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Short Communication

# Design principle of single- or double-layer wave-absorbers containing left-handed materials

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ABSTRACT

The physical models of single- or double-layer microwave absorbers containing left-handed materials as well as their corresponding design principles and methods are proposed. The theoretically derived calculation formulae that may be used in the design procedures are also presented. Finally, a design example is given. The results obtained may be useful to the researchers and designers working in the area of microwave absorber theory, technology and engineering.



## 1. Introduction

In recent years, radar technologies have been improved drastically by the use of high powered large bandwidth transmitters, thereby the developments of the "Stealth" technologies for evading radar detection have become more and more important [1]. One of the stealth technologies is the use of microwave absorbers, which can absorb most of the energy of the incident wave impinged on their surfaces and transfer the energy into other forms that have little reflection to reduce the radar cross section (RCS) of the target [2]. Several techniques have been suggested for the RCS reduction, which are broadly classified into three categories: shaping of the target, radar absorbing materials (RAM) and radar absorbing structures (RAS) [1]. Among them, RAS are the most promising one, since RAS are the structures that have both the functions of load bearing and EM energy absorbing without interfering with the external profiles set by the aircraft designers [3].

The researchers and engineers have made a lot of efforts in the microwave absorbing techniques for some decades and it is estimated that thousands of papers have been published. We would not like to draw up the list of the literatures here, because the main purpose of this paper is to discuss the applications of left-handed materials to the absorbers.

At present, the properties of the absorbers reported so far are still not satisfactory in the aspects of absorbing frequency band width, efficiency of wave absorption, density or weight of the wave absorbing materials or structures. Thus, further efforts are still needed. A possible direction of making effort is on the use of wave-absorbers with compatibility, which can cover meter-, centimeter-, millimeter-, and even infrared wave bands. Besides, RAS is also a promising developing area. With the fast development of material science and engineering technology, some new concept and new electromagnetic materials, including metamaterials [4–6], frequency selective surface structures [7–9], and high impedance surface [10,11], etc. are also employed in the RSC reduction area. The earlier work of the authors of this article verified that the properties of the wave-absorbers may indeed be improved by means of inducing left-handed materials (LHM) in their structures [12]. Here, "LHM" are the materials with simultaneously negative dielectric permittivity and negative magnetic permeability, which were firstly defined by Veselago [13]. Correspondingly, conventional materials with simultaneously positive permittivity and positive permeability are called Right-handed Materials (RHM).

The applications of metamaterials, including LHMs, epsilon-negative materials and mu-negative materials, in microwave absorbers are an interesting area and some research results have been reported [14–18] including the design principles and methods, manufacture, modeling, etc.

In this paper, we will present the design principles of wave-absorbers with single- or double-layer structure containing LHMs.

## 2. Design principles of single-layer LHM wave-absorbers

The schematic diagram of a wave-absorber with single LHM layer is shown in Fig. 1, where the dielectric permittivity, the magnetic permeability, the eigen-impedance and complex wave number of the air, the layer 1 and the metal substrate plate are expressed as $\varepsilon_0$, $\mu_0$, $Z_0$, $k_0$; $\varepsilon_1$, $\mu_1$, $Z_1$, $k_1$; and $\varepsilon_2$, $\mu_2$, $Z_2$, $k_2$, respectively, and $Z_0 = \sqrt{\mu_0/\varepsilon_0}, Z_1 = Z_0\sqrt{\mu_1/\varepsilon_1}$ and $Z_2 = 0$. Here, it is assumed that the incident electromagnetic field is expressed as $E = E_0 e^{i(\omega t - kx)}$; the incident angle is $\theta$; and the refractive angle is $\alpha$ (not drawn out in the figure). It is also supposed that all the layers discussed in this paper are isotropic and homogeneous. In the

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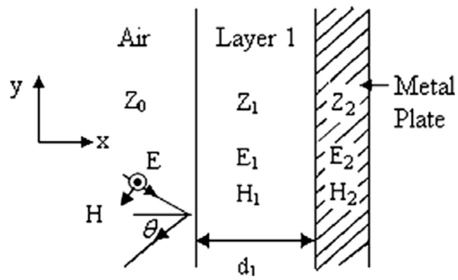


**Fig. 1.** Schematic diagram of a single-layer absorber's structure.

following text, we only consider the TE wave incidence case. The results for the TM wave incidence case can be derived employing a similar method.

