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Chromaticity and Color Temperature for Architectural Lighting

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ABSTRACT

HBLEDs bring a new dimension to architectural lighting design: color. Compared to traditional "white" light sources such as fluorescent and metal halide lamps, HBLEDs offer lighting designers unprecedented control over color temperature and lamp chromaticity. This raises two important questions: 1) How should the correlated color temperature metric used by lamp manufacturers be applied to HBLED-based light sources; and 2) What are appropriate limits on chromaticity variances for LED clusters and arrays? The answers to these questions will influence the acceptance of HBLEDs by architectural lighting designers.

Keywords: Lamp chromaticity, correlated color temperature, white light, generalized CCT metric, LED clusters

1. INTRODUCTION

One of the goals of high-brightness LED (HBLED) research is to develop energy-efficient replacements for traditional “white” light sources such as fluorescent and metal halide lamps. If we look beyond the immediate goal of luminous efficacy (i.e., lumens / input watt), we encounter two metrics of critical importance to architectural lighting designers: *correlated color temperature* and *lamp chromaticity*.

This is not to say that lighting designers are necessarily familiar with the scientific basis of these metrics. For most applications, it is necessary only to specify the lamp color temperature; the underlying issue of lamp chromaticity is addressed by the lamp manufacturer. All the lighting designer sees is the rated color temperature as listed in the manufacturer’s catalog.

If however HBLEDs are to replace fluorescent and metal halide lamps, LED manufacturers will need to address the same issues that have been previously addressed by the lamp manufacturers. In particular, LED manufacturers will need to fully understand the correlated color temperature metric and its relation to lamp chromaticity. This understanding must then be considered in HBLED product design and communicated to the architectural lighting design community.

2. COLOR TEMPERATURE

The correlated color temperature metric is based on the concept of *color temperature*, which the International Commission on Lighting (CIE) defines in its International Lighting Vocabulary[5] as:

> *Colour temperature: The temperature of a Planckian radiator whose radiation has the same chromaticity as that of a given stimulus.*

A Planckian radiator, also referred to as a *blackbody radiator*, is a theoretical object with zero reflectance. Its spectral radiant exitance distribution is determined by Planck’s radiation law:

$$M(\lambda,T) = c_1 \lambda^{-5} \left(\exp(c_2/\lambda T) - 1\right)^{-1} \tag{1}$$

where:

$c_1 = 3.74183 \times 10^{-16}$ W m^2
$c_2 = 1.4388 \times 10^{-2}$ m °K

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and where λ is the wavelength in meters and T is the blackbody temperature in degrees Kelvin.

The spectral radiant exitance distribution $M(\lambda,T)$ is directly proportional with respect to λ and a given color temperature T to the *relative color stimulus* function $\Phi(\lambda)$. Applying the CIE color-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ of the CIE standard colorimetric observer[4] yields the CIE tristimulus values:

$$\begin{aligned} X &= k\sum_{\lambda}\Phi(\lambda)\bar{x}(\lambda)\Delta\lambda \\ Y &= k\sum_{\lambda}\Phi(\lambda)\bar{y}(\lambda)\Delta\lambda \\ Z &= k\sum_{\lambda}\Phi(\lambda)\bar{z}(\lambda)\Delta\lambda \end{aligned} \tag{2}$$

and the CIE 1931 chromaticity diagram coordinates:

$$\begin{aligned} x &= \frac{X}{X+Y+Z} \\ y &= \frac{Y}{X+Y+Z} \end{aligned} \tag{3}$$

Plotting these chromaticity coordinates versus color temperature on the CIE 1931 chromaticity diagram yields the *Planckian locus* (Figure 1). A given color temperature T therefore has a unique color that is defined by its chromaticity coordinates (x, y).

