Requirements for soil-specific correlation between shear wave velocity and liquefaction resistance of sands

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**Abstract**

The application of the simplified method for evaluating the liquefaction potential based on shear wave velocity measurements has increased substantially due to its advantages, especially for microzonation of liquefaction potential. In the simplified method, a curve is proposed to correlate the cyclic resistance ratio (CRR) with overburden stress-corrected shear wave velocity ($V_s$). However, the uniqueness of this curve for all types of soils is questionable. The objective of this research is to study whether the correlation between CRR and $V_s$ is unique or not. Besides, the necessity of developing the soil-specific correlations is also investigated. Based on laboratory test data, a new semi-empirical method is proposed to establish the soil-specific CRR–$V_s$ correlation. To validate the proposed method, a number of undrained cyclic triaxial tests along with bender element tests were performed on two types of sands. Similar experimental data for six other types of sands reported in the literature was also compiled. Applying the proposed method, soil-specific CRR–$V_s$ correlation curves were developed for these eight types of sands. It is shown that the correlation is not unique for different types of sands and the boundary curve proposed in the available simplified method can only be used as an initial estimation of liquefaction resistance. Finally, using the results of this study as well as previous ones, a chart is suggested to be used in engineering practice showing the conditions for which a detailed soil-specific CRR–$V_s$ correlation study needs to be performed.

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1. Introduction

Evaluation of seismic liquefaction potential of soils is an important and challenging issue in geotechnical earthquake engineering. The simplified method, initially developed by Seed and Idriss [1], has been used in engineering practice and evolved as a standard method in the liquefaction resistance evaluation. In this method, based on standard penetration test (SPT), cone penetration test (CPT), or shear wave velocity ($V_s$) measurements, a boundary curve is proposed to separate the liquefiable and non-liquefiable soil zones.

To evaluate liquefaction resistance of sandy soils, $V_s$ measurements offer geotechnical engineers a promising alternative and a supplementary tool compared to penetration-based methods, such as SPT and CPT [2]. Many parameters can affect the $V_s$ of a soil, among which effective confining stress, void ratio, stress history, grain characteristics, aging effects and soil structure can be mentioned as the most important. These parameters can also affect the liquefaction resistance of soils [3,4]. Considering this fact, the idea of using $V_s$ to evaluate the liquefaction resistance of soils has received considerable attention. De Alba et al. [5] were among the first researchers who proposed the correlation between $V_s$ and liquefaction resistance (CRR–$V_s$ curve). CRR stands for cyclic resistance ratio which is defined as the cyclic liquefaction resistance normalized by initial overburden effective stress.

The advantages of using shear wave velocity for evaluation of liquefaction potential, compared to penetration-based methods, are as follows:

- $V_s$ can be measured in soils, such as gravelly soils, that are hard to sample. In these types of soils penetration tests may be unreliable [2,4].
- $V_s$ is one of the few parameters that are measurable both in the laboratory and in the field. If a relationship is obtained for shear wave velocity in the laboratory, it can also be used on site [4].
- There is no need for drilling boreholes in this method, and it can predict the liquefaction resistance more rapidly and economically than SPT or CPT; therefore, it can be used in microzonation of liquefaction potential [2,4].
- The cyclic resistances of intact and reconstituted samples are quite different [6]. However, if a reconstituted sample is
prepared to the same \(V_s\) as an intact sample, then the liquefaction resistance of both samples is equal. In other words, the reconstituted samples can be used to establish the correlation between \(V_s\) and CRR [3,7]. This issue is expected to be valid for uncedmented Holocene-age soils (< 10,000 years) or freshly deposited soils. For older deposits, its validity is open to question and aging factor may need to be used [8].

- The correlation between \(V_s\) and CRR is independent of the stress history of the soil as well as the method used to prepare the sample for cyclic testing [3].
- \(V_s\) is a significant soil property in earthquake site response and soil-structure interaction analyses. It has a clear physical meaning, whereas the penetration resistance cannot directly correspond to any of such properties [4].

Use of \(V_s\) to evaluate the liquefaction potential has some limitations. These include:

- No samples are routinely obtained as part of the testing procedure for soil classification and identification of non-liquefiable materials [2].
- Thin, low \(V_s\) liquefiable strata may not be detected if the measurement interval is too large [9,10].
- \(V_s\) measurements are made at small-strains, whereas pore water pressure buildup and liquefaction are medium to high-strain phenomena [11]. This can be a significant disadvantage for cemented soils. This is because small-strain measurements are highly sensitive to weak inter-particle bonds that are eliminated at medium to high strains. It can also be significant in silty soils above the water table where negative pore water pressures may increase the shear wave velocity [2].
- \(V_s\) may not be expected to reflect the critical friction angle and dilatancy of the soil, both of which are known to affect liquefaction resistance [12]. Critical friction angle and dilatancy increase with increasing non-plastic fines, whereas the shear wave velocity decreases [13].
- The different methods of measuring/estimating \(V_s\) sample different volumes of soil (e.g. SASW, MASW, CPT–\(V_s\), cross-hole, and suspension logger) and it can be difficult to compare laboratory and field data.