If the layer 1 is made of LHM, the structure is named as a LHM single-layer wave-absorber. According to Fresnel formulae, its amplitude reflective coefficient is given by:

$$R = \frac{Z_{\lambda 1\mathrm{eff}} - Z_{0\mathrm{eff}}}{Z_{\lambda 1\mathrm{eff}} + Z_{0\mathrm{eff}}} \tag{1}$$

where $Z_{1\mathrm{eff}} = Z_0\sqrt{\mu_1/\varepsilon_1}/\cos\alpha$ is the total effective wave impedance of the single-layer absorber, $Z_{0\mathrm{eff}} = \sqrt{\mu_0/\varepsilon_0}/\cos\theta$ is the effective impedance to the TE wave in the region of the air near the interface between the air and the absorber. The transmission equation for the structure is:

$$\begin{bmatrix} E_0 \\ H_0 \end{bmatrix} = \begin{bmatrix} \cos\delta_1 & \mathrm{i}Z_{1\mathrm{eff}}\sin\delta_1 \\ \mathrm{i}\sin\delta_1/Z_{1\mathrm{eff}} & \cos\delta_1 \end{bmatrix} \times \begin{bmatrix} E_1 \\ H_1 \end{bmatrix} \tag{2}$$

where $\delta_1 = -k_1 d_1 \cos\alpha$. Denoting $k_{1\mathrm{eff}}$ as $k_{1\mathrm{eff}} = k_1\cos\alpha$, then $\delta_1 = -k_{1\mathrm{eff}} d_1$. According to the definition of the effective wave impedance, one can know that, for TE waves, $Z_{1\mathrm{eff}} = Z_0\sqrt{\mu_1/\varepsilon_1}/\cos\alpha$. The metal substrate's effective impedance is $Z_2 = 0$, then one has

$$Z_{\lambda 1\mathrm{eff}} = \frac{E_0}{H_0} = Z_{1\mathrm{eff}}\frac{Z_{2\mathrm{eff}}\cos\delta_1 + \mathrm{i}Z_{1\mathrm{eff}}\sin\delta_1}{\mathrm{i}Z_{2\mathrm{eff}}\sin\delta_1 + Z_{1\mathrm{eff}}\cos\delta_1} = Z_{1\mathrm{eff}}\frac{\mathrm{i}\sin\delta_1}{\cos\delta_1} \tag{3}$$

Substituting Eq. (3) for the $Z_{\lambda 1\mathrm{eff}}$ in Eq. (1), we obtain

$$R = \frac{Z_{\lambda 1\mathrm{eff}} - Z_{0\mathrm{eff}}}{Z_{\lambda 1\mathrm{eff}} + Z_{0\mathrm{eff}}} = \frac{\mathrm{i}Z_{1\mathrm{eff}}\frac{\sin\delta_1}{\cos\delta_1} - Z_{0\mathrm{eff}}}{\mathrm{i}Z_{1\mathrm{eff}}\frac{\sin\delta_1}{\cos\delta_1} + Z_{0\mathrm{eff}}} \tag{4}$$

For lossy LHMs, we have $Z_1 = Z_1' - \mathrm{i}Z'', k_1 = k_1' - \mathrm{i}k_1''$ and $Z_{\lambda\mathrm{leff}} = Z'_{\lambda\mathrm{leff}} - \mathrm{i}Z''_{\lambda\mathrm{leff}}$. Then Eq. (1) can be rewritten as:

$$R = \frac{(Z'_{\lambda\mathrm{leff}} - Z_{0\mathrm{eff}}) - \mathrm{i}Z''_{\lambda\mathrm{leff}}}{(Z'_{\lambda\mathrm{leff}} + Z_{0\mathrm{eff}}) - \mathrm{i}Z''_{\lambda\mathrm{leff}}} \tag{5}$$

