Bistability and Switching Properties of Semiconductor Ring Lasers With External Optical Injection

Guohui Yuan and Siyuan Yu, *Member, IEEE*

***Abstract*—We investigate both analytically and numerically the switching, locking and stability properties of a bistable semiconductor ring laser subject to an external optical injection. Minimum optical power required for the injected signal at certain frequency to switch the lasing direction of a bistable semiconductor ring laser from its initially lasing direction to initially nonlasing direction is determined. Locking to the injected signal and stability of the switched laser are investigated to give an area of reliable switching operation. Correspondingly, numerical simulation has been carried out to find successful switching and stable locking region with variable injection power and frequency, and is compared with the analytical results. The region obtained from simulation coincides well with the intersection of switching, locking and stable locking regions. The relation between switching speed and parameters of injected source is also studied numerically.**

***Index Terms*—Bistability, injection locking, mode competition, semiconductor ring laser, switching.**

I. Introduction

SEMICONDUCTOR ring lasers (SRLs) have gained more and more attention owing to their unique feature of directional bistablility [1], [2], i.e., their ability to operate in two distinctive stable lasing directions (the clockwise (CW) and the counter-clockwise (CCW) directions) as shown in Fig. 1, and their potential applications in photonic systems for all-optical logic, optical switch, and optical memory applications [3], [4]. Switching from one lasing direction to the other can be triggered by externally injecting optical energy into the previously nonlasing direction, which starts a mode competition process favoring this direction for it to become the lasing direction at the end of the process.

Although the possibility and realization of achieving bistability in two mode lasers through mode competition by an external optical injection has been not only elegantly demonstrated mathematically using an idealized model but also experimentally [5]–[7], there are many questions about the switching process that need to be answered for any practical application. For the external optical injection source used to trigger the switching, relevant unknown aspects include the optical power or energy needed to induce switching, allowed detuning frequency range in which not only switching will happen, but the laser will stay stably locked to the external injection at the final state. In this paper, we extend the general theory to a more realistic and rigorous one for an SRL with two counter-propagating modes, and study the required conditions for realizing reliable switching in a SRL by an external optical signal.

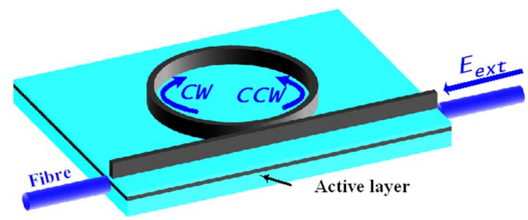

Fig. 1. Circular SRL with an external optical injection. The laser operates in the CCW direction before the optical injection (E_{ext}) is added. It is switched to the CW direction after the injection.

The formula for the conditions needed for an externally injected continuous optical wave with certain power and detuning frequency to the free running frequency of the SRL is derived first. Locking and stability analysis is then carried out supposing that lasing direction has been switched to be the same as the external injection source. Results from numerical simulation are compared with the one obtained from theoretical study later. Detuning frequency and injection power's effects on switching speed are studied finally.

II. Theoretical Studies

A. Two-Mode Model

A two-mode model that analyzes the competitions between a pair of counter-propagating longitudinal modes at a single cavity resonance has successfully described SRLs as gyroscopes [8] and explained the observed alternating oscillation regime in the L–I characteristics of the SRL [9]. Our model is based on the basic two-mode model but includes an external injection term to account for the injected optical signal.

In single longitudinal mode operation, the electric field inside the ring cavity can be expressed as

$$E(x,t)=E_1(t)\exp[-i(\omega_{o1}t-kx)]+E_2(t)\exp[-i(\omega_{o2}t+kx)] \quad (1)$$

where E_1 and E_2 are the mean-field slowly varying complex amplitudes of the electric field associated with the two propagation direction modes, i.e., mode 1 is CCW and mode 2 is CW; x is the longitudinal spatial coordinate along the ring circumference, assumed positive in the CCW direction, and $\omega_{o1,2}$ is the optical frequency of the lasing longitudinal modes as the two directional modes should have a small detune between them [10].

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The authors are with the Deptartment of Electrical and Electronic Engineering, University of Bristol, Bristol BS8 1TR, U.K. (e-mail: gh.yuan@bristol.ac.uk; S.Yu@bristol.ac.uk).

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The electric field of the external optical injection adding to the CW direction can be written as

$$E_{\rm ext}(x,t)=E_{\rm ext}(t)\exp\left[-i\left((\omega_{o2}t+\Delta\omega_{\rm ext}t+\Delta\phi_{\rm ext})+kx\right)\right] \quad (2)$$

where $\Delta\omega_{\rm ext}$ is the angular frequency detuning between the external injection and the free-running angular frequency ω_{o2} of the ring laser mode, and $\Delta\phi_{\rm ext}$ is the phase difference between them. The complex amplitudes of the electric fields are normalized so that $|E|^2$ equals to the density of photons. The time evolution of the fields in the cavity can be described by the following set of rate equations [9], [11], [12]:

$$\frac{dE_1}{dt}=\frac{1}{2}(1-i\alpha)\left(\Gamma v_g g_n(N-N_{\rm tr})\times\left(1-\varepsilon_s|E_1|^2-\varepsilon_c|E_2|^2\right)-\frac{1}{\tau_{p1}}\right)E_1 + i(\omega_{o1}-\omega_{th})E_1 \quad (3)$$

