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Heuristic method for reactive energy management in distribution feeders

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ABSTRACT

This paper proposes an efficient method for solving shunt capacitors sizing problem in radial distribution feeders. For this typical multi-objective optimisation problem, the optimal number, sizes and locations of fixed shunt capacitors are determined using a sensitivity-based heuristic solution. In this method, based on Markov chains, the number of variables to be optimised is reduced by using the node sensitivities to locate capacitors. Only a limited number of critical nodes satisfy the problem constraints and are considered for receiving standard shunt capacitors that maximise a net saving function. In order to overcome any over-compensation, the voltage admissible limits, imposed by many authors, are substituted by a new constraint on the branch reactive currents. To demonstrate the effectiveness and feasibility of the proposed approach, comparative studies were conducted on several test systems. The results we got were promising compared to those given by previous published techniques.

1. Introduction

Electrical power losses in distribution systems are considerable. They correspond to about 70% of total losses in electric power systems [1]. To minimise this power losses, increase feeder power transmission and improve power factor as well as voltage profile, shunt capacitors are widely used. However, the scope of these benefits greatly depends on how the capacitors are installed and dispatched in distribution feeders. This problem, called general capacitor scheduling, consists in determining the capacitors number, sizes and locations so that the objective function defined for the problem is maximised.

Many techniques have been used to solve the capacitors sizing and locating problem. The early used solutions, in which no assumptions are made, are analytical [2–5]. Developed by Grainger et al. [2–4], in these solutions, the authors have introduced the uniform-normalised feeder concept. Nevertheless, the problem has been considered as an unconstrained optimisation problem with continuous control variables. Come then, numerical methods, heuristic methods and artificial intelligence-based methods among of which we cite genetic algorithms, simulated annealing, fuzzy logic and neural networks.

In this work we focalise on the heuristic methods [6–10]. The goal of developing heuristic methods is the reduction of the search space and the number of variables while keeping the final solution to a near optimal solution. These techniques were first introduced by Civanlar et al. [6] for estimating the power losses change following the reconfiguration of distribution feeders. Taylor and Lubkeman [7] have subsequently applied the heuristic reconfiguration method for the removal of transformer overloads and feeder constraint problems. Salama and Chikhani [8] have adapted the method for the capacitors placement problem and have considered the system power loss reduction as objective function. In this method, the sensitive node at which optimal capacitor size should be placed is that whose load reactive current has the highest impact on the branch having the largest power losses. This process is repeated for the next candidate node until no further power loss reduction can be achieved. However, the nodes having the highest impact on the feeder branches with the largest losses are not necessary those having the highest effect on the power losses of the whole system. Chis et al. [9], in their heuristic method for solving the capacitors sizing and locating problem, have considered as an objective function, the savings function. According to Chis et al. [9], the sensitive nodes are selected based on the system power losses caused by each load reactive current. The node whose load reactive current has the largest impact on the feeder power losses is selected for receiving an optimal capacitor to be determined. Nevertheless, the authors have approximated the piecewise linear variation of the capacitor cost by a continuous function and have considered voltage constraint ratios equal to ±3% for urban areas and ±6% for rural ones. In the method presented by Mekhamer et al. [10], for each load node, an optimal capacitor size is calculated. The most sensitive node is that providing the largest cost reduction while placing on it the calculated optimal capacitor. As in Ref. [9], the authors in [10] have considered a voltage constraint of ±5% in addition to that made for the power loss and cost reductions. But, small voltage permissible limits, like that applied by

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authors in Refs. [9] and [10], could be not satisfied and thus, no solution is possible for the capacitor sizing problem. In order to overcome this situation, in the proposed heuristic method, the voltage constraint is substituted by a new one where the solution is guaranteed. This heuristic technique leads to a near global solution and require a small number of standard capacitors to achieve a large power loss reduction with optimum savings.

2. Formulation of the problem

The branch reactive currents transit limitation is formulated as a problem of maximisation of the defined objective function. It consists in determining shunt capacitor banks optimal number, sizes and locations so that we achieve a maximal net annual dollars saving. This objective should be met subject to some constraints. It may be stated as it follows:

$$\begin{cases} \max F(X,U) \quad \textit{subject to}: \\ g(X,U) = 0 \\ X_{\min} \leqslant X_i \leqslant X_{\max} \\ U_{\min} \leqslant U_i \leqslant U_{\max} \end{cases} \tag{1}$$

where, "F" is the objective function to be maximised and "g" is the load flow equation. "X" and "U" are, respectively, the control and state variables which are both limited by their minimum and maximum values.

