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FGP Approach for Solving Fractional Multiobjective Decision Making Problems using GA with Tournament Selection and Arithmetic Crossover

Debjani Chakraborti[a]*, Papun Biswas[b], Bijay Baran Pal[c]

[a]*Department of Mathematics, Narula Institute of Technology, Kolkata-700109, W.B., India*
[b]*Department of Electrical Engineering, JIS College of Engineering, Kalyani-741235, W.B, India*
[c]*Department of Mathematics, University of Kalyani, Kalyani-741235, West Bengal, India*

Abstract

This article presents an effective genetic algorithm (GA) based fuzzy goal programming (FGP) for modelling and solving multiobjective decision making (MODM) problems with fractional criteria. In the proposed approach, GA, inspired by the natural selection and population genetics, is introduced first for searching of solutions at different stages and thereby solving the problem. In the proposed GA scheme, tournament selection scheme, arithmetic crossover and uniform mutation are adopted to search a satisfactory solution in complex decision making environment. To illustrate the potential use of the approach, a numerical example is solved and compared with the solutions obtained in previous study.

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* Corresponding author. Tel.: +919432137013
E-mail address: debjani_333@yahoo.co.in.

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1. Introduction

Fractional programming, introduced by Charnes and Cooper [1], as a special field of study in the area of non-linear programming has been studied extensively by Bitran and Novaes [2] , Craven [3] , and others in the past. Also considering the multiobjective nature of most of the real-life decision problems, fractional programming with multiplicity of objectives has been studied by Kornbluth and Steuer [4] and other pioneer researchers in the field. In most of the approaches developed so far in the past, the linearization method discussed by Charnes and Cooper [1] has been extended to solve fractional programming problems.

In contrast to single objective fractional programming problems, multiobjective fractional programming (MOFP) problems are yet to widely circulate in the literature.

The goal programming (GP) approaches [5], [6], as prominent tools for solving MODM problems, have been studied [7] for decision analysis with fractional objectives in crisp decision making environment.

However, in most of the real-life multiobjective decision situation, decision makers (DMs) often faced the problem of setting exact aspiration levels to their objectives due to the imprecise nature of model parameters involved with the practical problems. To overcome such a situation, the fuzzy set theory (FST) [8], [9], has been used to decision making problems which involve imprecise data.

Now, the linear approximation approaches [7] are conventionally used to single objective as well as multiobjective decision problems with fractional objectives. But, a great computational difficulty arises in the solution process due to the big data type of the problems and as such inherent approximation errors occur in the decision search process of the MODM problems.

To overcome such a situation in a decision making environment, GA [10], [11], a bio-inspired computational technique, have appeared as prominent tool for multiobjective decision analysis. The GA approaches to different real-world problems have been investigated in the past. The uses of GAs to different frameworks of several problems as well as implementation to real-life problems with fractional criteria have been studied by Pal et al. [12], [13] in the past. But, exploration to the potential use of GA to MODM problems is at an early stage. Furthermore, the methodological development of GA based approaches to general MOFP problems is yet to be circulated in the literature.

This article demonstrates the efficient application of the GA method to the general framework of FGP formulation of an MOFP problem. In the proposed model, first the fractional objectives are transformed into fuzzy goals by assigning fuzzy aspiration levels to each of them using the GA scheme. Then, the membership functions for measuring the degree of achievement of fuzzy goals by defining the tolerance ranges for goal achievement are constructed. In the executable FGP model, minimization of the under-deviational variables of the defined membership goals with highest membership value (unity) as the aspiration levels of them on the basis of relative weights of importance of achieving the objective is taken into consideration. In the solution process, the GA scheme is iteratively used to achieve a balanced solution of the objectives in the MODM situation.

In the solution search process, the optimal decision of the problem under the framework of weighted FGP is determined through the proposed GA scheme.

A numerical example is illustrated to expound the potential use of the proposed approach.

