IIMB Management Review (2012) 24, 137—142

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IIMB Management Review

Multi-criteria decision making with overlapping criteria

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Available online 13 September 2012

KEYWORDS
Multi-criteria decision making (MCDM);
Group decision making;
Decision support systems;
Dempster—Schafer theory;
Probability

Abstract The evidential reasoning (ER) algorithm for multi-criteria decision making (MCDM) performs aggregation of the assessments of multiple experts, one each for every attribute (or subsystem or criterion) of a given system. Two variants of ER are proposed, that handle a scenario where more than one expert assesses an attribute. The first algorithm handles the case of multiple experts who assess an attribute of a larger system. Experiments compare a modification of ER for this scenario which results in poorer detection. The second algorithm is used when experts have overlapping areas of expertise among the subsystems. A comparison is made with a variant of ER in the literature. Both algorithms are examples of novel 'exclusive' and 'inclusive' ER.
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Introduction and problem outline

The Dempster rule of combination is a mathematical rule to combine the opinions of multiple experts on the same subject. This rule is illustrated using a simple example: suppose there are two experts, $i = 1, 2$ assessing whether a product can be termed 'good'. They use the *assessment levels* 'good', 'bad', and 'can't say' corresponding to indices $l = 1, 2, 3$ respectively, and attach *belief masses* m_l^i, $l = 1, 0.3(\text{s.t. } m_l^i \in [0,1]$ and $\sum_{l=1}^{3} m_l^i = 1)$. The 'can't say' assessment level is considered a special case in the Dempster rule of combination, as it represents ignorance or incomplete information.

Using the Dempster rule, an aggregated assessment m_l, $l = 1, 2$ is obtained using:

$$m_l = \frac{m_l^1 \cdot m_l^2 + m_l^1 \cdot m_3^2 + m_3^1 \cdot m_l^2}{\sum_{r=1,s=1}^{3,3} m_r^1 \cdot m_s^2} \quad (1)$$

The intuition behind this formula is that a system is graded 'good' if both experts assess it to this level, or if (any) one expert assesses it as 'good', while the other grades it as 'can't say'. In continuation with (1), the aggregated opinion for 'can't say' is computed as $m_3 = m_3^1 \cdot m_3^2 / \sum_{r=1,s=1}^{3,3} m_l^1 \cdot m_s^2$. The term in the denominator of the expression for $m_l = 1$. It is useful to interpret the Dempster rule as an experiment involving repeated events where both experts are sampled for an assessment. In each such sample, an expert i responds with either 'good', 'bad' or 'can't say' with the probabilities $\{m_l^i\}$. All events which indicate that a 'good' assessment is *plausible* for the overall system would occur with probability m_1 as calculated in (1).

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http://dx.doi.org/10.1016/j.iimb.2012.06.002

Multi-criteria decision making (MCDM) is the term applied when experts assess individual criteria (or subsystems) of the system and produce belief masses of the form $\{m_l^i\}$. These criteria may be hierarchical. MCDM algorithms follow four common sense 'synthesis' axioms:

1. If an assessment level has value 0 for each criteria, then it has value 0 for the higher criterion.
2. If an assessment level has value x for each criteria, then it has value x for the higher criterion.
3. If a subset of assessment level has non-zero values, then the higher criterion will have non-zero values for exactly the same subset.
4. If the assessment of any attribute is incomplete (i.e. 'can't say' has non-zero value), then an assessment obtained by aggregating the values of all criteria will also be incomplete.

Brief review of ER and MCDM

The evidential reasoning (ER) algorithm (Yang & Xu, 2002) has been widely used to solve MCDM problems in engineering design assessment (Yang, 2001), software safety analysis (Wang and Yang, 2001) and many other applications (software based on ER is available commercially). Evidential reasoning may be considered an advanced form of the Dempster rule (1). Evidential reasoning assumes that a product has N subsystems with non-zero weights w_n of importance such that $\sum_{n=1}^{n} w_n = 1$. There is one expert to assess one subsystem, issuing assessments $\{m_l^n\}$ for L assessment levels: the assessments of all N experts being aggregated to obtain a common assessment for the product. In order to process conflicting evidence rationally, ER assigns a global weight $w_l^n = w_n \cdot m_l^n$ for assessment $\{m_l^n\}$ from expert n. The weight $w_{L+1}^n = 1 - w_n \sum_{l=1}^{L} m_l^n$ is not allocated to any assessment level, and is a 'can't speak' mass with $l = L+1$. Thus, suppose $N = 2$, for $l = 1 \ldots L-1$, ER adapts (1) into:

$$w_l^{1,2} = \frac{w_l^1 w_l^2 + w_l^1 \left(w_L^2 + w_{L+1}^2\right) + \left(w_L^1 + w_{L+1}^1\right) w_l^2}{\sum_{r=1,s=1}^{L+1,L+1} w_r^1 w_s^2}$$