Besides these disadvantages, it should also be noted that both \(V_s\) and CRR are affected by soil type, and therefore, the correlation of these two parameters may also be affected by soil type. Based on database of various sandy soils, different CRR–\(V_s\) correlations have been proposed by different researchers [2,4,14–19]. Therefore, one of the major challenges associated with the use of \(V_s\) to evaluate the liquefaction resistance, is the uniqueness of \(V_s\)-CRR correlation for all sands.

Tokimatsu and Uchida [4] showed that the correlation between small-strain shear modulus, \(G_0\), and CRR (and consequently between \(V_s\) and CRR) are different for two types of sand, Niigata sand and Toyoura sand (Fig. 1(a)). Their attempt to provide unique curve for different sands, has led to curve (b) in Fig. 1. In this curve, the effect of sand type is intended by \(R \cdot e_{\text{min}}\) which is a function of the minimum void ratio (\(R(e_{\text{min}}) = (2.17 – e_{\text{min}})^2(1 + e_{\text{min}})\)). This is also indicative of the correlation dependence on sand type. This argument is reinforced because recent studies also have shown that CRR–\(V_s\) correlation is soil-specific [3,19,20]. On the other hand, the simplified and widely used field based procedure of Andrus and Stokoe [2] recommended by NCER is considered to be independent of soil type. However, the relationship proposed by Andrus and Stokoe [2] which is presented in detail in Section 7.2, is highly based on limiting upper value of \(V_s\) for cyclic liquefaction occurrence. Since cyclic stress ratio (CSR) values above about 0.35 are limited in the case history data gathered by Andrus and Stokoe, current estimates of upper values of \(V_s\) rely on penetration–\(V_s\) correlations [2]. However many penetration–\(V_s\) correlations have been proposed so far for different sandy soils. If these different correlations are used, different values for upper value of \(V_s\) will be obtained which in turn, result in different curves for CRR–\(V_s\) correlations. It means that the CRR–\(V_s\) correlation suggested by Andrus and Stokoe [2] also depends on the soil type and therefore, the CRR–\(V_s\) correlation may not be unique. Due to these contradictions and ambiguities, one of the aims of this study is to clarify whether the correlation between CRR and \(V_s\) is unique or not. Also, the reliability

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**Nomenclature**

- \(a, b\): curve fitting parameters for the data presented by Andrus and Stoke
- \(C_c\): coefficient of curvature
- \(C_{g}, a_g, n_g\): intrinsic soil parameters in the small-strain shear modulus evaluation from \(e\) and \(\sigma_{\text{ov}}\)
- \(\text{CRR, CSR, CSR}_{\text{Mw}}, \text{CSR}_{\text{tx}}\): cyclic liquefaction resistance ratio
- \(\text{CRR}_{\text{tx-15}}\): cyclic liquefaction resistance ratio from triaxial test
- \(\text{CSR}\): cyclic stress ratio
- \(\text{CSR}_{\text{Mw}}\): cyclic stress ratio for an earthquake magnitude of \(M_w\)
- \(\text{CSR}_{\text{tx}}\): cyclic shear stress ratio from triaxial test
- \(\text{CSR}_{\text{tx}}\): coefficient of uniformity
- \(D_{50}\): mean grain size
- \(\text{DA}\): double amplitude axial strain
- \(e\): void ratio
- \(e_{\text{max}}, e_{\text{min}}\): maximum void ratio, minimum void ratio
- \(G_0\): small-strain shear modulus
- \(G_{\text{tx}}\): laboratory small-strain shear modulus at a vertical effective stress of 100 kPa
- \(G_{0, \text{tx}}\): small-strain shear modulus at an effective confining stress of 100 kPa
- \(\sigma_{\text{s}}\): specific gravity of solids
- \(K_0\): coefficient of lateral earth pressure at rest
- \(K_s, n_c\): proposed CRR–\(V_{15}\) field correlation coefficients
- \(L\): tip-to-tip distance of bender elements
- \(\text{MSF}\): earthquake magnitude scaling factor
- \(M_w\): earthquake magnitude
- \(P_A\): reference stress (= 100 kPa)
- \(q\): deviatoric stress
- \(R^2\): correlation coefficient
- \(t\): travel time of the wave in bender element tests
- \(V_{s1}\): limiting upper value of \(V_s\) for cyclic liquefaction occurrence by the Andrus and Stoke method
- \(V_s\): shear wave velocity
- \(V_{s1, \text{tx}}\): in situ measured shear wave velocity
- \(V_{s1}\): field overburden stress normalized shear wave velocity
- \(V_{s1, \text{tx}}\): normalized shear wave velocity to confining stress of 100 kPa
- \(\alpha\): coefficient in Eq. (2) to correlate cyclic liquefaction resistance ratio and void ratio
- \(\beta\): a power in Eq. (2) to correlate cyclic liquefaction resistance ratio and void ratio
- \(\Delta\sigma_d\): cyclic deviator stress
- \(\rho\): total density
- \(\sigma_{\text{g0}}\): initial effective confining stress
- \(\sigma_{\text{m}}\): mean effective stress
- \(\sigma_{\text{v}}\): vertical effective stress
2. Method for establishing CRR–$V_s$ correlation

Many studies have been conducted to correlate the $V_s$ with liquefaction resistance of sands based on general format of the Seed–I DRiss simplified procedure [1]. This simplified procedure is the introduction of an empirical correlation to evaluate the cyclic resistance ratio (CRR) from field test results (e.g. $N$ from SPT, $q_c$ from CPT, or $V_s$). Following this simplified procedure; various methods for evaluation of the seismic liquefaction potential of soils from $V_s$ measurement have been developed. These include:

- Methods based on a combination of laboratory measurement of $V_s$ and liquefaction resistance [4,5,7,18,19,21,22].
- Methods based on in-situ measurements of $V_s$ and an appropriate correlation between liquefaction resistance and $V_s$ (i.e. field performance observations) [2,15,16,23,24]. Most of the in-situ shear wave measurements used in this method are post earthquake properties, and do not exactly reflect the initial state of the soil before earthquake [19]. On the other hand, liquefaction can only be judged from surface observations such as sand boils, or large settlements. Factors such as the existence of a superficial non-liquefiable thick layer would prevent the appearance of these surface indications. Also, local information from sites that have been liquefied previously and the shear wave velocities have been measured on them before the occurrence of liquefaction is very limited. Conditions such as the lack of information about the type and percentage of fine materials, and the lack of awareness about whether the soil layer is fully saturated or not, can also severely affect this local information. Therefore, these in-situ $V_s$-based methods are still less well defined [10,16].
- Methods based on analytical investigations [25].
- Methods based on penetration–$V_s$ correlations [17].
- Probability-based methods using statistical analysis [26,27].

These methods are applied to the results of the other above four methods.

Because of the more accurate and well-controlled conditions of laboratory methods, this method is used in this study to develop the CRR–$V_s$ correlation. For this purpose, laboratory measurements of $V_s$, using bender elements and liquefaction resistance estimations using cyclic triaxial tests were performed.

In order to develop a correlation between liquefaction resistance and $V_s$ from experimental data (CRR–$V_s$ relationship), a soil-specific relationship between liquefaction resistance and void ratio ($e$) is proposed (CRR–$e$ relationship). The small-strain shear modulus ($G_0$) can be calculated from measured $V_s$ ($G_0$–$V_s$ relationship). Also, $G_0$ is related to the void ratio through the existing empirical soil-specific relationships ($G_0$–$e$ relationship). Eliminating the void ratio between CRR–$e$ and $G_0$–$e$ relationships and using $G_0$–$V_s$ relationship, the liquefaction resistance is correlated with $V_s$ and hence, a new CRR–$V_s$ relationship is developed. This method has been verified by using the data obtained from sands tested in this study and was used for experimental data collected for six other types of sands from previous studies. Using the proposed method, the soil-specific CRR–$V_s$ correlations for these eight different types of sands are developed.

3. Tested materials

Two types of clean sands were used in this study. The first was Babolsar sand, a beach sand from north of Iran with sub-rounded grains. The second one was Firoozkooh no. 161 crushed silica sand with angular grains. Firoozkooh sand is a commercially available material from Firoozkooh mine in north-east of Tehran. The scanning electron microscope (SEM) images in Fig. 2 show different grain shapes of these two sands.
Both sands are used as standard sands in geotechnical testing in Iran. A summary of the physical properties of these sands is given in Table 1, and the corresponding grain size distribution curves are shown in Fig. 3. These sands have nearly identical particle size distributions. ASTM D 4253 and ASTM D 4254 standard test methods were used to determine the minimum and maximum void ratios, respectively.

4. Cyclic triaxial testing

The cyclic resistance of the sands tested was determined using undrained stress-controlled cyclic triaxial tests performed on reconstituted specimens. The tested specimens were 70 mm in diameter and 140 mm in height. In order to measure the $V_s$ and liquefaction resistance on a single sample, the bender elements were assembled on the cyclic triaxial apparatus. These bender elements were installed at the top and bottom pedestal of the triaxial cell. The cyclic triaxial apparatus used in this testing program is manufactured by Wykeham Farrance England, Ltd. Furthermore, some modifications were made to this apparatus. This apparatus is a digitally controlled, servopneumatic, closed-loop system, which controls three parameters: axial stress, confining pressure, and back pressure. A diagram of the automated triaxial system equipped with bender elements is shown schematically in Fig. 4.

4.1. Specimen preparation

Tokimatsu and Uchida [4] and also Huang et al. [22] argued that specimen preparation method does not affect the liquefaction resistance–$V_s$ correlations. Among the different kinds of specimen reconstitution methods, the moist tamping method gives the widest range of void ratios [28]. Huang et al. [22] showed that the specimens prepared by moist tamping are reasonably uniform and give sufficiently repeatable $V_s$ values. Therefore, moist tamping method of sample reconstitution with 5% water content was used to homogeneously prepare the samples. In order to obtain a uniform density, the specimens were made in seven layers and the under-compaction method, proposed by Ladd [29], was used.

4.2. Testing procedure

ASTM D 5311 standard testing procedure for load controlled cyclic triaxial strength of soil was used to conduct the experiments. To facilitate the saturation process, carbon dioxide (CO₂) was first passed through the samples. Subsequently deaired water was allowed to flow in the specimens. Samples were then saturated by applying proper back pressure in successive steps. The Skempton pore pressure parameter ($B$) was determined as a means of degree of saturation. According to ASTM D 5311, samples are considered to be fully saturated if $B$ value is at least equal to or greater than 0.95. Saturated samples were then consistently consolidated uniformly in steps of 10–30 kPa. The consolidation process continued until the effective confining stress reached a value of 200 kPa. Since increasing the consolidation stress can improve the vertical load controlling in the triaxial apparatus (Fig. 5), a consolidation stress of 200 kPa was chosen to perform the cyclic triaxial tests. Also, as is mentioned in Section 5.1, by increasing the consolidation stress, more bender elements tests can be conducted. Thus, possible errors are decreased and accuracy of the results is increased. On the other hand, Boulanger [30] pointed out that confining stress will be critical only for liquefaction evaluations at high overburden stresses. Therefore, liquefaction resistance of samples at consolidation stress of 100 or 200 kPa would not be much different.

