care insurance market and find positive correlation between perceived risk and coverage and zero correlation between coverage and actual risk.

\(^{30}\) Appendix C provides a sufficient condition on the degree of optimism for the existence of separating equilibria exhibiting zero or negative correlation that violate the prediction of Lemma 1.
This paper, by introducing heterogeneity in risk perceptions in a competitive model of asymmetric information, can simultaneously explain all these empirical findings.\footnote{In the absence of proportional loadings/taxes, standard asymmetric information models cannot simultaneously explain these empirical findings.} The more optimistic agents (the Os) underestimate their accident probability both in absolute terms and relative to the less optimistic ones (the Rs) and so purchase less insurance. They also tend to be less willing to take precautions. This gives rise to separating equilibria exhibiting negative or zero correlation both between coverage and risk and between coverage and per unit premiums.

These results have several interesting implications. First, they explain both the negative correlation between coverage and risk and the fact that insurance premiums display quantity discounts reported by Cawley and Philipson (1999).

Second, they are consistent with the puzzling empirical findings of Finkelstein and McGarry (2004): the positive correlation between perceived risk and coverage and the simultaneous lack of a positive relation between coverage and actual risk.

Third, they suggest that a lack of a positive correlation between coverage and risk occurrence can be consistent with the presence of informational barriers to trade in the insurance markets under study.

Finally, in conjunction with the revealed preference argument of Lemma 1, the predictions of the separating equilibria of Propositions 1 and 2 allow us to empirically distinguish our approach from standard asymmetric information models. A rejection of this revealed preference argument by the data is consistent with our model but not with standard asymmetric information models. That is, if the revealed preference test fails the data, our model can be empirically distinguished from standard asymmetric information models even if the latter models also predict negative or zero correlation between coverage and risk.

Chiappori et al. (2004) perform this test using data from the French car insurance market and their findings are consistent with the prediction of Lemma 1. They also find zero correlation between coverage and risk. Two remarks should be made here. First, these findings are not inconsistent with our approach. The violation of the revealed preference prediction is a sufficient but not necessary condition for the presence of optimism. Although, in this case, our model cannot be distinguished empirically from standard asymmetric information models, it can potentially explain
the empirical findings of Chiappori et al. Second, the empirical validity of the revealed preference argument should be tested in each market.

Appendix A: Proof of Lemma 1

Consider a contract $z'(\lambda_2 y_2, y')$ with premium: $y' = y_1 + (1 - p(z_1))(\lambda_2 y_2 - \lambda_1 y_1)$. We will show that the agent prefers $z'$ to $z_1$. Notice that if the agent still has ex post risk $1 - p(z_1)$ under $z'$ ($1 - p(z_1) = 1 - p(z') = 1 - p$), then he faces the following lottery:

$$L' = (-D_\theta + \lambda_2 y_2 - y', 1 - p; -y', p)$$

(A1)

The expectation of this lottery is:

$$(1 - p)(-D_\theta + \lambda_2 y_2 - y') - py' = (1 - p)(-D_\theta + \lambda_1 y_1 - y_1) - p\lambda_1 y_1$$

(A2)

Clearly, it is equal to the expectation of the lottery

$$L_1 = (-D_\theta + \lambda_1 y_1 - y_1, 1 - p; \lambda_1 y_1, p)$$

(A3)

which the agent faces under $z_1$. Since $0 \leq \lambda_1 y_1 < \lambda_2 y_2$ and contracts do not overinsure, lottery $L_1$ is a mean-preserving spread of $L'$. Thus, given risk aversion, the agent strictly prefers $L'$ to $L_1$. Furthermore, since under $z'$, if he wishes so, he can choose another $1 - p(z') \neq 1 - p(z_1)$, he strictly prefers $z'$ to $z_1$ and hence to $z_2$ (by assumption, $z_1$ is preferred to $z_2$). However, contracts $z'$ and $z_2$ offer the same coverage. Therefore, since $z'$ is strictly preferred to $z_1$, it must be the case that

$$y_2 > y' = y_1 + (1 - p(z_1))(\lambda_2 y_2 - \lambda_1 y_1) \quad \implies \quad 1 - p(z_1) < \frac{y_2 - y_1}{\lambda_2 y_2 - \lambda_1 y_1}$$

(A4)

Q.E.D.