$$\frac{dE_2}{dt}=\frac{1}{2}(1-i\alpha)\left(\Gamma v_g g_n(N-N_{\rm tr})\times\left(1-\varepsilon_s|E_2|^2-\varepsilon_c|E_1|^2\right)-\frac{1}{\tau_{p2}}\right)E_2 + i(\omega_{o2}-\omega_{th})E_2 + K_{\rm ext}E_{\rm ext}\exp\left[-i(\Delta\omega_{\rm ext}t+\Delta\phi_{\rm ext})\right] \quad (4)$$

$$\frac{dN}{dt}=\frac{\eta_i I}{eV}-\frac{N}{\tau_e}-v_g g_n(N-N_{\rm tr})\cdot\left(\left(1-\varepsilon_s|E_1|^2-\varepsilon_c|E_2|^2\right)|E_1|^2+\left(1-\varepsilon_s|E_2|^2-\varepsilon_c|E_1|^2\right)|E_2|^2\right) \quad (5)$$

where v_g is the group velocity, g_n is the differential gain at transparency, $N_{\rm tr}$ is the carrier density at transparency, ε_s and ε_c are the self-and cross-gain saturation coefficients respectively. α is the linewidth enhancement factor accounting for phase-amplitude coupling in the semiconductor medium, Γ represents the optical confinement factor which gives the spatial overlap between the active gain volume and the optical mode volume, $\tau_{p1,2}$ is the photon lifetime in the ring cavity for mode 1, 2 respectively, ω_{th} is the longitudinal resonant frequency at threshold. $K_{\rm ext}$ is the coupling parameter of the external injection. $E_{\rm ext}$ gives the field amplitude of injection. $\Delta\phi_{\rm ext}$ is always set to be 0, because the external field is always injected into the nonlasing direction which has little power initially. N is the carrier density in the active region. I is the bias current and η_i is the injection efficiency, e is the electronic charge. V is the volume for the quantum well active region, and τ_e is the carrier lifetime.

Linear coupling between the counter-propagating modes is caused by internal backscattering and external feedback [13] from the output waveguide facets, and can dominate SRL's operation at low bias current [9]. It is neglected because we are interested in the laser being biased high above threshold where nonlinear coupling is the fundamental physical mechanism that plays a much more important role as described in the (3) and (4) This is justifiable as relevant study [13] shows that only reflections with long time delay are a major concern. For short delay time (a few picoseconds as from integrated output waveguide facet), there is no feedback induced dynamics and the operating regimes mentioned in [9] can be reproduced. The reflection would only visibly reduce the extinction ratio between the lasing and nonlasing directions if facet reflection exceeds 10^{-3}, a level that is easily achievable with antireflection techniques such as coating.

Spontaneous emission noise is also neglected considering the SRL is biased high above the threshold current, and the operating point is chosen to be in the middle of a robust unidirectional region [9]. However, it may be important for the points at boundaries of a stable unidirectional region.

Equation (5) holds for the uniform carrier density N when any standing-wave pattern has a spatial period much smaller than carrier diffusion length and longitudinal variations of the carrier density are neglected.

Separating the magnitude part and phase part of the two-normalized-field (3)–(5) with $E(t)=A(t)e^{-j\phi}$, we have

$$\frac{dA_1}{dt}=\frac{1}{2}\left(\Gamma G_n(N-N_{\rm tr})\left(1-\varepsilon_s A_1^2-\varepsilon_c A_2^2\right)-\frac{1}{\tau_{p1}}\right)A_1 \quad (6)$$

$$\frac{d\phi_1}{dt}=\frac{\alpha}{2}\left(\Gamma G_n(N-N_{\rm tr})\left(1-\varepsilon_s A_1^2-\varepsilon_c A_2^2\right)-\frac{1}{\tau_{p1}}\right)-(\omega_{o1}-\omega_{th}) \quad (7)$$

$$\frac{dA_2}{dt}=\frac{1}{2}\left(\Gamma G_n(N-N_{\rm tr})\left(1-\varepsilon_s A_2^2-\varepsilon_c A_1^2\right)-\frac{1}{\tau_{p2}}\right)A_2+K_{\rm ext}A_{\rm ext}\cos\phi_2 \quad (8)$$

$$\frac{d\phi_2}{dt}=\frac{\alpha}{2}\left(\Gamma G_n(N-N_{\rm tr})\left(1-\varepsilon_s A_2^2-\varepsilon_c A_1^2\right)-\frac{1}{\tau_{p2}}\right)-(\omega_{\rm ext}-\omega_{th})-\frac{K_{\rm ext}A_{\rm ext}}{A_2}\sin\phi_2 \quad (9)$$

$$\frac{dN}{dt}=\frac{\eta_i I}{eV}-\frac{N}{\tau_e}-G_n(N-N_{\rm tr})\cdot\left(\left(1-\varepsilon_s A_1^2-\varepsilon_c A_2^2\right)A_1^2+\left(1-\varepsilon_s A_2^2-\varepsilon_c A_1^2\right)A_2^2\right) \quad (10)$$

where $G_n=v_g * g_n$. We have set the frequency of mode 2 to be the same as the external injected signal since there is little power in the initial nonlasing direction and finally it will be locked to the external source if locking conditions are satisfied. Although optical loss is related to the frequency of the mode and the losses for the two counter-propagating modes may be slightly different depending on the detuning frequency between mode 1 and the external injected source. However, they are assumed to be the same because the detuning involved is very small, therefore $\tau_{\rm p1}=\tau_{\rm p2}=\tau_{\rm p}$ in this paper. We also assume that the SRL is initially lasing in the CCW direction (mode 1).

B. Switching Conditions

As is well known that bistability can be achieved through gain saturation [5]–[7], and switching can be realized by external injection [7], but no specific switching conditions have been given. In this section we will discuss under what conditions the external optical continuous wave injected to the nonlasing direction can switch the lasing direction.