Table 1

<table>
<thead>
<tr>
<th>Sand type</th>
<th>$D_{50}$</th>
<th>$C_u$</th>
<th>$C_c$</th>
<th>$e_{max}$</th>
<th>$e_{min}$</th>
<th>$G_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Babolsar</td>
<td>0.24</td>
<td>1.80</td>
<td>1.00</td>
<td>0.825</td>
<td>0.546</td>
<td>2.78</td>
</tr>
<tr>
<td>Firoozkooh</td>
<td>0.23</td>
<td>1.32</td>
<td>0.92</td>
<td>0.886</td>
<td>0.637</td>
<td>2.65</td>
</tr>
</tbody>
</table>
At the end of consolidation process, a sinusoidal loading of 1 Hz frequency was applied to the sample having a specified cyclic shear stress ratio (CSR\textsubscript{tx}). CSR\textsubscript{tx} is defined as:

\[
\text{CSR}_{tx} = \frac{\Delta \sigma_d}{2 \sigma_{c0}}
\]  

where \(\Delta \sigma_d\) is the maximum cyclic deviator stress; and \(\sigma_{c0}\) is the initial effective confining stress.

For each test, the number of cycles required for the initial liquefaction of the sample, when pore water pressure firstly reaches the initial consolidation stress, has been recorded. Cyclic resistance ratio in triaxial tests (CRR\textsubscript{tx-15 cycles}) is defined as the applied CSR\textsubscript{tx} required to cause initial liquefaction in 15 cycles of loading (representing an earthquake magnitude of \(M_w = 7.5\)). At least three cyclic triaxial tests were performed to obtain CRR\textsubscript{tx-15 cycles} for a soil sample having a specified void ratio. All parameters except CSR\textsubscript{tx} were kept constant in these tests.

The after consolidation void ratio of the samples was determined by accurately measuring the water content of the entire sample at the end of the experiment when the cyclic loading has been applied. To measure the water content of the sample, at the first step, the bottom valve of the sample was opened and a confining stress of approximately 50 kPa was applied. The free water weight, expelled from the sandy sample, was accurately measured. At the second step, the entire moist sample was carefully taken out of the apparatus and its moisture content was also measured. The water content of the sample is the sum of the free water measured in the first step and the moist content measured in the second step.

### 4.3. Test result

Sample records of a cyclic triaxial test on Firoozkooh sand with a void ratio of \(e = 0.77\) and cyclic stress ratio of CSR\textsubscript{tx} = 0.231 are presented in Fig. 5. In this figure, cyclic deviatoric stress, excess pore pressure ratio, and axial strain are all plotted against the number of loading cycles. The effective stress path for the test is also plotted in this figure. For this test, the specimen reached its initial liquefaction at the 39th cycle.

Tests results, in the form of CSR\textsubscript{tx} versus the number of cycles for different void ratios, are shown in Fig. 6. As expected, the liquefaction resistance decreases as the void ratio increases.

It is worth noting that in Fig. 6(b), the single point specified by a dotted circle corresponds to the results previously presented test in Fig. 5. According to Fig. 6(b), the liquefaction resistance of Firoozkooh sand with a void ratio of \(e = 0.77\) is equal to CRR\textsubscript{tx-15 cycles} = 0.255. The same way, the liquefaction resistance of a soil at different void ratios can be obtained from the results of the conducted tests presented in Fig. 6. If the liquefaction resistance (CRR\textsubscript{tx-15 cycles}) versus void ratio is plotted, a power curve with the following expression (Eq. (2)) can be fitted to the data points of different sand types. This is shown in Fig. 7 indicating that for identical void ratios, the liquefaction resistances of both tested sands are almost the same.

\[
\frac{\text{CRR}_{tx-15 \text{ cycles}}}{C_0} = \alpha e^\beta
\]  

where \(e\) is the void ratio, and \(\alpha\) and \(\beta\) are constants for a given material in a specified test conditions.

According to Fig. 7, \(\alpha = 0.101\), \(\beta = -3.618\) and \(\alpha = 0.0897\), and \(\beta = -3.799\) yield the best fit curves for Babolsar and Firoozkooh sands, respectively. The values of correlation coefficient, \(R^2\), for both sand types are very close to 1.0, indicating that the correlation is satisfactory and Eq. (2) works well.

### 5. Bender element tests

Conducting an experiment with bender elements, \(V_s\) can be calculated using Eq. (3). In this equation, \(L\) is the distance between two bender elements and \(t\) is the travel time which is determined by recording the input and output voltages.

\[
V_s = \frac{L}{t}
\]
5.1. Testing procedure

In the bender element tests performed in this study, a single sinusoidal pulse having a frequency of 5 kHz was used as the transmitted signal. The value of \( L \) in Eq. (3) is the tip-to-tip distance of the bender elements [31]. The method of first arrival time was used to obtain the travel time from source to receiver. In this method, the initial portion of the weak signal is ignored, and the time interval between the start of the source signal and the start of the major cycle of the received signal is measured. This weak signal indicates the presence of the near field effect, and should be eliminated [31,32].