The product is assessed 'good' if both experts assess it as 'good' (in proportion to the importance of their respective subsystem), or if any one expert assesses it as 'good' (again, in proportion) while the other observes either 'can't say' or 'can't speak'. The equations for $w_L^{1,2}$ and $w_{L+1}^{1,2}$ are:

$$w_L^{1,2} = \frac{w_L^1 w_L^2 + w_L^1 w_{L+1}^2 + w_{L+1}^1 w_L^2}{\sum_{r=1,s=1}^{L+1,L+1} w_r^1 w_s^2} \quad \text{and} \quad w_{L+1}^{1,2} = \frac{w_{L+1}^1 w_{L+1}^2}{\sum_{r=1,s=1}^{L+1,L+1} w_r^1 w_s^2}$$

For the case $N = 2$, a final normalisation is applied: for $1 \leq l \leq L$,

$$m_l = \frac{W_l^{1,2}}{1 - W_{L+1}^{1,2}}$$

If $N > 2$, consider $\{w_l^{I(2)}\}_{l=1}^{L+1}$ to be the weights of the combination expert $I(2) = \{1,2\}$ and combine further with $\{w_l^3\}_{l=1}^{L+1}$ terms recursively.

Mere weighted addition of the assessment using weights wn will produce an overall assessment for the system. However, one may fail to identify any strengths/weaknesses within the *higher* criteria along the hierarchy. Evidential reasoning solves the MCDM problem since a decision-maker can pick the alternative with maximum utility, e.g. a higher total mass for the 'good' assessment. A clarification must be made regarding the criterion weights used in ER and general forms of MCDM: weights inferred in a general MCDM technique like AHP may be explicitly problem-dependant. The weights depend on what alternatives are being evaluated and can change when any new alternative joins the group. In ER, each criterion (and subcriterion, in turn) has a pre-determined weight, a measure of importance to the overall system. ER is incremental in that new alternatives can be assessed and ranked by merely calculating their total utility value, rather than recalculate the utilities of any alternatives assessed thus far.

Many MCDM algorithms, including ER, assume *utility independence* among the attributes: i.e. the utility over multiple criteria can be added to indicate the total utility measure of the product (also called multi-attribute utility theory (MAUT)). Classical MCDM algorithms assume that the L assessment levels are mutually exclusive and collectively exhaustive. One objection with Dempster's rule or algorithms derived from it, e.g. ER, is that it reflects the minority of experts if there is a high *degree of conflict* (i.e. denominator in (1) above). This is the finding of the work in (Dezert and Tacnet, 2011), where evidential reasoning refers to a different algorithm. In MCDM as handled by ER, this problem is avoided: the non-zero $1 - w_i$ mass for 'can't speak', apart from a possible 'can't say' weight in the assessment, ensures a low degree of conflict.

Contributions of this article

This article makes two contributions which are as follows:

- Classical ER does not consider a scenario where multiple experts assess a given subsystem. This scenario occurs in the subsystems of certain products, such as 'design' where, due to the risk associated with keeping one expert, an additional expert is posted. An algorithm is proposed that fits the frequentist view of Dempster rule and produces better results than a naive variant of ER.
- ER is adapted for a scenario where some of the experts have overlapping areas of expertise among the subsystems. Comparison against the scenario presented in Abdulla and Raghavan (2009) is made and better results obtained.

These are explained in the framework of exclusive and inclusive ER families of algorithms.