To reach the fixed goal and reduce the number of control variables, a heuristic search method is proposed. In this method, the proper capacitor locations are selected based on the sensitivity of each node of the feeder. The sensitivity of a node depends on the effect of its load reactive current on the whole feeder losses. The one that has the largest effect on the system losses is called the most sensitive node and is selected first to receive a near optimal standard capacitor. The capacitor optimal sizes are determined while solving the derivative of the objective function.

2.1. Selection of the sensitive node

For a distribution feeder having "n" branches, the total active power losses are given by:

$$P^b_{T_{loss}} = \sum_{k=1}^{n} r_k F^2_{d_k} + \sum_{k=1}^{n} r_k F^2_{q_k} \tag{2}$$

where r_k is the resistance of the branch k, F_{d_k} and F_{q_k} are, respectively, the active and reactive components of kth branch current determined when performing the load flow.

To find the effect on the system power losses of the load reactive currents, we put each of them equal to zero. The power losses, without the reactive current component "$I_{q_{L_j}}$" of the load at node j, can be written as it follows:

$$P^a_{T_{loss}} = \sum_{k=1}^{n} r_k F^2_{d_k} + \sum_{k=1}^{j} r_k (F_{q_k} - I_{q_{L_j}})^2 + \sum_{k=j+1}^{n} r_k F^2_{q_k} \tag{3}$$

Subtracting (3) from (2), we obtain the system power loss reduction due to the jth load reactive current:

$$\Delta P_j = 2I_{q_{L_j}} \sum_{k=1}^{j} r_k F_{q_k} - I^2_{q_{L_j}} \sum_{k=1}^{j} r_k \tag{4}$$

The node whose load reactive current has the largest ΔP_j is denoted as the most sensitive. It is selected first for receiving a near optimal standard capacitor size.

2.2. Capacitors sizing

The capacitor sizes should be such that the objective function is maximised. The capacitors being installed one by one then, the objective function becomes markovian. This means that the objective function at the state "l" depends only on the state "$l-1$". As objective function, we have considered the cost reduction function [10] which, for a given capacitor "k", has the following expression:

$$\Delta S_k = K_p \Delta P_k - K_{c_k} Q_{c_k} \tag{5}$$

where, K_p and K_{c_k} are, respectively, the kilowatt annual cost (\$/kW) and the kVar annual cost (\$/kVar) of the kth capacitor.

And where, the kth capacitor size (Q_{c_k}) and the corresponding power loss reduction (ΔP_k) are given by:

$$Q_{c_k} = V_{c_k} I_{cq_k} / \cos\varphi_{c_k} \tag{6}$$

$$\Delta P_k = 2I_{cq_k} \sum_{j=1}^{k} r_j F_{q_j} - I^2_{cq_k} \sum_{j=1}^{k} r_j \tag{7}$$

I_{cqk}, V_{ck} and φ_{ck} are, respectively, the current, voltage rms value and phase-angle of the capacitor "k".

Substituting Q_{c_k} and ΔP_k by their expressions (6) and (7) in (5), we obtain for the kth capacitor cost reduction (ΔS_k) the following formula:

$$\Delta S_k = 2K_p I_{cq_k} \sum_{j=1}^{k} r_j F_{q_j} - K_p I^2_{cq_k} \sum_{j=1}^{k} r_j - K_{c_k}(V_{c_k} I_{cq_k} / \cos\varphi_{c_k}) \tag{8}$$

The optimal capacitor size that maximises this cost function is obtained by solving the following equation.