For each sample, prior to the application of the cyclic loading, shear wave velocities were measured immediately after the end of each consolidation step for confining effective stresses ranging from 30 to 200 kPa.

Sample result of a bender element test on Babolsar sand with a void ratio of 0.806 at an effective confining stress of 70 kPa is represented in Fig. 8 in which the first arrival time is shown. The void ratio as well as the height of the samples changes in each consolidation step as the confining stress increases. To calculate the changes in the void ratio, the amount of water expelled from the specimen during consolidation stage was measured using a sensitive volume change apparatus. Also, as mentioned in Section 4.2, the water content of the entire sample was carefully measured at the end of the experiment. Given that the sample is saturated prior to the consolidation phase, the void ratios at the earlier steps of consolidation can be back-calculated from these measured values. The settlement of the sample was also measured during the saturation and consolidation phase using a displacement transducer, and the change in the height of the samples was accordingly used in calculating the shear wave velocity using Eq. (3). An example of the results obtained for a single sample is presented in Fig. 9.

In this research, a total number of 350 bender element tests were carried out on 35 different samples of two sands, and a series of void ratio, confining effective stress, and \( V_s \) data were obtained.
For a granular soil, the small-strain shear modulus can be obtained using Eq. (4). In this equation, $G_0$ is the total density of the soil, $\rho$ is a function of its void ratio and effective consolidation stress, and can be obtained from the empirical equation $G_0 = \rho V_s^2$.

Fig. 7. Laboratory liquefaction resistance against void ratio for tested sands.

Fig. 8. Representative result of bender element tests.

Fig. 9. An example of shear wave velocity measurement at different steps of consolidation.

for each type of sands. Fig. 6 shows the average measured shear wave velocities of samples with the same void ratios at the confining effective stress of 100 kPa along with the liquefaction curves.

5.2. Small-strain shear modulus calculation

In the bender element test, shear waves propagate through the sample with induced maximum shear strain of less than $10^{-3}$. Thus, having measured the $V_s$, the small-strain shear modulus can be obtained using Eq. (4). In this equation, $G_0$ is the small-strain shear modulus and $\rho$ is the total density of the soil.

$$G_0 = \rho V_s^2 \quad (4)$$

For a granular soil, $G_0$ is a function of its void ratio and effective confining stress, and can be obtained from the empirical equation $G_0 = \rho V_s^2$.

Table 2: Intrinsic parameters of Eq. (5) for tested sands.

<table>
<thead>
<tr>
<th>Sand type</th>
<th>$C_g$</th>
<th>$n_g$</th>
<th>$d_g$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Babolsar</td>
<td>449.7</td>
<td>0.453</td>
<td>−1.885</td>
<td>0.986</td>
</tr>
<tr>
<td>Firoozkooh</td>
<td>389.1</td>
<td>0.478</td>
<td>−1.835</td>
<td>0.955</td>
</tr>
</tbody>
</table>

Fig. 10. Small-strain shear modulus versus void ratio for tested sands at different consolidation stresses.

like the one developed by Jamiolkowski et al. [33], as introduced in Eq. (5).

$$G_0 = C_g \rho^{1-n_g} \dot{e}^n_g \sigma_m^n \quad (5)$$

where $P_0$ is the reference atmospheric pressure equal to 100 kPa, $\sigma_m$ is the mean effective stress in the same units as the reference stress and can be calculated using Eq. (6). The parameters $d_g$, $n_g$ and $C_g$ are intrinsic parameters associated with each type of material.

$$\sigma_m = 1 + 2K_0 \sigma_v \quad (6)$$

In Eq. (6), $\sigma_v$ is the vertical effective stress and $K_0$ is the ratio of effective horizontal stress to effective vertical stress.

The intrinsic parameters of Eq. (5) for Firoozkooh and Babolsar sands were obtained by fitting the results of the all bender element tests conducted at different consolidation stresses and void ratios. These parameters are listed in Table 2. The table shows that the values of correlation coefficient, $R^2$, for both sand types are very close to 1.0, indicating that the correlation is satisfactory.

The small-strain shear modulus of Firoozkooh and Babolsar sands at 100 and 200 kPa consolidation stresses versus void ratio are illustrated in Fig. 10, using Eq. (5) and the corresponding intrinsic parameters for each sand type from Table 2. Obviously, similar curves can be drawn at other isotropic effective stresses.

The data points of measured $G_0$ values at effective confining stresses of 100 and 200 kPa are also provided in Fig. 10. As expected, $G_0$ decreases with increasing void ratio and decreasing the confining effective stress.