Exclusive ER variants

Assume a system S with two subsystems and that subsystem 1, called S_1, has a single expert E_1, whereas subsystem 2 (S_2) has two experts $E_2{}^a$ and $E_2{}^b$ appointed to assess it. The proposed algorithm uses a formula to translate from S_1 (respectively. S_2) current weight w_1 (resp. w_2) to a corrected weight x_1 (resp. x_2) by solving:

$$\frac{x_1(1-x_2)^2}{w_1}=\frac{(1-x_1)x_2^2+2(1-x_1)x_2(1-x_2)}{w_2}, \quad (2)$$

$$x_1+x_2=1. \quad (3)$$

This is explained as follows: the probability $x_1(1-x_2)^2/z$, where Z is a normalisation term, is of the event that a sampled opinion for the entire system S originates solely from S_1, which is ruled upon by E_1. Hence, it is required that $x_1(1-x_2)^2/z$ be proportional to w_1. Similarly, $(1-x_1).(x_2^2+2x_2(1-x_2))/z$ must be proportional (with the same constant factor) to w_2, and hence x_1 and x_2 are solutions to equation (2). Constraint $x_1+x_2=1$ is used to maintain similarity with ER, and will be relaxed later.

Dempster-like frequentist view in exclusive ER

Exclusive ER can be explained as a sequence of sampling experiments on system S. Take repeated events where all three experts are sampled and respond with a certain probability indicating 'can't speak'. This probability is $1-x_1$ for E_1 and $1-x_2$ for both E_2^a and E_2^b. Each expert also produces an assessment vector $\{m_l^k\}$ (where k is in {1, {2,a}, {2,b}}) of the respective subsystem with probability x_1 for E_1 and x_2 for E_2^a and E_2^b. The expected number of events where all three experts produce an assessment is proportional to $x_1x_2^2$. The expected number of events where two out of three experts have an assessment is proportional to $(1-x_1)x_2^2+2x_1x_2(1-x_2)$, whilst the number of events where only one of the experts has an assessment is proportional to $2(1-x_1)x_2(1-x_2)+x_1(1-x_2)^2$. To complete the set, the number of events where no judge has an assessment is proportional to $(1-x_1)(1-x_2)^2$. The term $(1-x_1)x_2^2$ when two experts have an assessment, represents S_2 alone (i.e. exclusively) as does $2(1-x_1)\,x_2\,(1-x_2)$, when only one of three judges has an assessment. Term $x_1\,(1-x_2)^2$ is representative of subsystem S_1 alone. Terms $x_1x_2^2$, $2x_1x_2(1-x_2)$, and $(1-x_1)(1-x_2)^2$ are not considered as these represent both subsystems. Now solve for x_1 and x_2:

$$\frac{x_1(1-x_2)^2}{(1-x_1)x_2^2+2(1-x_1)x_2(1-x_2)}=\frac{w_1}{w_2} \quad (4)$$

There are infinitely many solutions to the above. One possible solution is when the constraint $x_1+x_2=1$ is employed. The solution $x_1=w_1$ has the advantage that x_2 is a constant $1-1/1.44$. An experiment was run where one million 'good' or 'bad' products were presented to the three experts. Experts E_2^a and E_2^b were of low proficience i.e. they could detect the correct state of the subsystem only with probability 0.55. The state of each subsystem was the same as that of the system. Expert E_1 had a high proficience (0.95), and in the experiment the weightage w_1 of S_1 was gradually increased. The comparison of the two solutions of (4) is in Fig. 1.

Regular ER adapted with exclusive events

The method of adjusting weights based on how many events are strictly representative of the subsystem can be used in regular ER (i.e. the scenario of S_2 having only one expert E_2 assigned to it). Thus, $x_1(1-x_2)$ is representative of S_1 while $(1-x_1)x_2$ represents S_2. Solve for $x_1(1-x_2)/(1-x_1)x_2=w_1/w_2$ and let $x_1=w_1$, to get $x_2=0.5$.

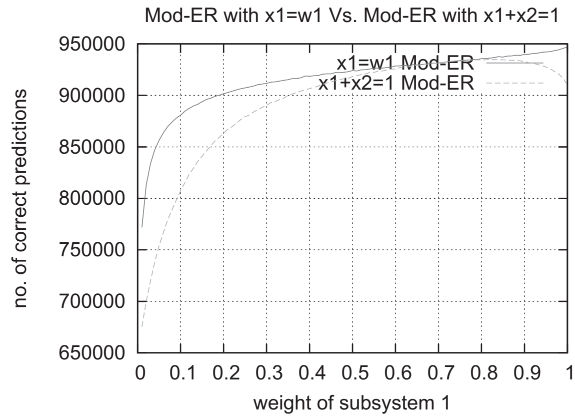

Fig. 1 Mod-ER with $x_1=w_1$ Vs. Mod-ER with $x_1+x_2=1$.