$$\frac{\partial \Delta S_k}{\partial I_{cq_k}} = 0 \tag{9}$$

However, each optimal capacitor size, solution of (9), the cost reduction that achieves as well as the corresponding power loss reduction must all be positive. Moreover, these capacitors must satisfy the voltage limits constraints. Nevertheless, the latter, in spite its validity, could not be satisfied or could make the problem non-solvable if small voltage permissible limits are considered as in [9] and [10]. For this raison, the voltage constraint is substituted by a new one relating to the branch reactive currents. This new constraint relies on the fact that distribution systems compensation involves node voltages improvement which at most could exceed their upper limit. In this case, the feeder (or branch) is said over-compensated and thus, the feeder (or branch) reactive current becomes negative. To overcome any over-compensation and therefore any over-voltage, we impose a branch reactive current equal or greater than zero to the capacitor sizing problem. So, the capacitor sizing problem should satisfy the power flow equation as well as the constraints on the power loss reduction, cost reduction, reactive branch currents and available capacitor sizes. It can be mathematically stated as given below:

$$\begin{cases} \max \Delta S(X,U) \ \textit{subject to}: \\ \Delta P(X,U) \succ 0 \\ \Delta S(X,U) \succ 0 \\ g(X,U) = 0 \\ Q_{\min} \leqslant Q_{ci} \leqslant Q_{\max} \\ F_{qk}(X,U) \geqslant 0 \end{cases} \tag{10}$$

From (9), we get the optimal capacitor current:

$$I_{cqk} = \frac{2k_p \sum_{i=1}^{k} r_i F_{qi} - (k_{ck} V_{ck} / \cos\varphi_{ck})}{2k_p \sum_{i=1}^{k} r_i} \tag{11}$$

The corresponding optimal capacitor size is calculated using expression (6). The maximum cost reduction and the corresponding power loss reduction are given by:

$$\Delta S_{k\max} = \frac{\left[2k_p\sum_{i=1}^{k} r_i F_{qi} - (k_{ck}V_{ck}/\cos\varphi_{ck})\right]^2}{4k_p\sum_{i=1}^{k} r_i} \tag{12}$$

$$\Delta P_{\Delta S_{k\max}} = \frac{4k_p^2\left(\sum_{i=1}^{k} r_i F_{qi}\right)^2 - (k_{ck}V_{ck}/\cos\varphi_{ck})^2}{4k_p^2\sum_{i=1}^{k} r_i} \tag{13}$$

Eq. (12) shows that $\Delta S_{k\max}$ is always positive whereas $\Delta P_{\Delta S_{\max}}$ (13) can be positive or negative. Then, the constraint on the cost reduction in (10) can be removed making the problem subject to only four constraints.

2.3. Proposed algorithm

To solve the capacitor sizing and locating constrained problem, the proposed algorithm follows the steps given below:

```
Step 1: Read the data of the feeder.
Step 2: Perform the load flow program for the uncompensated
    feeder to determine, the power losses, the branch currents
    components as well as the node voltage rms values and phase-
    angles.
Step 3: Initialise the power loss reduction.
Step 4: While the power loss reduction is positive:
  Step 4.1: Determine the node sensibilities according to (4) and
    rank them in a decreasing order.
  Step 4.2: Consider the node having the smallest range, discard it
    if it has already been considered and consider the next node.
  Step 4.3: Calculate for this node, the capacitor optimal size
    according to (11) and (6) assuming as initial capacitor cost, the
    average of the available standard capacitor costs of the studied
    feeder.
  Step 4.4: Calculate for this optimal capacitor the corresponding
    cost and power loss reductions according to (12) and (13).
  Step 4.5: Perform the load flow and update the branch currents
    components, the node voltage rms values and phase-angles.
  Step 4.6: Adjust the capacitor optimal size following the voltage
    changes.
  Step 4.7: If this capacitor is negative, smaller than the smallest
    standard capacitor or greater than the greatest standard one or,
    if the corresponding power loss reduction is negative then:
      Step 4.7.1: Remove this capacitor and set its value equal to
    zero.
      Step 4.7.2: Give to the branch active and reactive currents and
    node voltages their values before this capacitor.
  Step 4.8: Else, take as optimal capacitor size the standard one
    just smaller or greater according to the value of their cost
    reduction (the one having the greatest cost reduction is
    considered).
  Step 4.9: Perform again the load flow to update the branch active
    and reactive currents as well as the node voltage rms values and
    phase-angles.
  Step 4.10: Calculate again the power loss and cost reductions
    according to exact capacitor cost.
  Step 4.11: If the obtained standard capacitor produces any over-
    compensation ($F_q \prec 0$) then:
      Step 4.11.1: Replace it by a smaller standard one which does
    not produce over-compensation.
      Step 4.11.2: Check if the determined capacitor is not smaller
    then the smallest standard one.
      Step 4.11.3: Perform the load flow and update the values of
    the power loss and cost reductions.
      Step 4.11.4: Check if the just calculated power loss reduction
    is positive.
  Step 4.12: End if.
  Step 4.13: End if.
  Step 4.14: Go to Step.4.1
Step 5: End while.
Step 6: Write the results.
```