6. CRR–$V_s$ correlation

6.1. Establishment of the correlation

In this section, the detailed procedure for establishment of CRR–$V_s$ correlation is presented. Both the cyclic resistance and the $V_s$ values measured in the laboratory must be corrected to represent the field conditions. In a triaxial test, $K_0$ is equal to 1,
but for saturated normally consolidated sand in the field, $K_0$ is generally between 0.4 and 0.5. Also, seismic excitations in the field are multi-directional, while in a cyclic triaxial test, the cyclic load is applied in only one direction. Therefore, to account for these differences, the liquefaction resistance of the soil obtained from cyclic triaxial tests should be corrected. Several equations have been suggested in the literature regarding such corrections. In this study, the widely accepted equation proposed by Seed [34] was used in this respect as follow:

$$CRR = 0.9 \left( \frac{1 + 2K_0}{3} \right) CRR_{tx-15 \text{ cycles}}$$

(7)

where, $CRR$ is the actual liquefaction resistance in the field, and the value of 0.9 is a factor to account for the strength loss occurring in the field during the multidirectional shaking in real earthquakes compared with unidirectional laboratory testing.

On the other hand, using Eqs. (5) and (6), the actual on-site small-strain shear modulus ($G_0$) can be modified by the following equation to consider the effects of $K_0$:

$$G_0 = \left( \frac{1 + 2K_0}{3} \right) G_{01,tx}$$

(8)

where $G_0$ is the small-strain shear modulus at a vertical effective stress of 100 kPa in the field, and $G_{01,tx}$ is the small-strain shear modulus obtained in the laboratory at an confining effective stress of 100 kPa ($= P_A$).

According to Eq. (5):

$$C_{01} = C_g P_A e^{e_1}$$

Combining Eqs. (2) and (7) leads to the following equation:

$$CRR = 0.9 \left[ \left( \frac{1 + 2K_0}{3} \right) \left( \frac{1 + 2K_0}{3} \right)^{\alpha_g} \right]^{\beta N}$$

(10)

Also combining Eqs. (8) and (9) the following equation is obtained:

$$G_0 = C_g P_A e^{e_1} \left( \frac{1 + 2K_0}{3} \right)^{\alpha_g}$$

(11)

Using Eqs. (10) and (11), by eliminating the void ratio ($e$), the correlation between the field small-strain shear modulus at vertical effective stress of 100 kPa ($G_0$) and the field liquefaction resistance ($CRR$) can be established (Eq. (12)). Using Eq. (4), also the correlation between $CRR$ and $V_{s1}$ ($V_s$ at a vertical effective stress of 100 kPa) is obtained as follow:

$$CRR = (K_c P_A^{-1} G_0)^{\rho_1} = (K_c P_A^{-1} \rho V_{s1}^{K_c})^{\rho_1}$$

(12)

In this equation, $G_0$ and $P_A$ are in the same units. All parameters in Eq. (12), except $K_c$ and $n_c$ have been defined previously. $K_c$ and $n_c$ are defined as

$$K_c = (0.9 e^{\rho_1} \rho_1)^{\beta N} \left( \frac{1 + 2K_0}{3} \right)^{\alpha_g}$$

(13)

$$n_c = \frac{\beta}{\rho_1}$$

(14)

Using Eq. (12) and having the required values of intrinsic parameters for $G_0$ (i.e. $\alpha_g$, $n_g$, and $C_g$) and coefficients relating the cyclic liquefaction resistance ratio to void ratio (i.e. $\alpha$ and $\beta$), the $CRR-V_{s1}$ correlation can be obtained for any soil type. For Eq. (12), CSR=0.03 is also considered as the threshold of pore pressure generation [35].

In order to obtain the $V_s$ at an effective vertical stress of 100 kPa in the field ($V_{s1}$), the measured in-situ shear wave velocity ($V_{s, \text{field}}$) should be normalized with the effective overburden stress ($\sigma_c$).

The following equation can be used for this purpose.

$$V_{s1} = V_{s, \text{field}} \left( \frac{P_A}{\rho V_s} \right)^{0.25}$$

(15)

6.2. The $CRR-V_{s1}$ correlation for tested sands

Substituting the parameters $\alpha$ and $\beta$ from the cyclic triaxial tests (Fig. 7) and $\alpha_g$, $n_g$, and $C_g$ from the bender element tests (Table 2) in Eq. (12) and assuming $K_c$ to be 0.5, Eqs. (16) and (17) are derived to correlate $CRR$ and $V_{s1}$ values for Babolsar and Firoozkooh sands, respectively.

$$CRR = (6.2 \times 10^{-6} \rho V_{s1}^{K_c})^{1.92}$$

(16)

$$CRR = (7.6 \times 10^{-6} \rho V_{s1}^{K_c})^{2.07}$$

(17)

In these equations, $V_s$ is expressed in m/s and the soil density is in g/cm$^3$. Although the grain size distribution curves for Babolsar and Firoozkooh sands are rather similar (Fig. 2), Eqs. (16) and (17) show that the correlation between $V_s$ and liquefaction resistance for these two sands are different.

Using Eqs. (16) and (17), the predicted $CRR-V_{s1}$ curves of tested sands are plotted in Fig. 11. The experimental data points from Fig. 6 which are modified by Eqs. (7) and (8) to represent the field conditions are also provided in this figure. Fig. 11 shows that there is a very good agreement between predicted $CRR-V_{s1}$ correlation from Eq. (12) and the experimental results. It can be concluded that the suggested equation (Eq. (12)) can be used to develop the soil-specific $CRR-V_{s1}$ correlations.