One may solve with constraint $x_1+x_2=1$, but the particular solution above avoids any further calculations. Regular ER is called 'IonER' for reference, and exclusive variant of 'IonER' performs well in Fig. 2.

Inclusive ER variants

An alternative method for converting weights w_1, w_2 to x_1, x_2 can be employed. Consider the equation:

$$\frac{x_1x_2^2+2x_1x_2(1-x_2)+x_1(1-x_2)^2}{x_1x_2^2+(1-x_1)x_2^2+2(1-x_1)x_2(1-x_2)}=\frac{w_1}{w_2} \quad (5)$$

The numerator corresponds to events where subsystem S_1 is represented (irrespective of whether S_2 isn't). Earlier, (4) only considered events where subsystems S_1, S_2 are exclusively represented. All four synthesis axioms are satisfied here, as in (2). Besides, this scheme is applicable to many more overlap settings than (2).

The scheme of (5) is also used in Regular ER: note $w_1/w_2=x_1(1-x_2)+x_1x_2/(1-x_1)x_2+x_1x_2$, reducing $x_1=w_1$ and $x_2=w_2$ when constraint $x_1+x_2=1$ is employed. Without this constraint, we may use any x_1, x_2, that satisfy

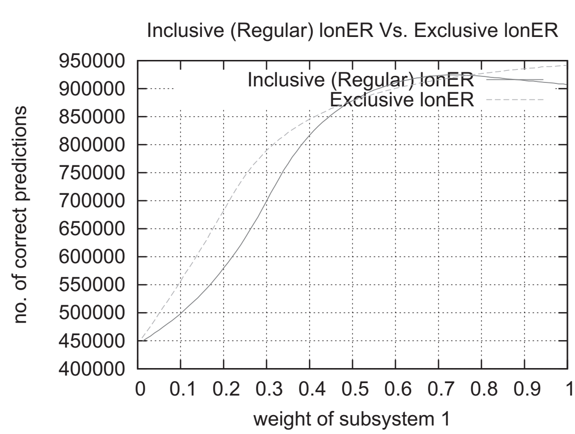

Fig. 2 Detection in exclusive IonER ($x_1=w_1$ and $x_2=0.5$).

$x_1/x_2 = w_1/w_2$. An experiment for $x_2 = 1$ and $x_1 = w_1/w_2$ indicates that this method is at par with Regular ER (Fig. 3).

Processing (5) gives $x_2^2(2x_1-1)+x_2(2-2x_1)-w_2/w_1x_1=0$. For the case of $w_1 \geq 0.5$, one can substitute $x_1 = 1$ and obtain $x_2 = \sqrt{w_2/w_1}$. For the case of $w_1 < 0.5$, use expression $-b \pm \sqrt{b^2} - 4ac/2a$ for the roots of generic equation $ax_2 + bx + c = 0$ to obtain

$$x_2 = \frac{-(2-2x_1) \pm \sqrt{(2-2x_1)^2 + 4(2x_1-1)\frac{w_2}{w_1}x_1}}{2(2x_1-1)}. \quad (6)$$

Take the positive root and substitute $x_1 = w_1/w_2$ to obtain $x_2 = 1$. A higher value of x_1 (than w_1) in both cases is preferred, as it maximises the effect of E_1 — the expert with higher proficience. Using these x_1 and x_2, inclusive ER brings only marginal improvement as seen in Fig. 4.

Inclusive ER is retained as a candidate method for modifying ER when multiple experts are assigned to each subsystem. Indeed, there are many scenarios in which inclusive ER is applicable whereas exclusive ER may not be.

1) Application 1 of inclusive ER: Assume, as before, that a system S has subsystems S_1 and S_2 (with weights w_1 and w_2, respectively). Unlike the scenario of having experts for each subsystem, there is now an expert E_1 (with adjusted weight x_1) tasked exclusively with assessing S_1 and another expert E_2 (with a corresponding x_2) tasked with assessing entire S. Every assessment from E_2 represents both S_1 and S_2, while an assessment of E_1 represents S_1 alone. Using inclusive ER:

$$\frac{x_1 + x_2 - x_1 \bullet x_2}{x_2} = \frac{w_1}{w_2}. \quad (7)$$

The numerator corresponds to the fraction of sampling events where S_1 is represented, whilst the denominator consists of events where S_2 is represented. From (7) a high effective weightage for the expert with higher proficience can be inferred. Suppose $w_1 > w_2$ and that E_1 has lower proficience; then a high effective value for x_1 is in the solution $x_1 = 1$, $x_2 = w_1/w_2$.