By deriving intensity equations with $dS/dt=dA^2/dt$ from amplitude rate equations, the rate equations without external injection are

$$\frac{dS_1}{dt}=\Gamma G_n(N-N_{\rm tr})S_1(\beta_1-\varepsilon_s S_1-\varepsilon_c S_2) \quad (11)$$

$$\frac{dS_2}{dt}=\Gamma G_n(N-N_{\rm tr})S_2(\beta_2-\varepsilon_s S_2-\varepsilon_c S_1) \quad (12)$$

where $\beta_{1,2} = 1-(1/\tau_{p1,2}\Gamma G_n(N-N_{\rm tr}))$, and following equation is obtained:

$$\frac{dS_2}{dS_1} = \frac{S_2(\beta_2 - \varepsilon_s S_2 - \varepsilon_c S_1)}{S_1(\beta_1 - \varepsilon_s S_1 - \varepsilon_c S_2)}. \tag{13}$$

The stability and final lasing direction can be analyzed in the phase plane by looking at the slope of the trajectories of the CCW power S_1 and CW power S_2 [5]–[7]. The trajectories for $dS_2/dS_1 = 0$ and $dS_2/dS_1 = \infty$ cross at the point $S = (S_1, S_2)$, where

$$S_1 = \frac{\varepsilon_c\beta_2 - \varepsilon_s\beta_1}{\varepsilon_c^2 - \varepsilon_s^2}, \quad S_2 = \frac{\varepsilon_c\beta_1 - \varepsilon_s\beta_2}{\varepsilon_c^2 - \varepsilon_s^2}. \tag{14}$$

Assuming $\varepsilon_c = 2\varepsilon_s$ for semiconductor ring lasers [9], they are bistable as the stationary direction could be either CCW or CW. Supposing that the SRL is initially lasing in the CCW direction (mode 1), the stationary solution is $S = (S_1 = \beta_2/\varepsilon_s, S_2 = 0)$. The external injection $S_{\rm ext}$ to the nonlasing direction will change the rate equation for S_2 to the following:

$$\frac{dS_2}{dt} = \Gamma G_n(N - N_{\rm tr})\sqrt{S_2} \cdot \left(\beta_2\sqrt{S_2} - \varepsilon_s\sqrt{S_2^3} - \varepsilon_c S_1\sqrt{S_2} + \kappa\sqrt{S_{\rm ext}}\right) \tag{15}$$

and thus changing the trajectory for $dS_2/dS_1 = 0$, with

$$\frac{dS_2}{dS_1} = \frac{\sqrt{S_2}\left(\beta_2\sqrt{S_2} - \varepsilon_s\sqrt{S_2^3} - \varepsilon_c S_1\sqrt{S_2} + \kappa\sqrt{S_{\rm ext}}\right)}{S_1(\beta_1 - \varepsilon_s S_1 - \varepsilon_c S_2)} \tag{16}$$

where $\kappa = 2K_{\rm ext}/\Gamma G_n(N - N_{\rm tr})$, $\sqrt{S_{\rm ext}} = A_{\rm ext}\cos\phi_2$.

As $S_{\rm ext2}$ increases, the laser behavior can be illustrated in Fig. 2 supposing carrier density is fixed. The SRL is bistable without external injection and for our case it is initially lasing in CCW direction $(S_2 = 0)$. With a low external injection power, the trajectory $dS_2/dS_1 = 0$ is distorted and more flows are attracted to the solution of $S_1 = 0$. However a solution with $S_2 \to 0$ still exists although it attracts fewer flows, hence the SRL is still bistable. With injection of adequate power, the SRL is no longer bistable and lasing direction is switched to the opposite as this is now the only stationary solution. In the case of an adequate external injection, the line for $dS_2/dS_1 = \infty$ should be below the line for $dS_2/dS_1 = 0$, which means that

$$A_{\rm ext} \geq \frac{\left(\varepsilon_s - \frac{\varepsilon_c^2}{\varepsilon_s}\right)\sqrt{S_2^3} + \frac{\varepsilon_c\beta_1 - \varepsilon_s\beta_2}{\varepsilon_s}\sqrt{S_2}}{\kappa\cos\phi_2} \tag{17}$$

and

$$\kappa A_{\rm ext}\cos\phi_2 = \frac{2K_{\rm ext}}{\Gamma G_n(N - N_{\rm tr})}A_{\rm ext}\cos\phi_2 \tag{18}$$

Therefore, we can determine if an external injection is sufficiently strong to flip the lasing direction by using (17), which gives the minimum required external optical injection power at certain detuning needed to cause switching. In order to obtain the parameters related to external source in (17), the steady state solutions without injection terms and those with injection terms are derived in Appendix A. The steady state solutions with injection terms are substituted into (17) to test the switching ability of the external injected optical continuous wave.