If there is no reflection occurred at the interface, i.e. $R = 0$, then we have

$$\begin{cases} Z'_{\lambda\mathrm{leff}} - Z_{0\mathrm{eff}} = 0 \\ Z''_{\lambda\mathrm{leff}} = 0 \end{cases} \tag{6}$$

From Eqs. (5) and (6) and noticing that $Z'' = A\sqrt{[-(1+\tan\delta_e\tan\delta_m) + \sqrt{(1+\tan^2\delta_e)(1+\tan^2\delta_m)}]/2}$ [19], we obtain the expressions of the design principles of the single-layer LHM wave-absorbers as

$$\tan\delta_{e1} = \tan\delta_{m1} = \tan\delta \tag{7}$$

$$d_1 = (2l+1)\frac{\lambda_{1eff}}{4} \quad l = -1, -2, -3\ldots \tag{8}$$

where $\tan\delta_{e1} = \varepsilon_1''/\varepsilon_1', \tan\delta_{m1} = \mu_1''/\mu_1'$ are the tangents of the electric loss angle and magnetic loss angle of the layer; $\lambda_{1\mathrm{eff}} = 2\pi/k_{1\mathrm{eff}} = \lambda_1/\cos\alpha$, where $\lambda_1$ is the wave length of the electromagnetic waves inside the layer 1, and $\lambda_1 = \lambda_0/n_1'$, where $\lambda_0$ is the wave length of the wave in vacuum, and $n_1' < 0$ is the real part of the refractive index of the LHM (layer 1). Eqs. (7) and (8) are the conditions should be fulfilled in the design procedure of the single-layer LHM wave-absorbers.

## 3. Design principles of double-layer wave- absorbers containing LHMs

The schematic diagram of the structure of double-layer wave-absorbers is shown in Fig. 2, where the dielectric permittivity, the magnetic permeability, the eigen-impedance and complex wave number of the air, the dielectric layer 1, the dielectric layer 2 and the metal substrate plate are expressed as $\varepsilon_0$, $\mu_0$, $Z_0$, $k_0$; $\varepsilon_1$, $\mu_1$, $Z_1$, $k_1$; $\varepsilon_2$, $\mu_2$, $Z_2$, $k_2$; $\varepsilon_3$, $\mu_3$, $Z_3$, $k_3$, respectively; $Z_0 = \sqrt{\mu_0/\varepsilon_0}, Z_1 = Z_0\sqrt{\mu_1/\varepsilon_1}, Z_2 = Z_0\sqrt{\mu_2/\varepsilon_2}$ and $Z_3 = 0$ .

### 3.1. Design principles of LHM/RHM double-layer wave-absorbers

When the layer 1 is made of a LHM and the Layer 2 is of a RHM, it is a LHM/RHM double-layer wave-absorber. The reflective coefficient of the absorber can be written as:

$$R = \frac{Z_{\lambda\mathrm{leff}} - Z_{0\mathrm{eff}}}{Z_{\lambda\mathrm{leff}} + Z_{0\mathrm{eff}}} = \frac{(Z'_{\lambda\mathrm{leff}} - Z_{0\mathrm{eff}}) - \mathrm{i}Z''_{\lambda\mathrm{leff}}}{(Z'_{\lambda\mathrm{leff}} + Z_{0\mathrm{eff}}) - \mathrm{i}Z''_{\lambda\mathrm{leff}}} \tag{9}$$

Using the transmission matrix method, it can be obtained that the effective impedance of the layer *j (j = 1,2)* is:

$$Z_{\lambda j\mathrm{eff}} = Z_{j\mathrm{eff}}\frac{Z_{\lambda(j+1)\mathrm{eff}}\cos\delta_j + \mathrm{i}Z_{j\mathrm{eff}}\sin\delta_j}{\mathrm{i}Z_{\lambda(j+1)\mathrm{eff}}\sin\delta_j + Z_{j\mathrm{eff}}\cos\delta_j} \tag{10}$$