If experts have a partial overlap, performing decision aggregation using inclusive events is easier. A simple method is to employ w_n of any expert E_n (w_n may be the sum of the weights of subsystems under review by E_n) into regular ER, even if the ER condition $\sum_{n=1}^{N} w_n = 1$ is violated. This is proposed in (Abdulla and Raghavan (2009)) as an alternative to regular ER, which cannot handle $N \neq N'$ experts. However, the algorithm in Abdulla and Raghavan (2009) is not as general as (7), which performs a correction of the x_n to be used.

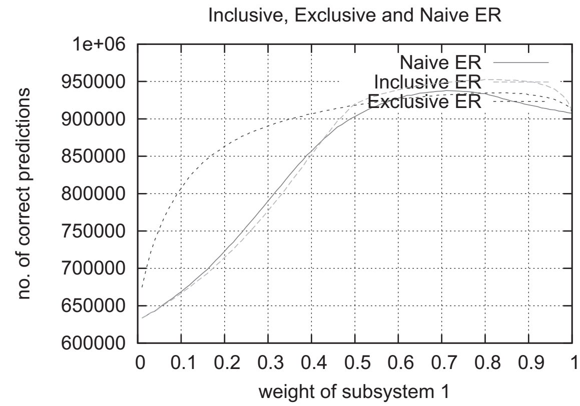

Fig. 4 Comparison of inclusive and naïve schemes.

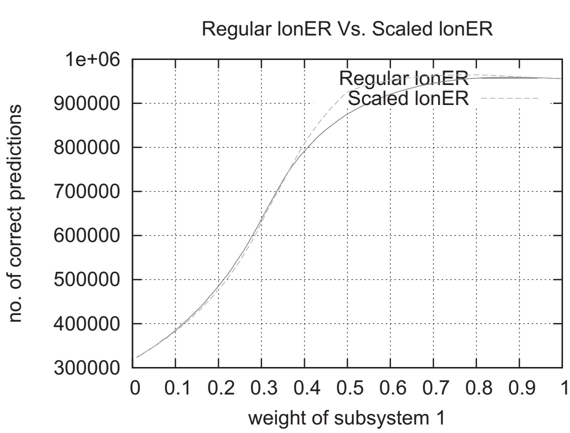

Fig. 3 Regular ER Vs. ER with $x_1 = w_1/w_2$ and $x_2 = 1$.

2) Application 2 of inclusive ER: Consider the system in Fig. 5: Three experts and three overlapping subsystems S_1, S_2 and S_3 with weights w_1, w_2 and w_3, respectively.

Infer the weights x_1, x_2 and x_3 to be used in the ER combination rule as follows:

$$\frac{x_1 \bullet x_2 + x_3 - x_1x_2x_3}{x_1} = \frac{w_3}{w_1}. \quad (8)$$

The intuition used above is that S_3 is represented both when expert E_3 is in 'speak' mode as well as when experts E_1 and E_2 jointly are in 'speak' mode. Substitute $x_1 = w_1$ and $x_2 = w_2$, to obtain a solution:

$$x_3 = \frac{w_3 - w_1w_2}{1 - w_1w_2}. \quad (9)$$

The constraint is that $w_3 > w_1^2$. Thus, the overlapping subsystem S_3 should be a subsystem with high importance. As $w_3 < 1$, it can be seen in (9) that $x_3 < 1$. This formula does not require the degree of *individual* overlap that S_3 may have with S_1 and S_2.

For comparison, we use the method developed in Abdulla and Raghavan (2009) where w_i is retained, in that $x_i = w_i$, $\forall i \in \{1,2,3\}$. The performance of the proposed algorithm is shown in Fig. 6, where non-corrected ER

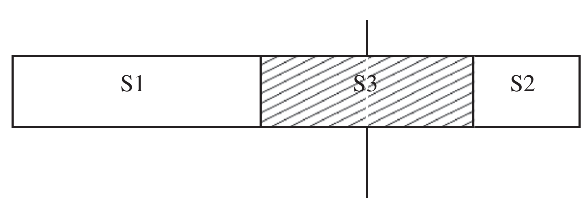

Fig. 5 Diagram of subsystem S_3 overlapping with subsystems S_1, S_2.