Appendix B: Proof of Corollary 1

Using the zero-profit conditions, we obtain:

$$\pi_i = p(z_i)y_i - (1 - p(z_i))(\lambda_i - 1)y_i = 0 \quad \implies \quad \lambda_i = 1/(1 - p(z_i)), \quad i = O, R$$

(B1)

In the separating equilibrium of Proposition 1 we have:

$$1 - p(z_o) = 1 - p_0 > 1 - p(z_R) = 1 - p_c \quad \implies \quad \lambda_R > \lambda_o$$

(B2)

Therefore,
In the separating equilibrium of Proposition 2 we have:

\[ 1 - p(z_O) = 1 - p(z_R) = 1 - p_c \quad \Rightarrow \quad \lambda_R = \lambda_O \quad \text{(B4)} \]

Therefore,

\[ \frac{y_R - y_O}{\lambda_R y_R - \lambda_O y_O} = \frac{y_R - y_O}{\lambda_R (y_R - y_O) + (\lambda_R - \lambda_O) y_R} < \frac{1}{\lambda_O} = 1 - p_c \quad \text{(B5)} \]

\[ Q.E.D. \]

Appendix C: A Sufficient Condition for the Existence of Equilibria that Violate the Prediction of Lemma 1

Let \( p^O_j(d) \) be the Os’ perceived probability of avoiding the loss where \( 0 \leq d \leq 1 \) is the degree of optimism. The perceived probability is a strictly increasing function with \( p^O_j(0) = p_j \), \( p^O_j(1) = 1 \), where \( p_j \), \( j = C, O \), is the true probability of avoiding the loss.

**Proposition C1:** Suppose that for \( d = \bar{d} \) the Os are indifferent between the contract chosen by the Rs under full information about types, \( z^fI_R \), and a contract offering lower coverage (possibly zero), \( z_O \). Under \( z_O \) the Os may take precautions or not. Then for each \( d > \bar{d} \), there exists a unique separating equilibrium exhibiting either zero or negative correlation between coverage and risk that violates the revealed preference argument of Lemma 1.33

**Proof:** Let \( I_O \) be the Os’ indifference curve corresponding to \( d = \bar{d} \) (see Fig. C1). For any \( d > \bar{d} \), the Os’ indifference curve becomes steeper than \( I_O \) and so the Os strictly prefer a contract offering less coverage than \( z_O \) to \( z^fI_R \). Also, the Rs strictly prefer \( z^fI_R \) to the lower coverage contract. That is, insurers can offer contracts that satisfy the revelation constraints of both types. Thus, in equilibrium, both types choose the contracts they would have chosen under full information about types. Therefore, for each \( d > \bar{d} \), there exists a unique separating equilibrium. The equilibrium contracts exhibit either zero (if the Os take precautions) or negative

\[ \text{If the accident probability is strictly positive (even if precautions are taken), it is always possible to find a } 0 < d < 1 \text{ so that the Os are indifferent between } z^fI_R \text{ and } z_O. \]

\[ \text{Notice that zero correlation equilibria that violate the revealed preference argument of Lemma 1 also exist even if } d < \bar{d} \text{ provided optimism discourages precautionary effort. However, in this case, the Rs are quantity-constrained in equilibrium (e.g. equilibrium of Proposition 2).} \]
correlation (if the Os do not take precautions). Competition among insurers then implies that the Os pay either the same or a higher per unit premium. Then, by Corollary 1, the per unit premium of the additional coverage offered by the high-coverage coverage is equal to or lower than the Os’ accident probability under the low-coverage contract. That is, for any \( d > \bar{d} \) the revealed preference argument of Lemma 1 fails. \( Q.E.D. \)

**Fig. C1.** A Sufficient Condition for the Existence of Equilibria that Violate the Prediction of Lemma 1.

**References**