Then, for the structure shown in Fig. 2, one has:

$$Z_{\lambda 2\mathrm{eff}} = \mathrm{i}Z_{2\mathrm{eff}}\frac{\sin\delta_2}{\cos\delta_2} \tag{11}$$

and

$$Z_{\lambda 1\mathrm{eff}} = Z_{1\mathrm{eff}}\frac{Z_{\lambda 2\mathrm{eff}}\cos\delta_1 + \mathrm{i}Z_{1\mathrm{eff}}\sin\delta_1}{\mathrm{i}Z_{\lambda 2\mathrm{eff}}\sin\delta_1 + Z_{1\mathrm{eff}}\cos\delta_1} \tag{12}$$

Eqs. (11) and (12) leads to:

$$Z_{\lambda 1\mathrm{eff}} = \mathrm{i}Z_{1\mathrm{eff}}\frac{Z_{2\mathrm{eff}}\cos\delta_1\sin\delta_2 + Z_{1\mathrm{eff}}\sin\delta_1\cos\delta_2}{Z_{1\mathrm{eff}}\cos\delta_1\cos\delta_2 - Z_{2\mathrm{eff}}\sin\delta_1\sin\delta_2} \tag{13}$$

For an ideal absorber, let Eq. (9) be zero, we obtain the conditions that a LHM/RHM double-layer wave-absorber should satisfy, which are expressed as:

$$\tan\delta_{e1} = \tan\delta_{m1} = \tan\phi_1 \tag{14}$$

$$\tan\delta_{e2} = \tan\delta_{m2} = \tan\phi_2 \tag{15}$$

$$d_1 = (2l_1+1)\frac{\lambda_{1\mathrm{eff}}}{4} \quad l_1 = -1, -2, -3\ldots \tag{16}$$

$$d_2 = (2l_2+1)\frac{\lambda_{2\mathrm{eff}}}{4} \quad l_2 = 0, 1, 2\ldots \tag{17}$$

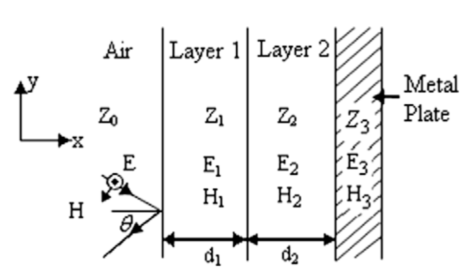


**Fig. 2.** Schematic diagram of a double-layer absorber's structure.

It should be noticed that $\tan\varphi_1 < 0$ and $\tan\varphi_2 > 0$ in this structure.

Let $l_1 = 0$, $l_2 = 0$, and noticing that [19]

$$K'_{LHM} = \frac{\omega}{c}\sqrt{\varepsilon'\mu'} \times \sqrt{\frac{1}{2}\left[1-\tan\delta_e\tan\delta_m+\sqrt{(1+\tan^2\delta_e)(1+\tan^2\delta_m)}\right]}$$
$$K''_{LHM} = \frac{\omega}{c}\sqrt{\varepsilon'\mu'} \times \sqrt{\frac{1}{2}\left[\tan\delta_e\tan\delta_m-1+\sqrt{(1+\tan^2\delta_e)(1+\tan^2\delta_m)}\right]} \tag{18}$$

$$K'_{RHM} = \frac{\omega}{c}\sqrt{\varepsilon'\mu'}\sqrt{\frac{1}{2}\left[1-\tan\delta_e\tan\delta_m+\sqrt{(1+\tan^2\delta_e)(1+\tan^2\delta_m)}\right]}$$
$$K''_{RHM} = \frac{\omega}{c}\sqrt{\varepsilon'\mu'}\sqrt{\frac{1}{2}\left[\tan\delta_e\tan\delta_m-1+\sqrt{(1+\tan^2\delta_e)(1+\tan^2\delta_m)}\right]} \tag{19}$$

the complex impedance and complex wave number can be simplified as:

$$k'_{1\text{eff}} = -\frac{\omega}{c}\sqrt{\mu'_1\varepsilon'_1}\cos\alpha_1 = -\frac{\pi}{2d_1} \tag{20}$$
$$K'_{1\text{eff}} = \frac{\omega}{c}\sqrt{\mu'_1\varepsilon'_1}\cos\alpha_1\tan\phi_1 = \frac{\pi}{2d_1}\tan\phi_1 \tag{21}$$
$$\sqrt{\mu'_1\varepsilon'_1}\cos\alpha_1 = \frac{\pi c}{2\omega d_1} \tag{22}$$
$$k'_{2\text{eff}} = \frac{\omega}{c}\sqrt{\mu'_2\varepsilon'_2} = \frac{\pi}{2d_2} \tag{23}$$
$$K''_{2\text{eff}} = \frac{\omega}{c}\sqrt{\mu'_2\varepsilon'_2}\tan\phi_2 = \frac{\pi}{2d_2}\tan\phi_2 \tag{24}$$
$$\sqrt{\mu'_2\varepsilon'_2}\cos\alpha_2 = \frac{\pi c}{2\omega d_2} \tag{25}$$
$$Z'_1 = Z_0\sqrt{\frac{\mu'_1}{\varepsilon'_1}}, \quad Z''_1 = 0 \tag{26}$$
$$Z'_2 = Z_0\sqrt{\frac{\mu'_2}{\varepsilon'_2}}, \quad Z''_2 = 0 \tag{27}$$

Substituting Eqs. (20)–(27) for the corresponding parameters in Eq. (13), we obtain:

$$Z_{\lambda 1\text{eff}} = Z'_{1\text{eff}}\frac{Z'_{2\text{eff}}\tan h\left(\frac{\pi}{2}\tan\phi_1\right)+Z'_{1\text{eff}}\tan h\left(\frac{\pi}{2}\tan\phi_2\right)}{Z'_{2\text{eff}}+Z'_{1\text{eff}}\tan h\left(\frac{\pi}{2}\tan\phi_1\right)\tan h\left(\frac{\pi}{2}\tan\phi_2\right)} \tag{28}$$

To ensure $R = 0$, it needs that $Z'_{\lambda 1\text{eff}} = Z_{0\text{eff}}$. Then, using Eqs. (26)–(28), we have:

$$\sqrt{\frac{\mu'_1}{\varepsilon'_1}} = \frac{\sqrt{\frac{\mu'_1}{\varepsilon'_2}}\cos\alpha_2\tan h\left(\frac{\pi}{2}\tan\phi_1\right)\tan h\left(\frac{\pi}{2}\tan\phi_2\right)-\sqrt{\frac{\mu'_2}{\varepsilon'_2}}\cos\alpha_1}{\sqrt{\frac{\mu'_2}{\varepsilon'_2}}\cos\alpha_1\tan h\left(\frac{\pi}{2}\tan\phi_1\right)-\sqrt{\frac{\mu'_1}{\varepsilon'_1}}\cos\alpha_2\tan h\left(\frac{\pi}{2}\tan\phi_2\right)} \times\frac{\cos\alpha_1}{\cos\theta} \tag{29}$$

$$\sqrt{\frac{\mu'_2}{\varepsilon'_2}} = \sqrt{\frac{\mu'_1}{\varepsilon'_1}}\frac{\cos\alpha_2}{\cos\alpha_1}\cdot\frac{\tan h\left(\frac{\pi}{2}\tan\phi_2\right)\left[\tan h\left(\frac{\pi}{2}\tan\phi_1\right)\frac{\cos\alpha_1}{\cos\theta}+\sqrt{\frac{\mu'_1}{\varepsilon'_1}}\right]}{\sqrt{\frac{\mu'_1}{\varepsilon'_1}}\tan h\left(\frac{\pi}{2}\tan\phi_1\right)+\frac{\cos\alpha_1}{\cos\theta}} \tag{30}$$

Eqs. (29) and (30) are the optimized laws of the electromagnetic parameters' values for the LHM/RHM double-layer wave-absorber. They are also called the matching conditions for the LHM/RHM double-layer wave-absorbers design.