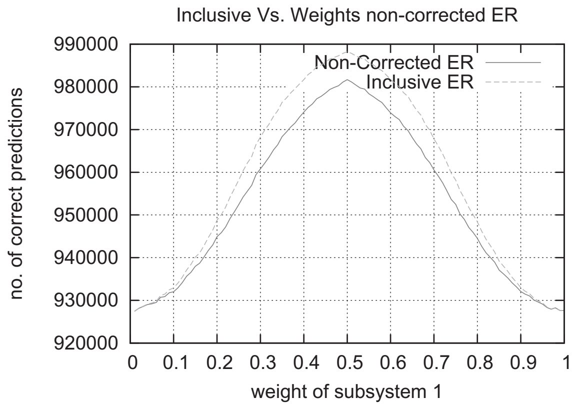

Fig. 6 Performance of inclusive ER vs. 'Weights non-corrected ER'of (Abdulla and Raghavan (2009)) for a typical case.

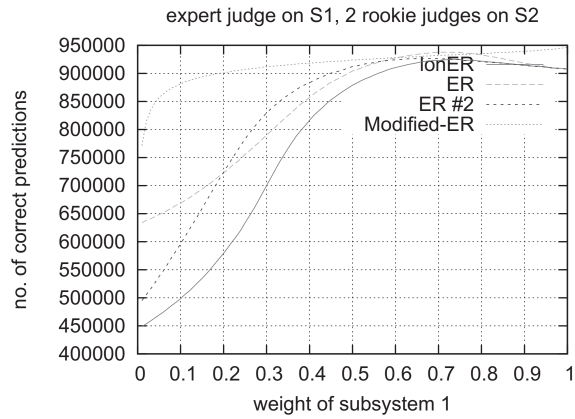

Fig. 8 Detection rate with modified ER as W_1 increases.

corresponds to the algorithm in Abdulla and Raghavan (2009). Choose $w_3 = \max(w_1, w_2)$ so as to avoid violating the constraint $w_3 > w_1 - w_1^2$. Expert E_3 is considered to have a lower proficience p_3 (0.6) than E_1 and E_2 (0.9). Such a value is chosen since $x_3 < w_3$ so that the algorithm in Abdulla and Raghavan (2009) associates a higher weight with the opinion of a less proficient expert.

Better results are obtained if x_1, x_2 and x_3 are modified depending on which of the experts have better proficience. Assume that p_3 is 0.9 while p_1 and p_2 are 0.6 with $w_3 \geq \max(w_1, w_2)$: then $x_3 = 1$, $x_1 = w_2/w_3$ is a solution that also gives a high effective weightage to proficient E_3. The resultant detection rate is better, as shown in Fig. 7.

Numerical results

Detailed experiments are conducted when $N = 2$ and $L = 3$ with, 'bad' and 'can't say' being the assessment levels. Assume that there is a product S with two subsystems S_1 and S_2 with a proficient expert E_1 on S_1 while experts E_2^a and E_2^b assess S_2. Expert E_1 is proficient because the true state of S_1 will be ruled correctly with probability 0.95. Also, if S is classed 'good' (which happens with probability 0.5), then both $S_1$and S_2 are also good a-priori and vice-versa. When presented with a stream of 'good' products, the assessment vector $\{m_l^1\}$ produced by E_1, say, has the mean value {0.95, 0.04, 0.01}. The mass assigned to 'good' is a random variable in (0,1) having mean 0.95. The masses assigned to 'bad' and 'can't say' are, on an average, 80% and 20%, respectively, of the mass left after assigning to 'good'. Varying importance of weights w_1 are considered in the experiments.