2. Problem Formulation

The general format of a real valued MOFP problem can be stated as:

Find $X(x_1, x_2, ..., x_n)$ so as to

$$\text{Maximize } Z_k(X), \qquad k \in K_1$$

$$\text{and Minimize } Z_k(X), \qquad k \in K_2$$

$$\text{Subject to } X \in S = \left\{ X \in R^n \mid AX \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, X \ge 0, b \in R^m \right\} \tag{1}$$

where A is a coefficient matrix and b is a resource vector. It is assumed that the feasible region S is nonempty $(S \neq \Phi)$, and $K_1 \bigcup K_2 = \{1,2,..,K\}$ with $K_1 \bigcap K_2 = \Phi$.

Now, in the field of fuzzy programming, an imprecise aspiration level is assigned to each of the objectives and certain tolerance limit for achievement of the respective aspired level is taken into account.

In the proposed problem, since the objectives are fractional in form, an GA scheme is introduced in the solution search process for assigning the fuzzy aspiration level and then the tolerance limit to each of them.

The GA scheme used in the process of solving the problem is presented in the following Section 3.

3. GA Scheme for MOFP Problems

In the literature of GAs, there is a variety of schemes [10,11] for generating new population with the use of different operators: selection, crossover and mutation. In the present GA scheme, real-valued representation of candidate solutions is considered in the evaluation process of the problem. The tournament selection scheme in [10], arithmetic crossover [11] and uniform mutation operations are adopted to generate offspring in new population in search domain defined in the decision making environment.

The basic steps of the GA scheme adopted in the solution search process are presented in the following algorithmic steps.

Step 1. Representation and initialization

In the present GA scheme, real-valued representation of candidate solutions, i.e., real coded chromosomes are considered in the evaluation process of the problem. Let '*E*' denote the chromosome in a population. The population size is defined by *pop_size*. The initial population is generated randomly in the domain of feasible set defined in (1).

Step 2. Fitness function

The fitness value of each chromosome is determined by evaluating an objective function. The fitness function is defined as:

$$\text{eval}\,(E_v) = (Z)_v = \sum_{k=1}^{K} \{w_k^- d_k^-\}_v, \quad v = 1,2,..., pop-size, \tag{2}$$

where $(Z)_v$ represents the objective function of the DM, and where the subscript v is used to indicate the fitness value of the v-th chromosome, $v = 1,2,...,$ *pop_size*.

The best chromosome with largest fitness value at each generation is determined as:

$$E^* = \max\{\text{eval}(E_v) \,|\, v = 1,2,..., pop\_size\}$$

$$\text{or, } E^* = \min\{\text{eval}(E_v) \,|\, v = 1,2,..., pop\_size\},$$

which depends on searching of the maximum or minimum value of an objective function.

Step 3. Selection

Selection is a method of selecting an individual from a population of individuals in a GA search process. In tournament selection [10] several "tournaments" running among the few individuals chosen at random from the population. The best cromosome of each tournament is selected for crossover. The efficiency of this scheme depends on the size of the tournament. With the increase of tournament size, weak individuals have a smaller chance to be selected. In the present scheme, the tournament selection with tournament size four is used for the selection of two parents for mating purpose in the genetic search process.

Step 4. Crossover

The parameter p_c is defined as the probability of crossover. The arithmetic crossover operation in [14] of a genetic system is applied here from the view point that the resulting offspring always satisfy the system constraints set *S*. Here, a chromosome is selected as a parent for a defined random number $r \in [0,1]$, if $r < p_C$ is satisfied.

The arithmetic crossover for the two parents E_1 and $E_2 \in S$ is defined as:

$$E_1^1 = \alpha_1 E_1 + \alpha_2 E_2, \; E_1^2 = \alpha_2 E_1 + \alpha_1 E_2,$$

for producing two offspring E_1^1 and E_1^2 (E_1^1 *and* $E_1^2 \in S$), where $\alpha_1, \alpha_2 \geq 0$ with $\alpha_1 + \alpha_2 = 1$.