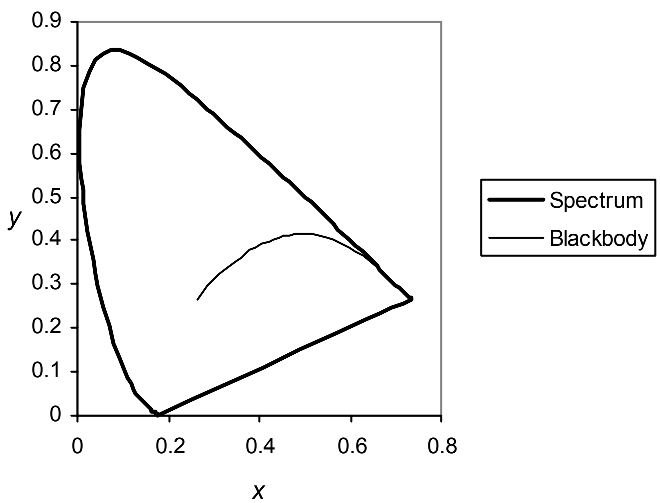

Figure 1 Planckian locus plotted on CIE 1931 chromaticity diagram

3. MIREDS AND MIREKS

The relationship between color temperature and chromaticity is highly nonlinear. For instance, the difference in chromaticity for one degree difference at 2,000º K corresponds to a difference of 100 degrees at 20,000º K. This makes it impractical to compare chromaticity differences on the basis of differences in color temperature.

The visual perceptibility of color temperature differences was investigated by Judd[12]. Based on the results of this study, Priest[28] proposed the use of reciprocal color temperature as a metric, noting that "a difference of one microreciprocal degree is fairly representative of the doubtfully perceptible difference in chromaticity under the most favorable conditions of observation." The unit of measurement is the *mirek* (reciprocal microkelvin), also known as the *mired*.

4. CORRELATED COLOR TEMPERATURE

The *correlated color temperature* (CCT) metric has long been used to characterize light sources. Originally developed for incandescent lamps and daylight[27], it is equally useful for any light sources that produce reasonably "white" light.

Most fluorescent and metal halide lamps designed for architectural applications exhibit chromaticities that are close to, but not necessarily coincident with, the Planckian locus. Davis[6] introduced the term "correlated color temperature," noting that the concept was first proposed by Hyde[9]. This is defined as the temperature of a Planckian radiator whose chromaticity is closest to that of the light source on a *perceptually uniform color space* diagram. In such a diagram, isotemperature lines are normal to the Planckian locus[14], as shown in Figure 2.

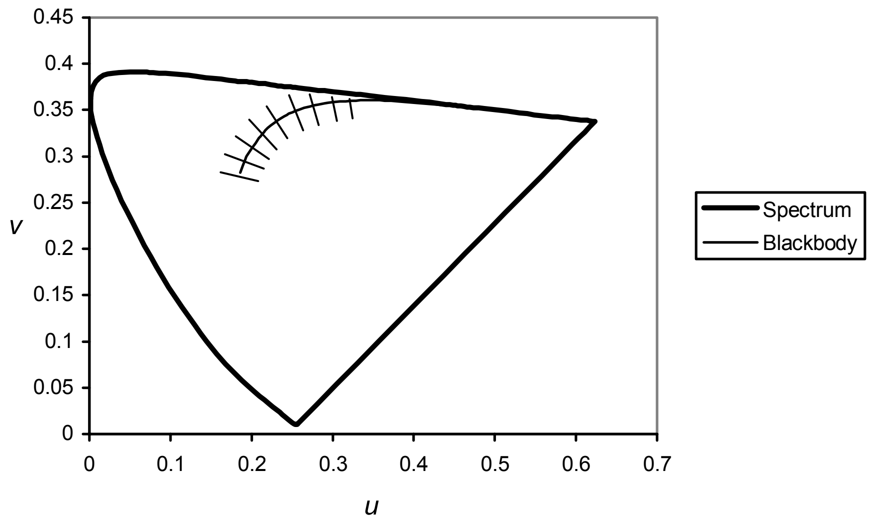

Figure 2 CCT normals to Planckian locus on CIE 1960 UCS diagram

Correlated color temperature was originally determined in terms of an approximately uniform color space diagram proposed by Judd[13]. This was replaced by the CIE 1960 Uniform Chromaticity Scale diagram, which has the projective transformation:

$$u = \frac{4X}{X + 15Y + 3Z}$$
$$v = \frac{6Y}{X + 15Y + 3Z} \quad (4)$$

where (u, v) are the transformed chromaticities. The CIE 1960 UCS diagram has since been deprecated by the International Commission on Illumination in favor of the CIE 1976 Uniform Color Space diagram (Figure 3), whose chromaticities (u', v') are related to (u, v) by the equations[4] $u = u'$ and $v = 2v'/3$. However, the CIE 1960 UCS diagram is still required for CCT determination.

Numerous methods have been proposed to determine the CCT of a light source with specified chromaticity values. Kelley[16] plotted equidistant isotemperature lines on both the CIE 1931 chromaticity diagram and the CIE 1960 UCS diagram. Other methods included Krystek[17], McCamy[22], Mori et al.[23], Robertson[30], Schanda et al.[31], and Xingzhong[36].

Because there is no CIE-approved or even recommended method, various photometric laboratories use slightly different algorithms. However, round-robin testing by the Council for Optical Radiation Measurement (CORM) found agreement to within ±2º K at lower CCTs and ±10º K at higher CCTs[25]. The differences were less than one mirek.

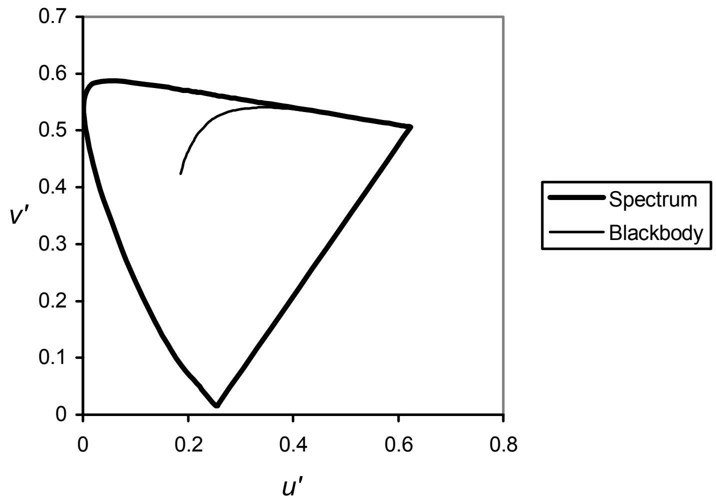

Figure 3 CIE 1976 uniform color space diagram

While the iterative method of Robertson is perhaps most commonly used, Kelley[16] observed that the isotemperature lines intersect in the purple region of the CIE 1931 chromaticity diagram, near ($x = 0.325$, $y = 0.154$). This led McCamy[22] to propose the following analytic equation for calculating the CCT of a light source[‡]:

$$n = (x - x_e)/(y_e - y) \tag{5}$$
$$CCT = 449.0 * n^3 + 3525.0 * n^2 + 6823.3 * n + 5520.33$$

where ($x_e = 0.3320$, $y_e = 0.1858$). The locus normals do not intersect at this point, but it is convenient for the formulation of the equation.

McCamy's method has a maximum absolute error of less than 2º K for color temperatures ranging from 2,856 to 6,500º K (corresponding to CIE illuminants A through D_{65}). In the absence of CIE recommendations for a particular method, this method may prove useful for implementation in real-time lighting control systems for multichip white light LEDs.

5. CCT LIMITATIONS

The ability of photometric laboratories to precisely determine the CCT of a given light source from its measured spectral power distribution belies its limitations as a metric. MacAdam[21] observed that the isotemperature lines are derived simply by drawing lines orthogonal to the Planckian locus in a "so-called uniform chromaticity scale (UCS) diagram."