The assessment vector of E_2^a and E_2^b has the notion $\{m_l^{2,a}\}$ and $\{m_l^{2,b}\}$. Because of the low proficience of experts E_2^a and E_2^b, the average assessment vector (for a string of 'good' products) is {0.55, 0.36, 0.09}. After aggregating assessments using exclusive ER (4), an assessment is ruled 'good' if $m_1 > 0.5$ (and not merely $m_1 > m_2$) with a similar procedure for 'bad'. This result is then compared with a-priori state of the product. Detection is said to have failed if the a-priori state does not match the result obtained after aggregation. We use ER with exclusive events, and the constraint $x_1 + x_2 = 1$, to compute the adjusted weights x_1 and x_2. Apart from naïve ER (ER1 in this discussion), a comparison is made against a pro-rata scheme of ER (ER2) where the two novice experts are assumed to be subsystems with weights $w_2/2$ each. Besides, a comparison against ER when the second novice judge on S_2 is left out (termed 'IonER') is also performed. In Fig. 8, the performance of all four algorithms is plotted.

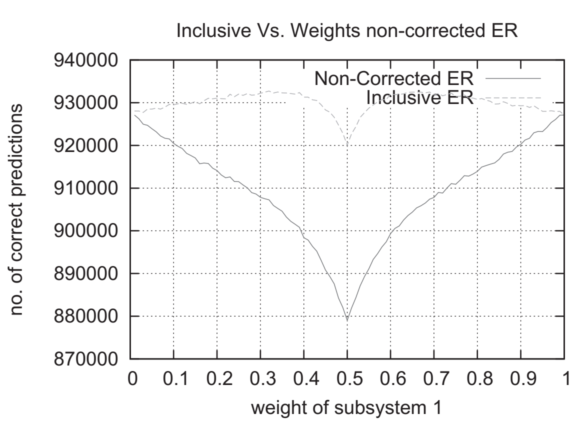

Fig. 7 Performance of inclusive ER vs. 'Weights non-corrected ER' with $ps_r = 0.75$, $r \in \{1,2,3\}$.

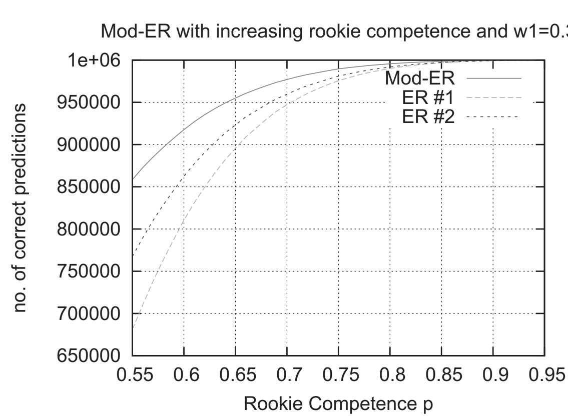

Fig. 9 Modified ER as proficience of experts on S_2 increases.

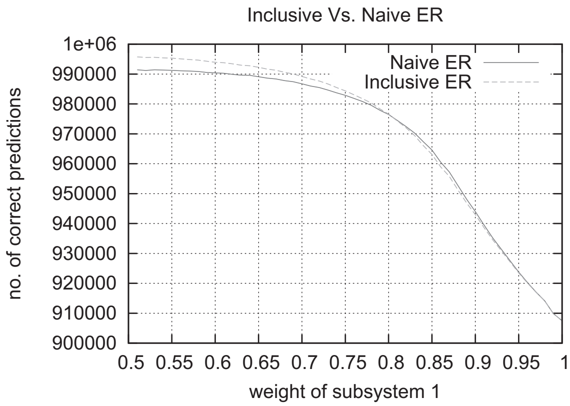

Fig. 10 ER for overlapping experts as in (7).

The next experiment measured the detection rate when the proficience of experts E_2^a and E_2^b is increased from a low value of 0.55—0.95. The weight of S_1 is held at $w_1 = 0.3$. The performance of the exclusive ER algorithm is shown in Fig. 9. In Fig. 10, the detection rate of (7) above is given. There is no equivalent of regular ER for this setting, hence $w_2 = 1 - w_1$ for all values of w_1 from 0.51 to 1.

Conclusions

This article presents variants of ER that handle the case of MCDM when multiple experts assess the same criteria or overlapping criteria. The framework of 'inclusive' and 'exclusive' ER algorithms is also introduced, and is of value since the algorithms can be suitably modified for any complicated cases not covered here (e.g., larger number of matching/overlapping criteria or experts). Numerical simulations for these variants of ER are also provided, with up to 20% better state-detection performance noticed vis-a-vis naive variants of ER.

Acknowledgements

The author would like to thank Ms Sunitha and Ms Renya of IT and Systems Area, IIM Kozhikode, for their typesetting and manuscript support.

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