3. Simulation results and discussion

The presented heuristic approach for reactive power optimisation has been applied to several practical test feeders. Only the results of two of these test systems are given here. It is about a radial distribution feeder having nine buses and a ramified distribution feeder with 69 buses.

3.1. First feeder

The first test feeder, whose data are given in reference [4], is a 23 kV non-uniform radial line having nine buses. Its one line diagram is shown in Fig. 1. For this feeder example, the annual kW cost k_p is taken equal to168 $/kW. The commercially available capacitors are shown in Table 1 [10]. If we consider that the maximum capacitor size should not exceed the total reactive load (i.e., 4186 kVar), the possible number of standard capacitors to be used in our compensation scheme is of 27. Their sizes and costs in dollars by kVar for a life expectancy of ten years are given in Table 2.

The load flow equation to be balanced every time in the capacitor sizing problem and considering distribution feeders configuration then, to solve this problem a backward and forward sweeps method given in [11] has been used. The voltage magnitudes base value is equal to that of source node (23 kV) which is also considered as reference. The powers base value is of 4186 kVA.

Without capacitors, the implementation of the proposed load flow solution has given total power losses of 845.41 kW where 761.22 kW are due to branch active currents and 84.19 kW to branch reactive currents. The branch active and reactive current distributions before compensation are shown in Fig. 2. In per-unit, maximum and minimum voltages are, respectively, of 0.832982 and 0.992875.

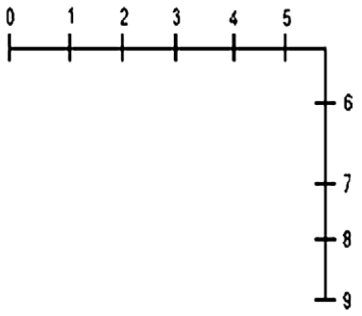

Fig. 1. One line diagram of the 9-nodes feeder.

Table 1
Available capacitors and costs.

Size (kVar)	150	300	450	600	900	1200
Cost ($)	750	975	1140	1320	1650	2040

Table 2
Possible standard capacitors and cost; 9-nodes feeder.

Q_c (kVar)	150	300	450	600	750	900	1050
$/kVar	0.500	0.350	0.253	0.220	0.276	0.183	0.228
Q_c (kVar)	1200	1350	1500	1650	1800	1950	2100
$/kVar	0.170	0.207	0.201	0.193	0.187	0.211	0.176
Q_c (kVar)	2250	2400	2550	2700	2850	3000	3150
$/kVar	0.197	0.170	0.189	0.187	0.183	0.180	0.195
Q_c (kVar)	3300	3450	3600	3750	3900	4050	
$/kVar	0.174	0.188	0.170	0.183	0.182	0.179	

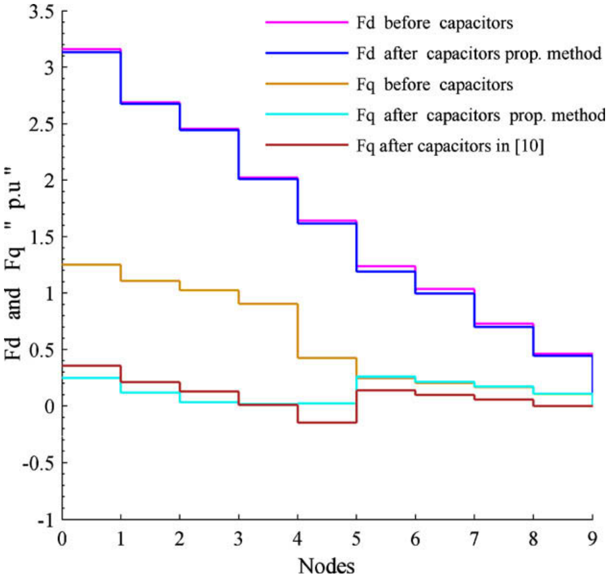

Fig. 2. Branch current distributions before and after capacitors: active (F_d) and reactive (F_q) components.