Step 5. Mutation

Mutation operation is applied over the population after performing crossover operation. It alters one or more genes of a selected chromosome to re-introduce the genetic material. As in the conventional GA scheme, a parameter p_m of the genetic system is defined as the probability of mutation. The uniform mutation operation is performed.

Step 6. Termination

The execution of the whole process terminates when the objective function reach with in a specific tolerance range in the solution search process.

The pseudo code of the standard genetic algorithm is presented as:

Initialize population of chromosomes $E(x)$

Evaluate the initialized population by computing its fitness measure

While not termination criteria **do**

$x := x + 1$

Select $E(x+1)$ from $E(x)$

Crossover $E(x+1)$

Mutate $E(x+1)$

Evaluate $E(x+1)$

End While

Now, the model formulation of the problem is described in the Section 4.

4. FGP Model Formulation

In the present decision situation, the individual best solution of each of the objectives is considered as the fuzzy aspiration levels of the objectives and they are determined by employing the proposed GA scheme.

Let, $Z^*_{B_{1k}}$ and $Z^*_{B_{2k}}$ be the best solutions of the two types of objectives (max and min), respectively,

$$\text{where} \quad Z^*_{B_{1k}} = \underset{X \in S}{\text{Max}}\, Z_k(X), \qquad k \in K_1 \tag{3}$$

$$\text{and} \quad Z^*_{B_{2k}} = \underset{X \in S}{\text{Min}}\, Z_k(X), \qquad k \in K_2 \tag{4}$$

Then, the fuzzy objective goals can be obtained as:

$$Z_k(X) \gtrsim Z^*_{B_{1k}}, \qquad k \in K_1 \tag{5}$$

$$\text{and} \quad Z_k(X) \lesssim Z^*_{B_{2k}}, \qquad k \in K_2 \tag{6}$$

where $\gtrsim$ and $\lesssim$ refers to the fuzziness of the aspiration levels in [7].

Now, in the multiobjective decision situation, since the objectives often conflict each other for individual goal achievement, a certain tolerance level for goal achievement need be given to make an overall satisfactory decision under the given system constraints in the decision making context.

To make a reasonable balance of goal achievement, the individual worst objective function values are considered as the lower tolerance limit of the objective goals.

Let, $Z_{L_{1k}}$ and $Z_{L_{2k}}$ be the worst objective function values of the respective objectives, where

$$Z_{L_{1k}} = \underset{X \in S}{\text{Min}}\ Z_k(X), \qquad k \in K_1$$

$$\text{and} \qquad Z_{L_{2k}} = \underset{X \in S}{\text{Max}}\ Z_k(X), \qquad k \in K_2 \tag{7}$$

Then, characterization of membership functions for goal achievement of the objectives within the tolerance ranges specified in the decision situation is presented in the following Section 4.1.

4.1. Characterization of membership function

Let $\mu_k(X)$ be the membership function representation of the k-th fuzzy goal.

Then, for $\gtrsim$ type of restriction, $\mu_k(X)$ takes the form:

$$\mu_k(X) = \begin{cases} 1 & , \quad \text{if } Z_k(X) \geq Z^*_{B_{1k}} \\ \dfrac{Z_k(X) - Z_{L_{1K}}}{t_{1k}} & , \quad \text{if } Z_{L_{1k}} \leq Z_k(X) < Z^*_{B_{1k}} \\ 0 & , \quad \text{if } Z_k(X) < Z_{L_{1k}} \end{cases} \tag{8}$$

where, $t_{1k} = (Z^*_{B_{1k}} - Z_{L_{1K}})$ is the tolerance range for achievement of the k-th fuzzy goal, $k \in K_1$.