6.3. $CRR-V_{s1}$ correlation for other sands

In this study, comprehensive investigations in the literature were performed to obtain similar laboratory data for six other sands. The physical properties of these sands are listed in Table 3. Compiling the experimental data obtained from the literature for these six types of sands on reconstituted specimens, the
liquefaction resistances at different void ratios were obtained accordingly. Triaxial liquefaction resistances versus void ratios for these sands are shown in Fig. 12. The corresponding values of $\alpha$ and $\beta$ obtained by fitting appropriate power curves to the laboratory data together with the reference papers from which these data are obtained are provided in Table 3. The table shows that the values of $R^2$ for all sand types are close to 1.0 indicating that Eq. (2) is a good correlation between void ratio ($e$) and liquefaction resistance ($C_{RR}$) at 15 cycles. For these sands, the values of intrinsic parameters of the $C_{0}$ correlation (i.e. $a_C$, $n_C$ and $C_y$ in Eq. (5)) were also calculated by fitting the provided data from the literature which are also presented in Table 3. In cases where the parameter $n_C$ was not provided, the value of $n_C$ was considered to be 0.5 which have been shown to be a good approximation.

The small-strain shear modulus curves versus void ratio for these sand types at a consolidation stress of 100 kPa, using Eq. (5) and the provided intrinsic parameters in Table 3, are also presented in Fig. 13. Finally, the $C_{RR}$–$V_s$ correlations for these sands were obtained using the data presented in Table 3 and Eq. (12). Assuming $K_0$ to be 0.5, intrinsic parameters of Eq. (12) ($K_i$ and $n_i$) for these sands together with those for the tested sands were calculated and are presented in Table 4.

It is shown that the power of $V_s$ in $C_{RR}$–$V_s$ correlation (which is $2n_i$) for these sands ranges from 3.84 to 10.3. It is worth noting that according to the studies conducted by Chen et al. [18], the suggested power of $V_s$ is 4. Also Zhou and Chen [19] concluded that the power of $V_s$ is in the range of 1.8 to 6. Noteworthy is that further studies are required to investigate the effects of different soil parameters on intrinsic parameters for $C_{RR}$–$V_s$ correlation (i.e. $K_i$ and $n_i$). This is considered to be beyond the scope of this study.

Using the data in Table 4 and Eq. (12), the $C_{RR}$–$V_s$ curves for these six types of sands together with two tested sands are shown in Fig. 14. It is noteworthy that the variations of $C_0$ or $C_{RR}$ with void ratio ($e$) and confining stresses are characterized by Eqs. (2) and (5) respectively. Since Eq. (12) is based on these two equations, it is possible to extrapolate somewhat the $C_{RR}$–$V_s$ correlation beyond the limits of the performed tests using this equation. In Fig. 14, to indicate the extrapolation parts of the $C_{RR}$–$V_s$ curves beyond the tested range, these parts of curves are shown with different style. As can be seen in Fig. 14, these curves are different for each type of sand. Therefore, it can be concluded that the $C_{RR}$–$V_s$ correlation for these eight sand types is highly soil-specific. As a result, for an accurate assessment of liquefaction potential for a given sand type, it is required to determine a soil-specific $C_{RR}$–$V_s$ correlation for that sand.

7. Discussion

7.1. Uniqueness of $C_{RR}$–$V_s$ correlation

As previously mentioned, several researchers have proposed curves for evaluating liquefaction resistance through $V_s$ measurements. Fig. 15
curves that have been developed based on field performance observations (i.e. proposed curves by Robertson et al. [15]; Kayen et al. [16]; Lodge [17]; and Andrus and Stokoe [24,2]) as well as those that have been developed based on laboratory tests (i.e. proposed curves by Tokimatsu and Uchida [4]; Chen et al. [18]; Zhou and Chen [19] and Zhou et al. [20]). The scattering of the curves in Fig. 15 suggest that the CRR–\( V_{s1} \) may not be unique. The different curves obtained for different sand types in the present study, including those that have been tested and those that have been compiled from the literature (Fig. 14) also confirm that there is not a unique CRR–\( V_{s1} \) curve for all types of sands.

7.2. Comparison with the existing procedure

As mentioned earlier in Section 1, Andrus and Stokoe [2] suggested a method for the evaluation of liquefaction potential based on field \( V_s \) measurements. This method is recommended by NCEER and is widely used in current practice. It follows the framework of the Seed–Idriss simplified procedure [1], correlating the overburden stress-corrected shear wave velocity (\( V_{s1} \)) to the magnitude-scaled cyclic stress ratio (CSR) induced by earthquakes. In this method, CRR is calculated from the following equation:

\[
\text{CRR} = \left\{ a \left( \frac{V_{s1}}{108} \right)^2 + b \left( \frac{1}{V_{s1} - V_{s1c}} - \frac{1}{V_{s1}} \right) \right\}
\]

(18)

where \( V_{s1c} \) is the limiting upper value of \( V_{s1} \) for cyclic liquefaction occurrence with an assumed value of 215 m/s for clean sand; and \( a \) and \( b \) are curve fitting parameters taken to be 0.022 and 2.8 respectively.

The CRR–\( V_{s1} \) curve suggested by Andrus and Stokoe [2] are also plotted in Figs. 14 and 15. As can be seen, although this proposed method can be used as an initial estimation of liquefaction resistance, it may underestimate or overestimate the liquefaction resistance of different sandy soils. For a more accurate assessment of the liquefaction resistance of a sandy soil, a soil-specific CRR–\( V_{s1} \) correlation is needed.