The proposed technique for determining optimal capacitor sizes and locations has given, three capacitors of ratings 1650 kVar, 1950 kVar and 450 kVar located, respectively, at nodes 5, 4 and 3 (Fig. 3). As shown in Fig. 4, these optimal capacitors lead to a power loss reduction of 11.80% (99.75 kW) on the total power losses (845.41 kW). The latter is the result of the depreciation of both branch reactive currents (F_q) under the capacitors effect (71.90 kW) and branch active currents (F_d) following the voltage improvement (27.85 kW). Consequently, the total cost is reduced by 16257 $ (Fig. 4) and the voltage profile is improved (Fig. 5). The voltage minimum and maximum values, which were, respectively, equal to 0.832982 and 0.992875 in p.u. (before capacitors), pass to 0.865044 and 0.996181 after the optimal compensation.

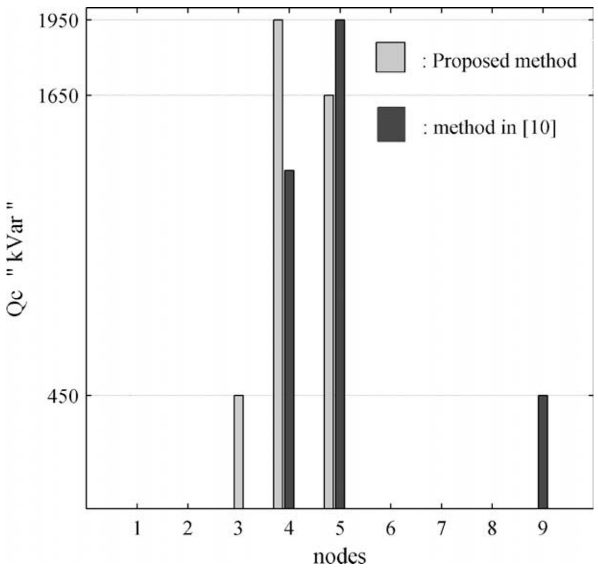

Fig. 3. Capacitors optimal sizes and locations: comparative graph.

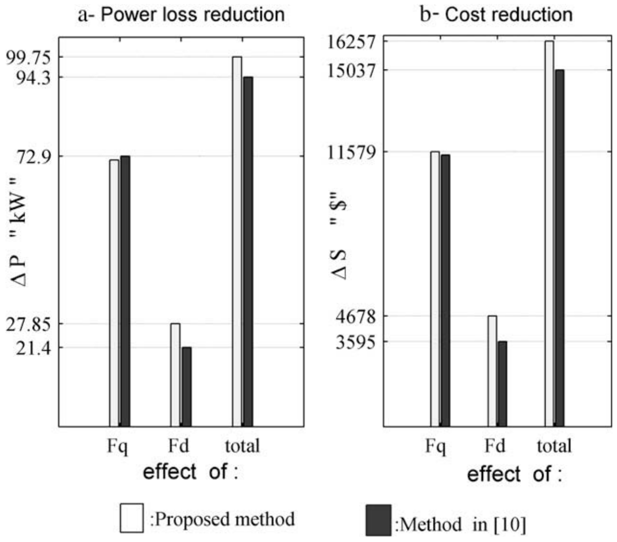

Fig. 4. Power loss and cost reductions: comparative graph.

Compared to the results given by Mekhamer in [10], the proposed method requires more kVar. Although the total of kVar is of 300 kVar greater than in [10], the power loss and cost reductions are better in our case (Fig. 4). They are, respectively, of 5.5% and 7.5% greater than those given in reference [10]. The third capacitor (i.e., 450 kVar) location is node 3 in the proposed method and node 9 in reference [10]. Applying this capacitor at node 9 causes an over-compensation in the branch located between nodes 4 and 5 where the reactive component F_q is negative (Fig. 2). In reference [10], the minimum voltage magnitude after compensation is of 0.8788 what means, that the voltage constraint of ±5% is violated. When we put all node reactive loads equal to zero, which is equivalent to a local satisfaction of reactive powers requirements, the voltage minimum rms value is equal to 0.874350 (Fig. 5). Even in reference [4] where an analytic method is used, this minimum value does not exceed 0.87320 which moreover, is a consequence of the high compensation level (4495 kVar). We can conclude from what have been mentioned above that, small voltage permissible limits cannot be satisfied.