MacAdam further noted that despite being in use for more than forty years, the concept of CCT was not supported by any reliable experimental data. This issue was later addressed by Grum et al.[8], and more recently and comprehensively by Borbély et al.[3].

Borbély et al. presented seven subjects with the task of matching 12 colors whose chromaticity coordinates were slightly off the Planckian locus with the closest color on the locus. They found that the scatter in the individual observations was "tremendously large," and stated:

> Results of our experiments show that the concept of correlated colour temperature cannot be quantified using visual observations. Correlated colour temperature is nothing more than a shorthand description of whether the light is bluish-white, neutral, or reddish white.

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‡ A typographical error in McCamy (1992) presents Equation 5 as $(x - x_e) / (y - y_e)$ rather than the correct $(x - x_e) / (y_e - y)$. The author thanks Peter Kan of TIR Systems for this observation.

The authors concluded that CCTs for light sources should not be reported with any greater precision than is currently used for fluorescent and metal halide lamps. For example, ANSLG[1] specifies CCTs of 2,700, 3,000, 3,500, 4,000, 5,000 and 6,500º K and their corresponding chromaticities for fluorescent lamps. (The International Electrotechnical Commission[10,11] has equivalent documents.)

A second limitation of the CCT as a metric is that there are no constraints of the distance from the Planckian locus. As noted by Judd[14], "The experimental setting of nearest color temperature when a considerable chromaticity difference exists between the illuminant in question and the Planckian radiator at any temperature becomes increasingly difficult and ambiguous as the chromaticity difference is increased." He then suggested that a CIE (x, y) chromaticity difference on the order of ±0.02 would be an acceptable maximum.

When Kelley[16] calculated data points in (u, v) space along normals to the Planckian locus, he extended the range to ±0.04 units while stating that "Each set of points was connected by a straight line, this length selected to be usably long but not so long as to invite the criticism that they represent notably non-Planckian colors."

6. MacADAM ELLIPSES

The CCT concept was developed some eleven years before MacAdam[20] investigated the sensitivity of the human visual system to color differences when adapted to average daylight (CIE Illuminant C, with a CCT of about 6,800º K).

One-step MacAdam ellipses (Figure 4) delineate the chromaticities of colors that are just noticeably different from the specified color under controlled laboratory conditions. While MacAdam's experimental data provided 25 such ellipses across the CIE 1931 chromaticity diagram, they can be reasonably interpolated for any given color. Plotted along the Planckian locus, they roughly correspond to one "doubtfully perceptible difference," or one mirek[27].

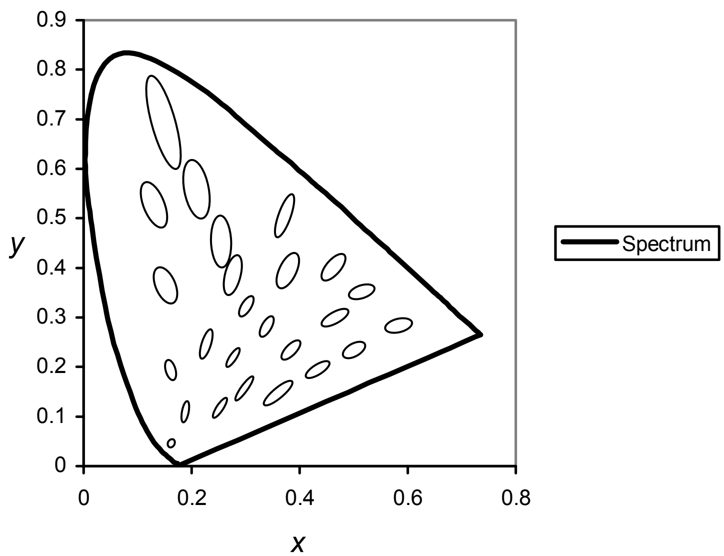

Figure 4 10-step MacAdam ellipses plotted on CIE 1931 chromaticity diagram

MacAdam ellipses are used in the specification of chromaticity tolerances for fluorescent and metal halide lamps. For example, ANSLG[1] defines such tolerances in terms of 4-step MacAdam ellipses centered on the Planckian locus, while major fluorescent lamp manufacturers generally maintain production tolerances based on 2-step or 3-step ellipses[2,32].