Similarly, for $\lesssim$ type of restriction, $\mu_k(X)$ appear as:

$$\mu_k(X) = \begin{cases} 1 & , \quad \text{if } Z_k(X) \leq Z^*_{B_{2k}} \\ \dfrac{Z_{L_{2K}} - Z_k(X)}{t_{2k}} & , \quad \text{if } Z^*_{B_{2k}} < Z_k(X) \leq Z_{L_{2k}} \\ 0 & , \quad \text{if } Z_k(X) > Z_{L_{2k}} \end{cases} \tag{9}$$

where, $t_{2k} = (Z_{L_{2k}} - Z^*_{B_{2k}})$ is the tolerance range for achievement of the k-th fuzzy goal, $k \in K_2$.

Now, the FGP model formulation of the problem for the defined membership functions is presented in the following Section 4.2.

4.2. Minsum FGP model

In the process of formulating FGP model of the problem, the membership functions are transformed into membership goals by assigning the highest membership value (unity) as the aspiration level and introducing under- and over-deviational variables to each of them. In *minsum* FGP, minimization of the sum of weighted under-deviational variables of the membership goals in the goal achievement function on the basis of relative weights of importance of achieving the aspired goal levels is considered.

The general *minsum* FGP model can be presented as [7]:

Find $X(x_1, x_2, ..., x_n)$ so as to

$$\text{Minimize } Z = \sum_{k=1}^{K} w_k^- d_k^-$$

$$\text{and satisfy} \quad \frac{Z_k(X) - Z_{L_{1K}}}{t_{1k}} + d_k^- - d_k^+ = 1$$

$$\frac{Z_{L_{2K}} - Z_k(X)}{t_{2k}} + d_k^- - d_k^+ = 1$$

$$d_k^-, d_k^+ \geq 0, \quad k=1,2......K$$

subject to the given system constraints in (1),

(10)

Here, d_k^-, d_k^+ are the under- and over-deviational variables, respectively, of the *k*-th membership goal, Z represents the goal achievement functions consisting the weighted under-deviational variables, where the numerical weights w_k^- represent the relative weights of importance of achieving the goals to their aspired levels, and they are determined as:

$$w_k^- = \frac{1}{t_{1k}}, \text{ defined for the goal expression in (5),}$$

$$w_k^- = \frac{1}{t_{2k}}, \text{ defined for the goal expression in (6).}$$

The efficient use of the proposed approach is illustrated by a numerical example.

5. Numerical Example

The following fractional MODM problem is considered:

Find $X(x_1, x_2)$ so as to:

$$\text{Minimize} \quad Z_1 = \frac{12x_1 - 10.95x_2 - 19.05}{x_1 - 2x_2 + 1},$$

$$\text{Minimize} \quad Z_2 = \frac{5x_1 + 6x_2 + 4}{x_1 + 2x_2},$$

$$\text{Maximize} \quad Z_3 = \frac{8x_1 + 5.9x_2}{x_1 - 2x_2 + 2},$$

$$\text{Maximize} \quad Z_4 = \frac{12x_1 - x_2 + 2}{x_1 + 1},$$

Subject to

$$x_1 + 2x_2 \leq 12,$$
$$x_1 \leq 9,$$
$$x_2 \leq 6,$$
$$x_1, x_2 \geq 0.$$

(11)

The GA is implemented using the Optimization Toolbox under MATLAB (MATLAB R 2010a) is employed at different stages for evaluation of the problem. The execution is made in Intel Pentium IV with 2.66 GHz. Clock-pulse and 3 GB RAM. The parameter values used in genetic algorithm solution are given in Table 1.

Table 1. The parameter values used in GA

Parameter	Value
Selection	Tournament
Crossover function	Arithmetic
Tournament size	4
Crossover probability	0.8
Mutation Probabiliy	0.06
Stopping criteria	Constraint tolerance

Then, following the procedure, the individual best and worst values of the successive objectives are obtained as:

(i) $Z^{*}_{B_{21}} = 8.5608$, $Z_{L_{21}} = 10.3607$

(ii) $Z^{*}_{B_{22}} = 4.833$, $Z_{L_{22}} = 5.4962$

(iii) $Z^{*}_{B_{13}} = 10.1062$, $Z_{L_{13}} = 6.4108$

(iv) $Z^{*}_{B_{14}} = 11.2308$, $Z_{L_{14}} = 10.7882$

Then, the fuzzy objective goals appear as:

$$Z_1 : \frac{12x_1 - 10.95x_2 - 19.05}{x_1 - 2x_2 + 1} \lesssim 8.5608$$

$$Z_2 : \frac{5x_1 + 6x_2 + 4}{x_1 + 2x_2} \lesssim 4.833$$

$$Z_3 : \frac{8x_1 + 5.9x_2}{x_1 - 2x_2 + 2} \gtrsim 10.1062$$

$$Z_4 : \frac{12x_1 - x_2 + 2}{x_1 + 1} \gtrsim 11.2308$$

Now, defining the tolerance limits for the worst values of the objectives and then following the procedure, the membership functions of the fuzzy objectives are successively obtained as:

$$\mu_{Z_1} = \frac{10.3607 - \dfrac{12x_1 - 10.95x_2 - 19.05}{x_1 - 2x_2 + 1}}{1.8},$$

$$\mu_{Z_2} = \frac{5.4962 - \dfrac{5x_1 + 6x_2 + 4}{x_1 + 2x_2}}{0.6632},$$

$$\mu_{Z_3} = \frac{\dfrac{8x_1 + 5.9x_2}{x_1 - 2x_2 + 2} - 6.4108}{3.7},$$

$$\mu_{Z_4} = \frac{\dfrac{12x_1 - x_2 + 2}{x_1 + 1} - 10.7882}{0.4426}. \tag{12}$$

Then the resultant *minsum* FGP model appears as:

Find (x_1, x_2) so as to

Minimize $Z = \frac{1}{1.8} d_1^- + \frac{1}{0.6632} d_2^- + \frac{1}{3.7} d_3^- + \frac{1}{0.4426} d_4^-$

and satisfy the membership goal constraints

$$\frac{10.3706-\dfrac{12x_1-10.95x_2-19.05}{x_1-2x_2+1}}{1.8}+d_1^- - d_1^+ =1\,,$$

$$\frac{5.4962-\dfrac{5x_1+6x_2+4}{x_1+2x_2}}{0.6632}+d_2^- - d_2^+ =1\,,$$

$$\frac{\dfrac{8x_1+5.9x_2}{x_1-2x_2+2}-6.4108}{3.7}+d_3^- - d_3^+ =1\,,$$

$$\frac{\dfrac{12x_1-x_2+2}{x_1+1}-10.7882}{0.4426}+d_4^- - d_4^+ =1\,.$$

subject to the given system constraints in (11)

(13)

The goal achievement function Z in (13) appears as the evaluation function in the GA search process of solving the problem.
The evaluation function to determine the fitness of a chromosome appears as:

$$\text{eval}\,(E_v)=(Z)_v=\sum_{k=1}^{4}\{w_k^- d_k^-\}_v,\quad v=1,2,\ldots,pop-size, \tag{14}$$

where $(Z)_v$ is used to represent the achievement function Z in (13) for measuring the fitness value of v-th chromosome in the decision process.

The best objective value (Z^*) for the fittest chromosome at a generation in the solution search process is determined as:

$$E^*=\min\{\text{eval}(E_v)\,|\,v=1,2,\ldots,pop\_size\}\,, \tag{15}$$

The achieved values of the objectives are (Z_1, Z_2, Z_3, Z_4) = (9.7493, 4.999, 8.6557, 10.8997),
with respective membership values are (0.3451, 0.7481, 0.6067, 0.2520).
The plotting of the best individual from MATLAB Optimization Toolbox in presented in Fig. 1.