3.2. Second feeder

The second test system is a ramified feeder having 69-nodes, bus source included (Fig. 6). The data of this feeder are given in reference [12]. As for the first feeder, k_p is equal to 168 $/kW. The base values of the voltage and power are, respectively, equal to 12.66 kV and 2667.8 kVA. The possible capacitor sizes and kVar cost with a life expectancy of 10 years are given in Table 3.

Before installing capacitors, the power losses are of 223.44 kW where, 151.92 kW are due to branch active currents and 71.52 kW to the branch reactive components. The node voltage magnitudes, source node not included, are between 0.909509 and 0.999966.

The implementation of the developed program shows that to streamline the reactive energy transit, three capacitors of 1050, 150 and 600 kVar located, respectively, at nodes 60, 58 and 8 are required. This optimal solution leads to voltage profile improvement. The minimum and maximum node voltage magnitudes pass from the values given above to 0.930544 and 0.999980. It also reduces both active (F_d) and reactive (F_q) components of the branch currents (Fig. 7). The distribution of the branch reactive current shows the absence of any over-compensation. The power losses pass from 223.44 kW (without capacitors) to 148.48 kW which means a power loss reduction of 74.96 kW (i.e., 33.54% of the total power losses) and a cost reduction equal to 12,419 $. Compared to the results given by references [12–14], the voltage minimum rms value is equal to that given in Ref. [13] and greater than those in

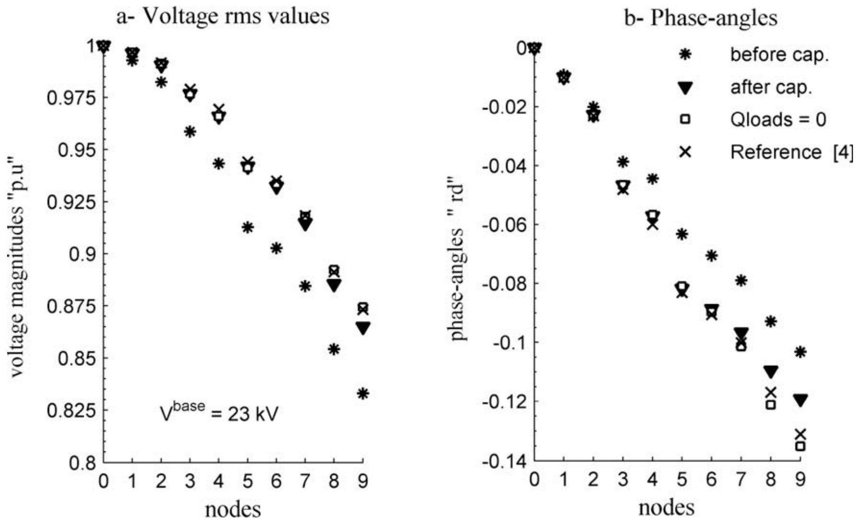

Fig. 5. Voltage profile improvement; 9-nodes feeder.

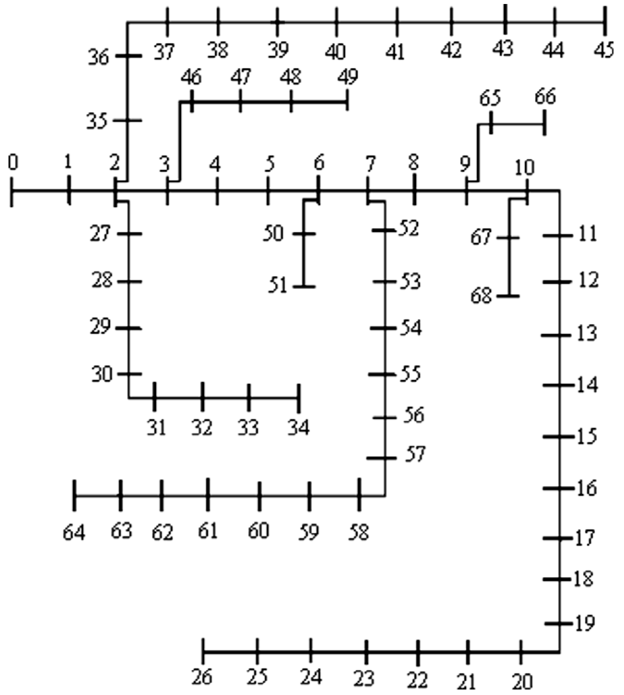

Fig. 6. 69-nodes feeder, one line diagram.