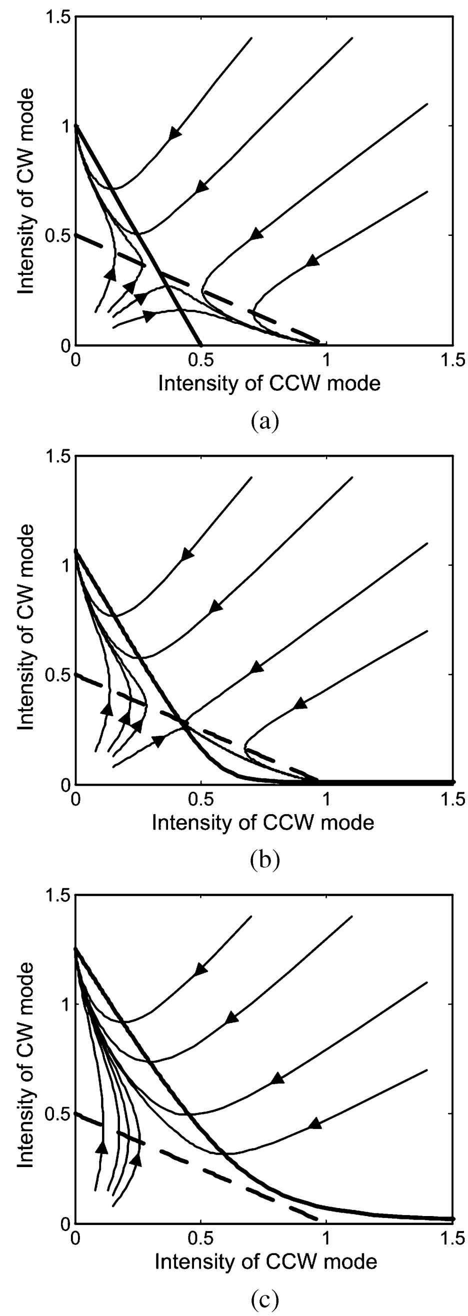

Fig. 2. Illustrations of trajectories of the intensities of CCW and CW mode (normalized to one) in the phase plane under the condition of $\varepsilon_c = 2\varepsilon_s$. Bold solid lines are the trajectories for $dS_2/dS_1 = 0$, and bold dashed lines are those for $dS_2/dS_1 = \infty$. (a) Without external injection. (b) With relatively low injection. (c) With adequate injection.

C. Locking Region

Having obtained the external injection levels needed for switching, this part aims to obtain the locking region in terms of detune within which the final lasing direction will be locked to the external injection. This is crucial for stable output after switching. Locking region can be obtained through stationary solutions of the rate equations (6)–(10) [14]–[16], supposing lasing direction has been switched to the initially nonlasing direction by the external injected signal. The steady state solutions without injection terms are derived by setting the derivatives and injection terms to zero supposing lasing direction is CW

$$\overline{A}_1 = 0 \tag{19}$$

$$(\overline{\omega}_{01} - \omega_{th}) = \frac{\alpha}{2}\left(\Gamma G_n(\overline{N} - N_{\rm tr}\left(1 - \varepsilon_c \overline{A}_2^2\right) - \frac{1}{\tau_{p1}}\right) \tag{20}$$

$$G_n(\overline{N} - N_{\rm tr})\left(1 - \varepsilon_s \overline{A}_2^2\right) = \frac{1}{\Gamma \tau_{p2}} \tag{21}$$

$$\frac{\eta_i I}{eV} - \frac{\overline{N}}{\tau_e} = \frac{\overline{A}_2^2}{\Gamma \tau_{p2}}. \tag{22}$$

In the same way, we can obtain the steady state solutions with injection terms in the form of those without injection terms, and

$$\hat{A}_1 = 0 \tag{23}$$

$$\hat{\omega}_{o1} - \omega_{th} = \frac{\alpha}{2}\left(\Gamma G_n(\hat{N} - N_{\rm tr})\left(1 - \varepsilon_c \hat{A}_2^2\right) - \frac{1}{\tau_{p1}}\right) \tag{24}$$

$$\hat{A}_2^2 = \frac{\overline{A}_2^2 - \frac{\tau_{p2}\Gamma \Delta \hat{N}}{\tau_e}}{1 + \tau_{p2}\Gamma G_n \Delta \hat{N}\left(1 - \varepsilon_s \hat{A}_2^2\right)} \tag{25}$$

$$\hat{\phi}_2 = \arcsin\left(-\frac{\omega_{\rm ext} - \omega_{th}}{\omega_m}\right) - \arctan\alpha \tag{26}$$

$$\Delta \hat{N} = -\frac{2K_{\rm ext}A_{\rm ext}}{\Gamma G_n\left(1 - \varepsilon_s \hat{A}_2^2\right)\hat{A}_2}\cos\hat{\phi}_2 \tag{27}$$

where $\omega_m = \sqrt{1+\alpha^2}K_{\rm ext}A_{\rm ext}/\hat{A}_2$, $\hat{N} = \overline{N} + \Delta\hat{N}$, $\Delta\hat{N}$ is the carrier density difference between the steady state solution without optical injection and the steady solution with optical injection.

From (26), we have

$$\sin(\hat{\phi}_2 + \arctan\alpha) = -\frac{(\omega_{\rm ext} - \omega_{th})\hat{A}_2}{\sqrt{1+\alpha^2}K_{\rm ext}A_{\rm ext}}. \tag{28}$$

Since the external optical signal should be a source instead of an obstacle, $\cos\hat{\phi}_2 > 0$ and

$$-\frac{1}{\sqrt{1+\alpha^2}} \le \sin(\hat{\phi}_2 + \arctan\alpha) = -\frac{(\omega_{\rm ext} - \omega_{th})\hat{A}_2}{\sqrt{1+\alpha^2}K_{\rm ext}A_{\rm ext}} \le 1.$$

Therefore, the locking region is

$$-\frac{\sqrt{1+\alpha^2}K_{\rm ext}A_{\rm ext}}{\hat{A}_2} \le \omega_{\rm ext} - \omega_{th} \le \frac{K_{\rm ext}A_{\rm ext}}{\hat{A}_2}. \tag{29}$$

Similar to injection locking of other semiconductor lasers [14]–[16], (29) shows the locking region is asymmetric due to the nonzero line-width enhancement factor α.