7. PHYSICAL CONSIDERATIONS

The use of CCT as a metric for characterizing light sources is further confounded by the physical characteristics of some lamps. High intensity discharge lamps in particular are prone to both short- and long-term color shifts. These color shifts typically include significant changes in correlated color temperature. For applications such as television studio lighting where color balance is a critical requirement, it may be necessary to color filter individual lamps[35].

More germane to this discussion are the physical characteristics of white light LEDs. For example, Nägele[24] reported that the CCTs of white light phosphor-based LEDs can vary, depending on the viewing angle. He measured ranges of 5,000 to 8,500º K for clear LEDs and 6,000 to 11,000º K for diffuse LEDs over a viewing angle range of 0 to ±90 degrees from the optical axis. (Although the LEDs under test were not reported, this appears to be primarily a function of the LED package design.)

There are also problems with multichip and discrete LEDs arranged as RGB arrays and clusters. For example, luminous flux output is a function of LED junction temperature, which in turn is a function of both the forward current and the ambient temperature. Nägele[24] reported that the relative luminous intensity of high-brightness LEDs decreased by approximately 0.5% (blue), 2.2% (green), and 14% (red) following a 14º C. increase in ambient temperature.

A second problem is that the LED peak wavelength and spectral bandwidth are also dependent on junction temperature. As an example, Schanda et al. reported a temperature change of 10º C. can produce a luminous intensity change of 10% or more and noticeable color shifts for green LEDs.

These are short-term problems, to which may be added the long-term degradation (yellowing) of encapsulating epoxy and acrylic resins due to exposure to blue light and ultraviolet radiation, as well as thermal degradation[18]. Fortunately, this problem is expected to be alleviated through the use of better resin formulations and possibly glass encapsulation[19] of LED dies.

These problems have important consequences for maintaining constant CCTs with white light LEDs. Current development efforts are focusing on ultraviolet LEDs with three phosphors designed to emit red, green, and blue light. Assuming that the problems of phosphor aging, encapsulant yellowing, and stray ultraviolet emission can be overcome, these devices should provide better quantum efficiencies and color rendering properties than current devices. It also means however that their chromaticity coordinates will be nominally fixed during manufacturing.

Multichip designs will likely be the most efficient because there are no losses associated with phosphor down-conversions from ultraviolet to visible light. However, there are still major technical hurdles to overcome, including the development of efficient green LEDs. There is also the problem of integrating on-chip feedback mechanisms to control the LED intensity and color balance as the junction temperatures change and the device ages.

In the absence of active color control mechanisms, it is therefore evident that the CCTs of white light LEDs may change over their usable lifetime, and vary according to their operating conditions.

8. COLOR ADAPTATION

In considering CCTs as a metric for white light LEDs, it is important to remember that the human visual system is capable of color adaptation under a wide range of illuminants. In particular, we tend to perceive the color of the dominant light source as being more or less "white."[§] This ability ranges from illumination due to low-wattage incandescent lamps to north skylight[7].

At the same time, we are remarkably tolerant of luminous environments with mixed illuminants. According to studies performed by Katoh[15] and others, we quickly adapt to the average illuminant CCT, typically within 50 milliseconds or so[29]. Computer monitors are a good example: these are commonly calibrated such that their white point has a CCT of 9300º K to take advantage of the efficiency of blue phosphors[26]. We rarely notice this color cast unless the monitor is situated in an environment where its display luminance is dominated by the average background (or surround) luminance.