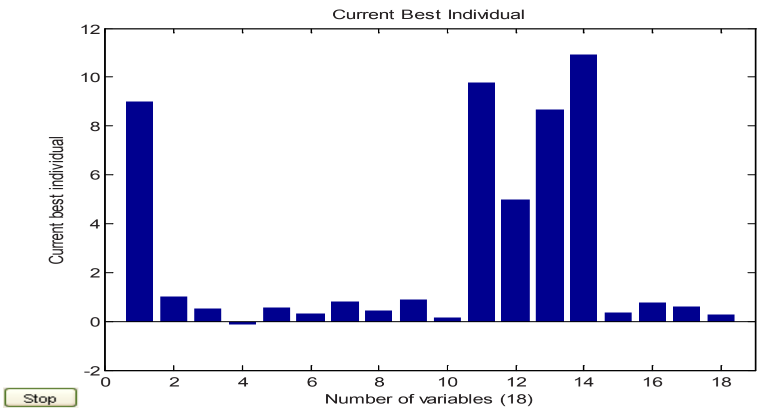

Fig. 1. Plotting of best individual from MATLAB optimization toolbox

Fig. 1 representing the output plot of best individual from optimization toolbox. First two bars in fig 1 are the decision variables, x_1 and x_2, respectively and last eight bars are objective values (Z_1, Z_2, Z_3, Z_4) and the achieved membership values $(\mu_{Z_1}, \mu_{Z_2}, \mu_{Z_3}, \mu_{Z_4})$ of the problem.

The membership functions obtained by pre-emptive priority structure in [15] are appearing as: (0.06, 0.96, 0.92, 0.15).

The graphical representation of membership values of the objectives achieved under the proposed method and priority-base approach is displayed in Figure 2.

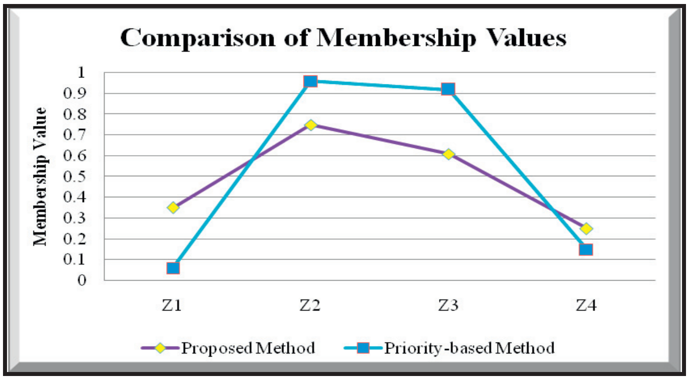

Fig. 2. Comparison of the achieved membership values of the objectives

The result indicates that a better distribution of decision is obtained here from the viewpoint of achieving the aspired goal values on the basis of the needs and desires of the DM in the decision making environment.

***Note 1*:** GA parameter values in Table 1are adopted from the test results. Here, the parameter setting is particularly adopted to avoid any early convergence with suboptimal decision and substantial increase in generation numbers in the decision making horizon. The experiments with the different values of probability of crossover (p_c) and probability of mutation (p_m) in the ranges $(0.6 \le p_c \le 0.9)$ and $(0.03 \le p_m \le 0.8)$ are made with the proposed GA scheme. It is found that $p_c = 0.8$ and $p_m = 0.06$ are most successful in the present decision search process. Again, the use of arithmetic crossover increase the efficiency of the GA as there is no need to convert chromosome to the binary, less memory and time required for the execution of the problem. However, it is worthy to mention here that the selection of GA parameter values highly depends on the characteristics as well as size of a problem in the decision making environment.

6. Conclusions

The main advantage of the GA solution approach to the *minsum* FGP model of the problem is that the higher degree of satisfaction of the solution can be searched on the basis of the needs and desires of the DMs in the decision making environment. Again, the computational load with the use of conventional linearization technique can be avoided here with the use of the proposed GA scheme. The approach can easily be extended to complex real-life MODM problems with quadratic as well as general non-linear form of objectives in the decision making horizon. In future study, the proposed approach can be extended to solve large scale hierarchical decision making problems in an uncertain environment. However, it is hoped that the proposed approach may open up many new areas for study in the current inexact MODM arena.

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