D. Stability Analysis

In some cases, the system represented by the steady state solutions is not stable and a perturbation such as noise will drive it into nonlinear dynamics. This part aims to define the stably locked region by stability analysis, which is carried out by adding a small perturbation to the stationary solutions with external injection [17], [18]

$$\begin{aligned} A_{1,2}(t) &= \hat{A}_{1,2} + e^{\lambda t}\delta A_{1,2} \\ \phi_{1,2} &= \hat{\phi}_{1,2} + e^{\lambda t}\delta\phi_{1,2} \\ N &= \hat{N} + e^{\lambda t}\delta N \end{aligned} \tag{30}$$

where λ is a complex number and supposed to be the eigenvalue of the system described by the rate equations (6)–(10). The condition for stability is that the real parts of all λ are negative, since the E field and carrier density will experience exponential growth if the real part of λ is positive. The imaginary part of λ represents the angular frequency of the damping oscillation.

By substituting (30) into the (6)–(10), and ignoring high order terms of the small perturbation,

$$\begin{pmatrix} \lambda - a_{11} & 0 & 0 & 0 & 0 \\ 0 & \lambda - a_{22} & -a_{23} & 0 & -a_{25} \\ 0 & 0 & \lambda - a_{33} & -a_{34} & -a_{35} \\ 0 & 0 & -a_{43} & \lambda - a_{44} & -a_{45} \\ 0 & 0 & -a_{53} & 0 & \lambda - a_{55} \end{pmatrix} \times \begin{pmatrix} \delta A_1 \\ \delta\phi_1 \\ \delta A_2 \\ \delta\phi_2 \\ \delta N \end{pmatrix} = 0 \tag{31}$$

where the coefficients in the above matrix can be found in Appendix B, and the determinant of the matrix is

$$\begin{aligned} \det(\lambda) = {} & \lambda^3 - (a_{33} + a_{44} + a_{55})\lambda^2 \\ & + (a_{33}a_{44} + a_{33}a_{55} + a_{44}a_{55} - a_{34}a_{43} - a_{35}a_{53})\lambda \\ & - a_{33}a_{44}a_{55} + a_{34}a_{43}a_{55} + a_{35}a_{53}a_{44} \\ & - a_{34}a_{45}a_{53} \end{aligned} \tag{32}$$

where λ is solved numerically and examined to determine the stable locked region. If the real part of λ is not positive, the system has a negative damping rate and is stable for a small perturbation. Otherwise it is unstable and the system would exhibits nonlinear dynamics, such as period-doubling bifurcation route to chaos, as experimentally demonstrated [19].

III. Calculation and Simulation Results

Calculated minimum optical power required to achieve switching is plotted along with the locking region and the stable locking region when the lasing direction is the same as the external injection source. On the other hand, simulations for the ring laser with external injection are obtained by directly solving rate equations (6)–(10) and compared with the calculated results. Calculation and simulation are based on a monolithic SRL, as depicted in Fig. 1. It is very similar to that of devices described and fabricated previously [9], which is a ridge waveguide circular laser with a straight output waveguide that is coupled to the ring via a directional coupler. The wafer structure and processing technology used to fabricate this 1.55 μm device is supposed to be almost the same as in [20] and [21] but with lower $v_g^* g_0 = 1.2 \times 10^{13}$ s^{-1} (instead of 1.6×10^{13} s^{-1}

TABLE I
SRL PARAMETERS

Symbol	Description	Value
R	SRL radius	100×10^{-6} m
α	linewidth enhancement factor	4
Γ	confinement factor for all wells	0.226
λ	lasing wavelength	1.55×10^{-6}m
v_g	group velocity	0.857×10^{-8} m·s^{-1}
N_{tr}	carrier density at transparency	0.78×10^{23} m^{-3}
T	coupler transmission	0.3
f_b	power coupling back ratio	0.5
g_n	differential gain	1.71×10^{-19} m^2
I_{th}	threshold current	27.5×10^{-3}A
I	bias current	110×10^{-3}A
τ_{p1}	photon lifetime for mode 1	4.85×10^{-12} s
τ_{p2}	photon lifetime for mode 2	4.85×10^{-12}s
τ_p	photon lifetime	4.85×10^{-12}s
τ_e	carrier lifetime	3.14×10^{-9}s
P_o	output power	2.3 mW
ε_s	self-nonlinear coefficient	3.16×10^{-23}m^3
ε_c	cross-nonlinear coefficient	$2*\varepsilon_s$

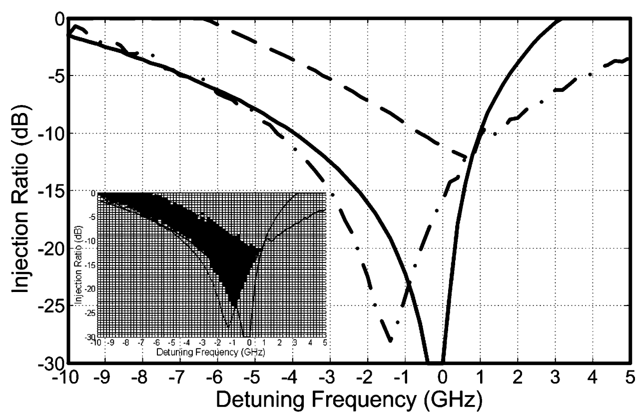

Fig. 3. Calculated minimum switched, the locking and stable locking regions. Dash–dotted line gives the minimum injection ratio required to achieve flipping for a certain detuning frequency, the locking region is outlined between two solid lines, and dashed line shows the maximum injection ratio before unstably locking. Inset figure indicates that the simulated results (middle black region) coincide with the intersection of those three regions and good agreement is achieved.

in [20] and [21]). The main basic parameters used in calculations and simulations are listed in Table I, and we assume that the internal loss experienced by the two counter-propagating modes are the same. The device is assumed to be biased at 110 mA (which would dissipate about 160 mW of electrical power).