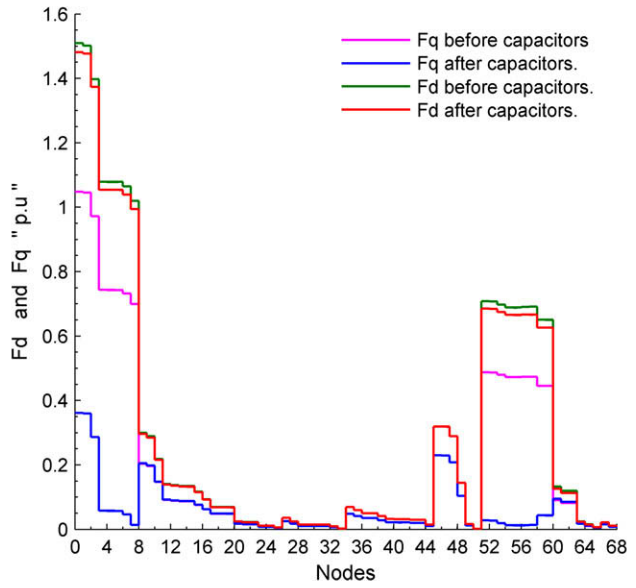

Fig. 7. Branch currents distributions before and after compensation; 69-nodes feeder.

Table 3
Possible standard capacitors and cost; 69-nodes feeder.

Q_c (kVar)	150	300	450	600	750	900
$/kVar	0.500	0.350	0.253	0.220	0.276	0.183
Q_c (kVar)	1050	1200	1350	1500	1650	1800
$/kVar	0.228	0.170	0.207	0.201	0.193	0.187
Q_c (kVar)	1950	2100	2250	2400	2550	
$/kVar	0.211	0.176	0.197	0.170	0.189	

references [12] and [14] (Fig. 8). If all loads reactive powers are set equal to zero, the feeder minimum and maximum voltage magnitudes are, respectively, equal to 0.931824 and 0.999987. All voltage minimum values (see Fig. 8) point out that permissible limits of ±5% cannot be met and, therefore, demonstrate the non-feasibility of the voltage constraint.

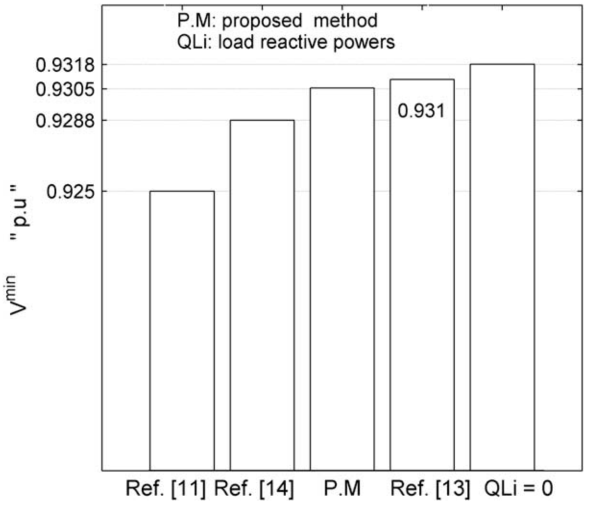

Fig. 8. Voltage minimum values after compensation.

4. Conclusion

A two stage heuristic method has been presented in this paper in order to rationally manage the reactive power transit in distribution feeders. In the first stage, the node sensitivities are used for determining the proper capacitor locations. In the second stage, we determine the optimal capacitor sizes by maximising the savings function. In the developed approach, the non-realisable voltage constraint is substituted by a new constraint which permits overcoming any over-compensation and the violation of the voltage upper limit. The proposed approach has been tested on several feeders and the substitution of the voltage constraints justified. The developed method has given better power loss and cost reductions compared to a number of available approaches.

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