If the values of the electromagnetic parameters of the layer 1 have previously been determined, then the dependence relations of the values of the electromagnetic parameters of the layer 2 on those of the layer 1 are:

$$\mu'_2 = \frac{\pi c}{2\omega d_2\cos\alpha_1}\sqrt{\frac{\mu'_1}{\varepsilon'_1}} \times\frac{\tan h\left(\frac{\pi}{2}\tan\phi_2\right)\left[\tan h\left(\frac{\pi}{2}\tan\phi_1\right)\frac{\cos\alpha_1}{\cos\theta}+\sqrt{\frac{\mu'_1}{\varepsilon'_1}}\right]}{\sqrt{\frac{\mu'_1}{\varepsilon'_1}}\tan h\left(\frac{\pi}{2}\tan\phi_1\right)+\frac{\cos\alpha_1}{\cos\theta}} \tag{31}$$
$$\mu''_2 = \mu'_2\tan\phi_2 \tag{32}$$
$$\varepsilon'_2 = \frac{\pi c\cos\alpha_1}{2\omega d_2\cos^2\alpha_2}\sqrt{\frac{\varepsilon'_1}{\mu'_1}} \times\frac{\sqrt{\frac{\mu'_1}{\varepsilon'_1}}\tan h\left(\frac{\pi}{2}\tan\phi_1\right)+\frac{\cos\alpha_1}{\cos\theta}}{\tan h\left(\frac{\pi}{2}\tan\phi_2\right)\left[\frac{\cos\alpha_1}{\cos\theta}\tan h\left(\frac{\pi}{2}\tan\phi_1\right)+\sqrt{\frac{\mu'_1}{\varepsilon'_1}}\right]} \tag{33}$$
$$\varepsilon''_2 = \varepsilon'_2\tan\phi_2 \tag{34}$$

### 3.2. Design principles of RHM/LHM double-layer wave-absorbers

When the layer 1 and layer 2 are made of RHM and LHM respectively, it becomes a RHM/LHM double-layer wave-absorber. Employing the same method used in Section 3.1, we know that the design principles and the concluding formulae have the same or similar forms with those for a LHM/RHM double-layer wave-absorber. The differences between them are $\tan\varphi_1 > 0$, $\tan\varphi_2 < 0$, $k'_{1\text{eff}} = \frac{\omega}{c}\sqrt{\mu'_1\varepsilon'_1}\cos\alpha_1 = \frac{\pi}{2d_1}$, and $k'_{2\text{eff}} = -\frac{\omega}{c}\sqrt{\mu'_2\varepsilon'_2} = -\frac{\pi}{2d_2}$. The signs of them are just opposite to those in Eqs. (20) and (23).

If the values of the electromagnetic parameters of the layer 1 have been previously determined, the dependence relations of the values of the electromagnetic parameters of the layer 2 on those of the layer 1 are:

$$\mu'_2 = \frac{\pi c}{2\omega d_2\cos\alpha_1}\sqrt{\frac{\mu'_1}{\varepsilon'_1}} \times\frac{\tan h\left(\frac{\pi}{2}\tan\phi_2\right)\left[\tan h\left(\frac{\pi}{2}\tan\phi_1\right)\frac{\cos\alpha_1}{\cos\theta}-\sqrt{\frac{\mu'_1}{\varepsilon'_1}}\right]}{\frac{\cos\alpha_1}{\cos\theta}-\sqrt{\frac{\mu'_1}{\varepsilon'_1}}\tan h\left(\frac{\pi}{2}\tan\phi_1\right)} \tag{35}$$
$$\mu''_2 = \mu'_2\tan\phi_2 \tag{36}$$
$$\varepsilon'_2 = \frac{\pi c\cos\alpha_1}{2\omega d_2\cos^2\alpha_2}\sqrt{\frac{\varepsilon'_1}{\mu'_1}} \times\frac{\cos\alpha_1\cos\theta-\sqrt{\frac{\mu'_1}{\varepsilon'_1}}\tan h\left(\frac{\pi}{2}\tan\phi_1\right)}{\tan h\left(\frac{\pi}{2}\tan\phi_2\right)\left[\frac{\cos\alpha_1}{\cos\theta}\tan h\left(\frac{\pi}{2}\tan\phi_1\right)-\sqrt{\frac{\mu'_1}{\varepsilon'_1}}\right]} \tag{37}$$
$$\varepsilon''_2 = \varepsilon'_2\tan\phi_2 \tag{38}$$