The calculated minimum injection ratio, which is defined as the power of external injection light compared to that of the output of the SRL in the lasing direction without injection, is plotted as a function of detuning frequency together with the locking and stable locking regions in Fig. 3. The overlap of those three regions is the area where successful switching and stable locking is possible.

Simulations are carried out by injecting a continuous wave in the initial nonlasing direction CW to find whether lasing direction is switched from CCW (initial lasing direction) to CW and stay stable locked to the external injection source when the

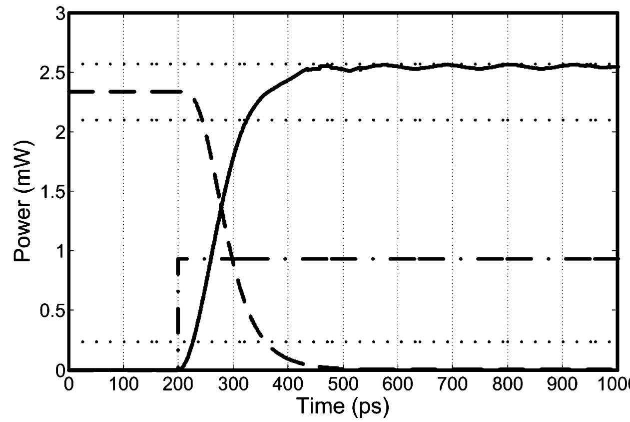

Fig. 4. Numerical transient out put power of CW (solid line) and CCW (dashed line) following external optical continuous wave (dash–dotted line) injected at 200 ps, and $\Delta\omega = -4$ GHz. The lasing direction changes from CCW (initial lasing direction) to CW (final lasing direction). The one lower and two upper dotted lines indicate the 10% margin of the initial power.

injection ratio of the injected source compared with that of the lasing mode varies from -40 to 0 dB, and detuning frequency of the injected signal from the oscillating mode changes from -10 to 5 GHz. Simulation results obtained by solving the nonlinear differential equations are compared with the calculated results in Fig. 3. The simulated results (middle black region) coincide with the intersection of those three regions and therefore, good agreement is achieved.

The numerically simulated dynamic transient following the external injected optical continuous wave is shown in Fig. 4. The SRL is initially lasing in CCW direction and its lasing direction switches to CW when an external optical continuous wave is injected to the initially nonlasing CW direction.

Settling time, which is defined as the time needed for the power in both directions, after switching, to settle to within 10% of the initial optical power in the CCW direction, is measured from simulation and its minimum value at certain detuning frequency and injection ratio are shown in Fig. 5(a) and (b).

From Fig. 5, we can see that minimum settling time decreases with increasing injection power, and the smallest value appears in the negative detuning frequency side. The sudden rise of the minimum settling time in CW direction compared with that in CCW direction at large negative detuning frequency and high injection sides is because that the final optical power in CW direction exceeds 110% of the initial optical power in the CCW direction.

IV. DISCUSSIONS AND CONCLUSION

We have analytically and numerically studied the behavior of a semiconductor ring laser subjected to an external optical injection to the initially nonlasing direction for the first time. The switching characteristics is studied as a function of detuning frequency and injected power of the external source relative to that of the free-running mode of the SRL. Analytically, we have first found a switching region, in which flipping of the lasing direction by external injection will happen. Assuming lasing direction has been switched to that of the external injection source, a locking region and a stable locking region, where the output