From this perspective, the recommendations of Borbély et al.[3] make sense: there is little reason to report the CCTs of white light LED light sources with any greater precision than is currently used for fluorescent and metal halide lamps. Following ANSLG[1], CCTs of 2,700, 3,000, 3,500, 4,000, 5,000 and 6,500º K are likely acceptable for general illumination applications.

[§] The *white point* (or *reference*) on the CIE 1931 Chromaticity Diagram has chromaticity coordinates ($x = 0.33$, $y = 0.33$) and represents CIE Illuminant E, a theoretical light source with a uniform spectral power distribution. More practically however, CIE Illuminant D_{65} is most often used in television as the standard white reference, along with D_{50} in the print industry and D_{55} in photography as compromises between incandescent and daylight light sources.

9. LED ARRAY CONSIDERATIONS

This generalization does not of course apply to situations where the individual LEDs in an LED array or cluster are normally visible. As shown by MacAdam ellipses, we are capable of distinguishing color differences of approximately one mirek when viewing two or more light sources side-by-side. It therefore seems reasonable for LED manufacturers to follow the recommendations of ANSLG[1] by adopting a 4-step MacAdam ellipse centered in the Planckian locus as an industry standard for LED arrays.

For applications where it is known that the LED arrays will be used with non-imaging optics or in luminaires providing indirect illumination, this recommendation could be stated in terms of the chromaticity coordinates of the entire LED array rather than its component LEDs. However, given the ever-increasing luminous efficacy of white light LEDs, it is likely that there will be too few LEDs in a typical array to justify a statistical approach.

10. COLOR PREFERENCES

At issue here is the unique ability of multichip white light LEDs to produce dynamically variable colors surrounding the Planckian locus. It is a design parameter with which the architectural lighting industry has no previous experience.

In many architectural environments, the interreflection of luminous flux from colored surfaces may significantly alter the spectral distribution of the ambient illumination. For example, a rich red carpet and dark wood walls in a hotel lobby will effectively lower the CCT of the overall illuminant and shift its color away from the Planckian locus. Lighting designers may find that future LED-based luminaires with non-Planckian chromaticity coordinates produce more pleasing illumination than traditional incandescent or fluorescent lamps. The ability to adjust these coordinates may be perceived as being a desirable feature, much as cinematographers use acetate filters to adjust the color balance of studio and location lighting.

A similar issue involves changes in natural illumination during the day and through the seasons. The illuminance and chromaticity coordinates of daylight vary according to the solar altitude and atmospheric conditions. Given the ability to dynamically alter the illuminant color without filtering, there may be psychological advantages in doing so during the day or night in indoor environments.

11. GENERALIZED CCT METRIC

With all of these issues in mind, it is evident that the concept of correlated color temperature is applicable to white light LEDs with fixed chromaticity coordinates. For general illumination applications where they are designed to supplement or replace fluorescent lamps, the recommendations of ANSLG[1] or IEC[11] for CCTs, chromaticity coordinates, and chromaticity tolerances can likely be applied directly.

The situation for multichip white light LEDs with dynamically variable chromaticity coordinates is more complex. Future white light LEDs or LED-based luminaires will likely include feedback mechanisms designed to maintain the apparent lamp color. They may offer the ability to automatically or manually adjust the nominal color temperature.

If the apparent lamp color remains on the Planckian locus, then it is sufficient to specify its CCT and a chromaticity tolerance corresponding to a four-step MacAdam ellipse whose parameters can be interpolated from the measured data presented in MacAdam[20].

As discussed above, it may be that the apparent lamp color is intentionally displaced from the Planckian locus for a particular environment or application. Admittedly, the CCT concept does not strictly apply beyond a certain displacement, suggested to be ±0.02 units by Judd[14] and ±0.04 units by Kelley[16]. However, the normal to the Planckian locus in the CIE 1960 uniform color space is mathematically convenient in that the displacement can be expressed as a scalar.