### 3.3. Design principles of LHM/LHM double-layer wave-absorbers

If both the layer 1 and layer 2 in Fig. 2 are made of LHMs, it is a LHM/LHM double-layer wave-absorber. Its design principles can also be written as:

$$\tan\delta_{e1} = \tan\delta_{m1} = \tan\phi_1 \quad (39)$$

$$\tan\delta_{e2} = \tan\delta_{m2} = \tan\phi_2 \quad (40)$$

$$d_1 = (2l_1 + 1)\lambda_{1\mathrm{eff}}/4 \quad l_1 = -1, -2, -3\ldots \quad (41)$$

$$d_2 = (2l_2 + 1)\lambda_{2\mathrm{eff}}/4 \quad l_2 = -1, -2, -3\ldots \quad (42)$$

The simplified complex wave numbers and complex impedance are:

$$k'_{1\mathrm{eff}} = -\frac{\omega}{c}\sqrt{\mu'_1\varepsilon'_1}\cos\alpha_1 = -\frac{\pi}{2d_1} \quad (43)$$

$$K''_{1\mathrm{eff}} = \frac{\omega}{c}\sqrt{\mu'_1\varepsilon'_1}\cos\alpha_1\tan\phi_1 = \frac{\pi}{2d1}\tan\phi_1 \quad (44)$$

$$\sqrt{\mu'_1\varepsilon'_1}\cos\alpha_1 = \frac{\pi c}{2\omega d_1} \quad (45)$$

$$k'_{2\mathrm{eff}} = -\frac{\omega}{c}\sqrt{\mu'_2\varepsilon'_2}\cos\alpha_2 = -\frac{\pi}{2d_2} \quad (46)$$

$$K''_{2\mathrm{eff}} = -\frac{\omega}{c}\sqrt{\mu'_2\varepsilon'_2}\cos\alpha_2\tan\phi_2 = -\frac{\pi}{2d_2}\tan\phi_2 \quad (47)$$

$$\sqrt{\mu'_2\varepsilon'_2}\cos\alpha_2 = \frac{\pi c}{2\omega d_2} \quad (48)$$

$$Z'_1 = Z_0\sqrt{\frac{\mu'_1}{\varepsilon'_1}}, \quad Z''_1 = 0 \quad (49)$$

$$Z'_2 = Z_0\sqrt{\frac{\mu'_2}{\varepsilon'_2}}, \quad Z''_2 = 0 \quad (50)$$

Substituting them for the parameters in the expression of $Z_{\lambda 1\mathrm{eff}}$, we have

$$Z_{\lambda 1\mathrm{eff}} = Z'_{1\mathrm{eff}}\frac{Z'_{2\mathrm{eff}}\tan h(\frac{\pi}{2}\tan\phi_1) + Z'_{1eff}\tan h(\frac{\pi}{2}\tan\phi_2)}{Z'_{2\mathrm{eff}} + Z'_{1\mathrm{eff}}\tan h(\frac{\pi}{2}\tan\phi_1)\tan h(\frac{\pi}{2}\tan\phi_2)} \quad (51)$$

The dependence of the values of the electromagnetic parameters of the layer 2 on those of the layer 1 is:

$$\mu'_2 = \frac{-\pi c}{2\omega d_2\cos\alpha_1}\sqrt{\frac{\mu'_1}{\varepsilon'_1}} \times \frac{\tan h(\frac{\pi}{2}\tan\phi_2)\left[\tan h(\frac{\pi}{2}\tan\phi_1)\frac{\cos\alpha_1}{\cos\theta} + \sqrt{\frac{\mu'_1}{\varepsilon'_1}}\right]}{\sqrt{\frac{\mu'_1}{\varepsilon'_1}}\tan h(\frac{\pi}{2}\tan\phi_1) + \frac{\cos\alpha_1}{\cos\theta}} \quad (52)$$