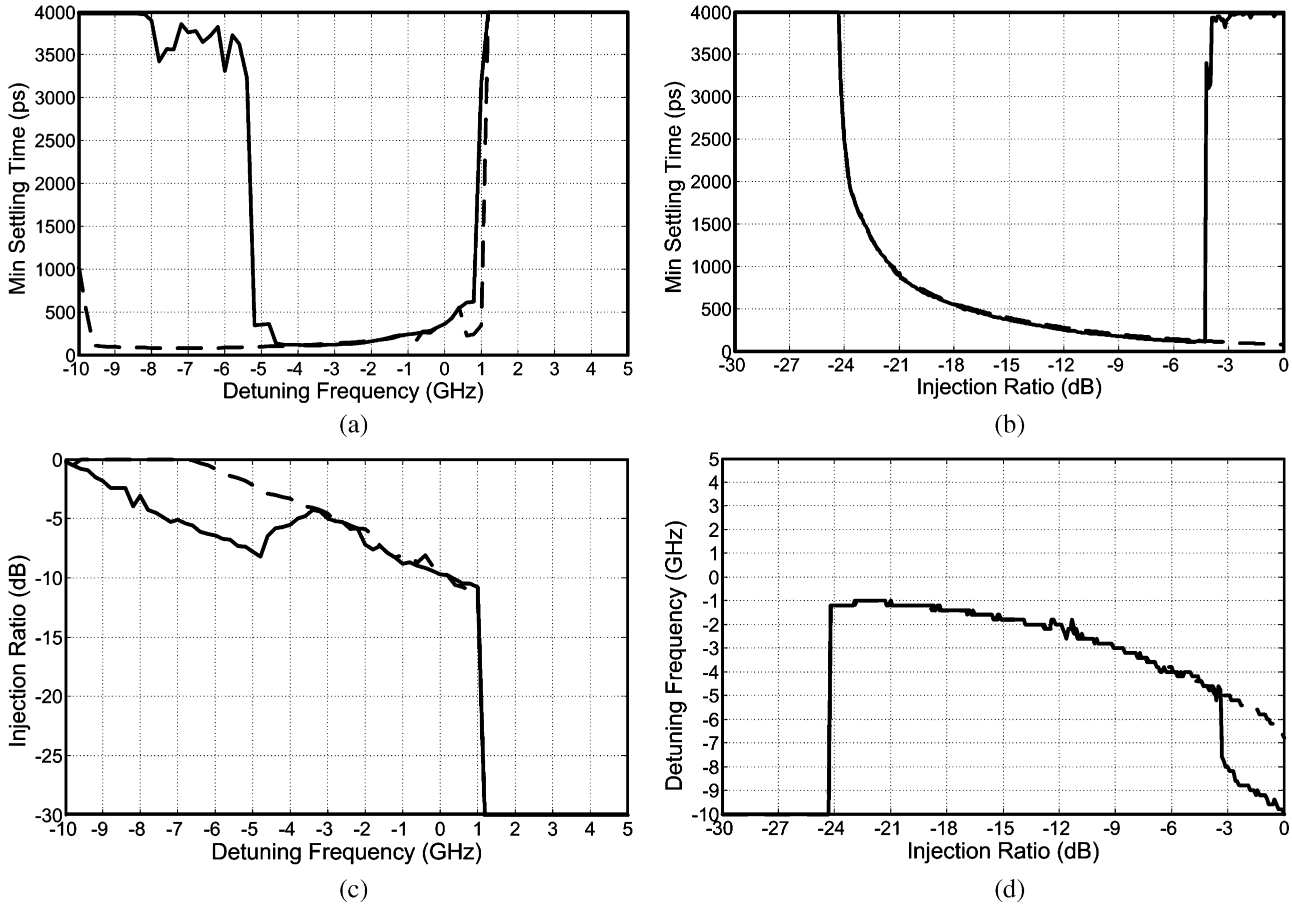

Fig. 5. Simulated minimum settling time for CW (dashed line) and CCW (solid line) (a) at different detuning frequency, (b) at different injection ratio, (c) injection ratio for minimum settling time at different detuning frequency, and (d) detuning frequency for minimum settling time at different injection ratio.

light from SRL is locked and stably locked to the external injection source respectively, have been obtained theoretically. Intersection of these three regions gives the area where successful and stable locked switching exists. Simulations for finding successful and stable switching region are carried out to verify analytical calculations. Numerically determined stable switching and locking region coincides with the intersection of the three regions mentioned above, showing good agreement with the analytical results. The results obtained should provide guidance to experimental applications such as all-optical data processing functions where to achieve reliable switching is essential.

Numerical results further show that the smallest of the minimum switching settling time (MSST) for both direction of SRL at certain detuning frequency lies in the negative detuning frequency side and MSST can be decreased by increasing the injection power.

The difference in the magnitude of self- and cross-saturation originates from the $\chi^{(3)}$ nonlinearity in the semiconductor gain medium [10]. The derivation of minimum switched, locking and stable locking regions in this paper applies to bistable semiconductor ring laser in general and the assumption of $\varepsilon_c = 2\varepsilon_s$ is not important. Bistability can be achieved through gain saturation [5]–[7] as long as $\varepsilon_c/\varepsilon_s > 1$, which is the case for semiconductor ring lasers with sufficient optical intensity in the cavity [9]. The relative strength of the saturation parameters $\varepsilon_c/\varepsilon_s$ however has a more profound effect on dynamic transient and settling time which also relies on many other parameters, such as device size, material gain parameters and working conditions. The dynamic behavior is the subject of further study.

Appendix A
Steady-State Soltions Without and With External Injection

The steady state solutions without injection terms are derived by setting the derivatives and injection terms of (6)–(10) to zero supposing lasing direction is the same as A_1

$$\overline{\beta} = 1 - \frac{1}{\tau_p \Gamma G_n(\overline{N} - N_{\mathrm{tr}})} \tag{A1}$$

$$\overline{A}_1^2 = \frac{1}{\varepsilon_s}\left(1 - \frac{1}{\Gamma G_n(\overline{N} - N_{\mathrm{tr}})\tau_{p1}}\right) \tag{A2}$$

$$\overline{A}_2 = 0 \tag{A3}$$

$$\frac{\eta_i I}{eV} - \frac{\overline{N}}{\tau_e} = \frac{\overline{A}_1^2}{\Gamma \tau_{p1}}. \tag{A4}$$

By setting the derivatives terms of (6)–(10) to zero, and combining the steady state solutions without injection terms of (A1)–(A4), we can obtain the steady state solutions with injection terms supposing lasing direction is the same as A_1

$$\widetilde{\beta} = 1 - \frac{1}{\tau_p \Gamma G_n(\widetilde{N} - N_{\mathrm{tr}})} \tag{A5}$$

$$\widetilde{A}_1^2 = \frac{\left(1-\varepsilon_c\widetilde{A}_2^2\right)\left(\frac{1}{\tau_{p2}} - \frac{2K_{\text{ext}}A_{\text{ext}}\cos\widetilde{\phi}_2}{\widetilde{A}_2}\right) - \left(1-\varepsilon_s\widetilde{A}_2^2\right)\frac{1}{\tau_{p1}}}{\left(\frac{\varepsilon_s}{\tau_{p2}} - \frac{2\varepsilon_s K_{\text{ext}}A_{\text{ext}}\cos\widetilde{\phi}_2}{\widetilde{A}_2} - \frac{\varepsilon_c}{\tau_{p1}}\right)} \quad \text{(A6)}$$