While the range of normals to the Planckian locus does not cover the color gamut achievable with multichip LEDs, it does cover the range of what could reasonably be considered “white.” As such, measuring the distance along a normal from the Planckian locus to a given color offers a useful generalization of the CCT for white light LEDs.

The units of this metric could be expressed in terms of Euclidean distance in the CIE 1960 UCS diagram. More usefully however, they could be expressed in terms of the one-step MacAdam ellipse centered on the intersection of the normal with the Planckian locus. While somewhat more difficult to calculate, this provides a more intuitive measure of the displacement.

Rather than developing a complex analytic expression for interpolated MacAdam ellipses along the locus, it is sufficient to note that these ellipses are approximately circular in the region of the Planckian locus when plotted on the CIE 1960 UCS diagram. As noted by Priest[28], a mirek is in retrospect "fairly representative" of a one-step MacAdam ellipse centered on the Planckian locus. It is therefore reasonable to adopt mireks as the basis for calculating the local unit of measurement along both the normals and the locus itself.

From this, we have the following simple procedure:

Generalized Correlated Color Temperature Calculation

1. Determine chromaticity coordinates (u_0, v_0) for given color temperature T (in mireks).
2. Determine chromaticity coordinates (u_1, v_1) and (u_2, v_2) for $(T - 0.5)$ and $(T + 0.5)$.
3. Local slope of Planckian locus = $(v_1 - v_2)/(u_1 - u_2)$
4. Local slope of displacement vector = $-(u_1 - u_2)/(v_1 - v_2)$
5. Local unit distance = $\sqrt{(u_1 - u_2)^2 + (v_1 - v_2)^2}$

The units of the "local unit distance" are mireks (MK^{-1}) only along the Planckian locus. Moreover, these vary in terms of color difference units according to the color temperature. In the local region of the given color temperature however, a unit distance in any direction corresponding to approximately the same color difference.

Whether this local unit distance is colloquially referred to as a "mirek" or some other term is not important. What is important is the underlying concept. The purpose of this generalized CCT – the CCT itself and the displacement of the color from the Planckian locus – is twofold. First, it eliminates the ambiguity in chromaticity inherent in the CCT. This is important for specifying the color of white LEDs and LED array without resorting to their less intuitive chromaticity coordinates.

Second, the generalized CCT provides a quantifiable basis for optoelectronic feedback controls. Interoperability between control equipment from different manufacturers will require an industry standard for measurement units. The CIE (x, y) and (u, v) chromaticity coordinates are inadequate in that they do not provide orthogonal directions with respect to color temperature. By comparison, the generalized CCT metric provides an orthogonal and intuitive scale for white light consisting of color temperature and color tint (i.e., displacement).

12. CONCLUSIONS

In consideration of the above, it is evident that the CCT concept is applicable to white light LEDs designed for general illumination applications. This is to be expected, as white light LEDs are just another light source.

Similarly, the fluorescent lamp chromaticity specifications expressed in ANSLG[1] and IEC[11] are equally applicable to white light LEDs and LED arrays with nominally constant CCTs.

Multichip white light LEDs present a more complex problem in that different lamp colors may have the same CCT. A generalized CCT has been proposed to address this issue. (It must be emphasized that this is only a proposal, and is intended as a basis for discussion between LED manufacturers and the International Commission on Illumination.)

It could be argued that this proposal is immature – the state of the art in white light LEDs is still based on blue LEDs with yellow phosphors. Multichip white light LEDs are possible but not (as of this writing) commercially available. If the LED industry standardizes on ultraviolet LEDs with tricolor phosphors, the proposal becomes moot.

However, it is still necessary to evaluate the applicability of the CCT concept to white light LEDs in general. Moreover, if multichip white light LEDs do become commercially viable, this paper offers a basis for discussion on how best to incorporate them in luminaires designed for general illumination.

13. ACKNOWLEDGEMENTS

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