$$\mu''_2 = \mu'_2\tan\phi_2 \quad (53)$$

$$\varepsilon'_2 = \frac{-\pi c\cos\alpha_1}{2\omega d_2\cos^2\alpha_2}\sqrt{\frac{\varepsilon'_1}{\mu'_1}} \times \frac{\sqrt{\frac{\mu'_1}{\varepsilon'_1}}\tan h(\frac{\pi}{2}\tan\phi_1) + \frac{\cos\alpha_1}{\cos\theta}}{\tan h(\frac{\pi}{2}\tan\phi_2)\left[\frac{\cos\alpha_1}{\cos\theta}\tan h(\frac{\pi}{2}\tan\phi_1) + \sqrt{\frac{\mu'_1}{\varepsilon'_1}}\right]} \quad (54)$$

$$\varepsilon''_2 = \varepsilon'_2\tan\phi_2 \quad (55)$$

## 4. A design example

We take a RHM/LHM double-layer wave-absorber's design as an example to demonstrate the feasible of the principles and methods reported in this paper. The real and imaginary part of the dielectric permittivity of the RHM (poly) ranges from 7 to 25 and from 2 to 35 at 10 GHz, respectively, which are taken from Ref. [20] and assumed that the values are linearly distributed with frequency, because no measured data or curves of them were provided by the reference. To simplify the design calculations, it is assumed that the real part of the relative permeability of the RHM is Re[$\mu_{r1}$] = 1, and the imaginary part of it is Im[$\mu_{r1}$] = Re[$\mu_1$]·tan$\varphi_1$, where $\tan\phi_1 = \varepsilon''_{r1}/\varepsilon'_{r1}$, so that the condition expressed by Eq. (14) is satisfied. Employing the constitutive parameters of the RHM we obtain the refractive index of the 1st layer. Supposing the refractive index of the LHM is just the negative value of that of the RHM layer, which is feasible, the refractive angle $\alpha$ can be calculated using Snell's low. It is also supposed that the thickness of the RHM is just a quarter of the wavelength inside the RHM at 10 GHz according to Eq. (16) but with + sign for the values of the $l$. To determine the parameters of the LHM layer, Eqs. (35)–(38) are used. Then exploiting the relationship between the structure parameters of the cell of LHM and the permittivity and the permeability, one can design and work out suitable LHM layer [21,22]. After this, employing Eq. (51), the values of $Z_{\lambda 1\mathrm{eff}}$ are obtained. Then exploiting Eq. (9), the amplitude reflective coefficient is work out. We define the reflection loss as RdB = −20log(|R|). Here, the absolute value of R is adopted for convenience, because the R has different signs for TE wave and TM wave, respectively. It should be noted that all these parameters are functions of frequency. The calculated reflection loss with 30° incidence for the frequency varying from 4 GHz to 18 GHz is shown in Fig. 3, from which one can see the reflection loss is very large. It should be pointed out that this phenomenon is impossible for practical cases because that it is very difficult to make the thicknesses of the layers and the constitutive parameters of the RHM and LHM just be the values satisfying the conditions making $R = 0$. However, these values may be used as references to practically available ones and the further optimization is suggested.

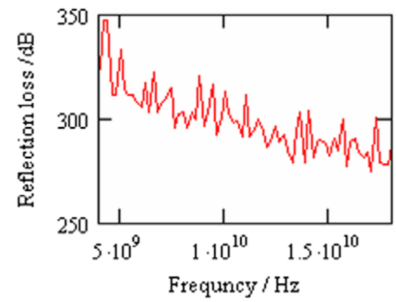


**Fig. 3.** The calculated curve of reflection loss vs. frequency for the design example.

## 5. Conclusions and discussion

The special electromagnetic properties of LHMs result in their promising applications in microwave absorbers. To broaden the design concept of wave-absorbers, we have established the physical models of LHM single-layer absorbers and double-layer absorbers containing LHMs. Their design principles have been presented, some useful formulae have been given, and a design example is demonstrated in this paper.

From the viewpoint of the properties of the absorbers discussed above, the four absorbers will have the similar performances, because all of them are designed based on the same standard, i.e. $R = 0$. However, considering the complexity of the design and practical manufacturing of LHMs, the single-layer LHM absorber is the simplest one, the combinations of LHM and RHM are simpler, and the LHM/LHM design is more complicated.

The authors hope that the work reported here may provide some references to the researchers and engineers working in wave-absorbers research and development area.

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