$$\widetilde{A}_2^2 = \frac{1}{\varepsilon_s}\left[1 - \varepsilon_c\widetilde{A}_1^2 - \frac{1}{\Gamma G_n(\widetilde{N} - N_{\text{tr}})} \times \left(\frac{1}{\tau_{p2}} - \frac{2K_{\text{ext}}A_{\text{ext}}\cos\widetilde{\phi}_2}{\widetilde{A}_2}\right)\right] \quad \text{(A7)}$$

$$\sin(\widetilde{\phi}_2 + \arctan\alpha) = -\frac{\omega_{\text{ext}} - \omega_{th}}{\omega_m} \quad \text{(A8)}$$

$$\Delta\widetilde{N} = \tau_e\left[\frac{\overline{A}_1^2}{\Gamma\tau_{p1}} - \frac{\widetilde{A}_1^2}{\Gamma\tau_{p1}} - \left(\frac{1}{\Gamma\tau_{p2}} - \frac{2K_{\text{ext}}A_{\text{ext}}\cos\widetilde{\phi}_2}{\Gamma\widetilde{A}_2}\right)\widetilde{A}_2^2\right] \quad \text{(A9)}$$

where $\omega_m = K_{\text{ext}}A_{\text{ext}}\sqrt{1+\alpha^2}/\widetilde{A}_2$, $\widetilde{N} = \overline{N} + \Delta\widetilde{N}$, $\Delta\hat{N}$ is the carrier density difference between the steady state solution without optical injection and the steady solution with optical injection. The steady state solutions with injection terms are found by solving the set of nonlinear equation (A5)–(A9).

Appendix B
Coefficients in the Perturbation Matrix

The coefficients are

$$a_{11} = \frac{1}{2}\left(\Gamma G_n(\hat{N} - N_{\text{tr}})\left(1 - \varepsilon_c\hat{A}_2^2\right) - \frac{1}{\tau_{p1}}\right) \quad \text{(B1)}$$

$$a_{22} = 0 \quad \text{(B2)}$$

$$a_{23} = -\alpha\Gamma G_n(\hat{N} - N_{\text{tr}})\varepsilon_c\hat{A}_2 \quad \text{(B3)}$$

$$a_{25} = \frac{\alpha}{2}\Gamma G_n\left(1 - \varepsilon_c\hat{A}_2^2\right) \quad \text{(B4)}$$

$$a_{33} = -\Gamma G_n(\hat{N} - N_{\text{tr}})\varepsilon_s\hat{A}_2^2 - z\cos\hat{\phi}_2 \quad \text{(B5)}$$

$$a_{34} = -z\hat{A}_2\sin\hat{\phi}_2 \quad \text{(B6)}$$

$$a_{35} = \frac{1}{2}\Gamma G_n\left(1 - \varepsilon_s\hat{A}_2^2\right)\hat{A}_2 \quad \text{(B7)}$$

$$a_{43} = \frac{z}{\hat{A}_2}\sin\hat{\phi}_2 - \alpha\Gamma G_n(\hat{N} - N_{\text{tr}})\varepsilon_s\hat{A}_2 \quad \text{(B8)}$$

$$a_{44} = -z\cos\hat{\phi}_2 \quad \text{(B9)}$$

$$a_{45} = \frac{\alpha}{2}\Gamma G_n\left(1 - \varepsilon_s\hat{A}_2^2\right) \quad \text{(B10)}$$

$$a_{53} = -\frac{2\hat{A}_2}{\Gamma}\left(\frac{1}{\tau_{p2}} - 2z\cos\hat{\phi}_2\right) + 2G_n(\hat{N} - N_{\text{tr}})\varepsilon_s\hat{A}_2^3 \quad \text{(B11)}$$

$$a_{55} = -\frac{1}{\tau_e} - G_n\left(1 - \varepsilon_s\hat{A}_2^2\right)\hat{A}_2^2 \quad \text{(B12)}$$

$$z = \frac{K_{\text{ext}}A_{\text{ext2}}}{\hat{A}_2}. \quad \text{(B13)}$$

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Guohui Yuan was born in Hunan Province, China, in 1982. She received the B.S. and M.S. degrees in electronics and information engineering from Tianjin University, Tianjin, China, in 2003 and 2006, respectively. She is currently working toward the Ph.D. degree with the Department of Electrical and Electronic Engineering, University of Bristol, Bristol, U.K., where she is engaged in modelling and simulation of semiconductor ring lasers.

Siyuan Yu was born in Nanchang, Jiangxi Province, China, in May 1963. He received the B.Eng. degree from Tsinghua University, Beijing, China, in 1984, and the M.Eng. degree in 1987 from Wuhan Research Institute of Post and Telecommunications, Wuhan, China, where he worked on the frequency stabilization of semiconductor lasers.

He joined the Department of Optoelectronic Engineering, Huazhong University of Science and Technology in 1987 and had worked on semiconductor optical amplifiers and other photonic devices. From 1993 to 1996 he studied for his Ph.D. degree at the Department of Electronics and Electrical Engineering, University of Glasgow, Glasgow, Scotland, UK, working on monolithically integrated mode-locked semiconductor ring lasers. In 1996 he joined the Department of Electrical and Electronic Engineering, University of Bristol, England, U.K., where he is currently a Reader. His research interest has been integrated photonic devices and technologies for optical networking, including optical packet switches, tunable lasers, wavelength converters, and all-optical logic devices. He is the (co-)author of more than 90 papers and (co-)inventor of several patents.