7.3. The region for soil-specific CRR–\( V_{s1} \) curves

Using the CRR–\( V_{s1} \) curves developed by different researchers (Fig. 15) and also those obtained for eight different sands in this study (Fig. 14), a region can be determined in which all the CRR–\( V_{s1} \) curves are located. In Fig. 16, this region is defined as the region of “Suspected to liquefaction”. All the soil-specific CRR–\( V_{s1} \) curves are most likely to be located in this region. Considering that all the curves presented in Figs. 14 and 15 are located within this region, the provided chart is based on field performance observations as well as the results of laboratory tests.

This region is bounded by two parallel lines. One line extending from the point of CRR=0 and \( V_{s1}=90 \) m/s to the point of CRR=0.5 and \( V_{s1}=180 \) m/s and the other line extending from the point of CRR=0 and \( V_{s1}=180 \) m/s to the point of CRR=0.5 and \( V_{s1}=270 \) m/s. Also as mentioned in Section 6.1, below CSR=0.03 soil is considered non-liquefiable.

7.4. Practical applications

For liquefaction assessment at a specific site, in addition to the resistance of the soil to liquefaction (CRR), obtained from the CRR–\( V_{s1} \) curves, the level of cyclic loading on the soil caused by the earthquake (CSR) should also be determined. Based on the simplified procedure, liquefaction may occur at points where CSR exceeds the CRR.

All the proposed curves for the estimation of CRR (Figs. 14–16) are for an earthquake moment magnitude of \( M_{w}=7.5 \). Therefore, the CRR derived from these curves would be scaled for other...
earthquake magnitudes by applying a magnitude scaling factor (MSF). Instead of scaling the CRR parameter, another approach is to scale the CSR parameter for a specified earthquake (CSRM\textsubscript{w}) other than M\textsubscript{w}=7.5 using Eq. (19) as below:

\[ \text{CSR} = \frac{\text{CSRM}\textsubscript{w}}{\text{MSF}} \]  

Eq. (19)

Corrections for consideration of other factors that affect the liquefaction resistance, e.g. effective overburden pressure, initial shear stresses, cementation and aging effects are applicable in the same manner.

The field measured value of V\textsubscript{s} corrected to V\textsubscript{s1}, together with the scaled value of CSR for a specified earthquake at a specific location in a sandy soil layer define a point in Fig. 16. If this point falls within the region specified as “No liquefaction” in Fig. 16, then the liquefaction will not occur in that specific location for the specified earthquake. On the other hand, if the point falls within the region specified as “Liquefaction”, then liquefaction will occur at that specific location due to the specified earthquake. In both cases described above, there is no need to establish the soil-specific CRR–V\textsubscript{s1} correlation. However if the point falls within the region specified as “Suspected to liquefaction”, then in order to determine if the soil in the specified location in that field is liquefiable or not by the specified earthquake, it is necessary to obtain the soil-specific CRR–V\textsubscript{s1} correlation for the soil of that field. In case, the soil-specific CRR–V\textsubscript{s1} correlation cannot be established, that specific location may be considered to liquefy for the given earthquake. In other words, to be conservative in such a case, the line drawn from the point of CRR=0 and V\textsubscript{s1}=180 m/s to the point of CRR=0.5 and V\textsubscript{s1}=270 m/s can be considered as the boundary line between liquefiable and non-liquefiable soil conditions.

Therefore, for the liquefaction potential microzonation using V\textsubscript{s} measurements, the mapped area can be classified as one of these three different zones: 1 – liquefiable zone, 2 – non-liquefiable zone, and 3 – zone with liquefaction susceptibility.

It is worth noting that for a zone with liquefaction susceptibility, one of these two approaches can be used: (1) preparation of soil-specific CRR–V\textsubscript{s1} curve or (2) adopting a conservative approach, and considering the zone to be liquefiable. The choice of one of these approaches can also be based on economic aspect of the project at hand.

8. Summary and conclusion

In this paper, a new semi-empirical equation is suggested to correlate the cyclic resistance ratio (CRR) and the overburden stress-corrected shear wave velocity (V\textsubscript{s1}) values from laboratory tests data. Laboratory measurements of V\textsubscript{s} using bender element tests and the liquefaction resistance estimation using cyclic triaxial tests were performed on two types of clean sands, namely Firoozkooh and Babolsar sands. Using the proposed method, the soil-specific CRR–V\textsubscript{s1} correlations were established for these two sands.

In addition, the laboratory test data for six other sands gathered from the literature, were compiled to obtain the soil-specific CRR–V\textsubscript{s1} correlations for these sands.

The results of this study show that although there is a good correlation between shear V\textsubscript{s} and liquefaction resistance, this correlation is not unique for different types of sandy soils and is soil-specific.

The accuracy of the widely used simplified method for evaluation of liquefaction potential from V\textsubscript{s} measurements was also investigated. It was found that the simplified method, proposed by Andrus and Stokoe [2], can only be used as an initial estimation of liquefaction resistance. To accurately evaluate the liquefaction potential from V\textsubscript{s} measurements, it may be necessary to develop the soil-specific CRR–V\textsubscript{s1} correlations by laboratory tests. The method proposed in this paper can be used for development of such soil-specific correlations from laboratory tests.

Using the CRR–V\textsubscript{s1} curves obtained for eight different sands in this study and the curves suggested by other investigators which are based on field performance observations or laboratory tests, a region is introduced in which all the soil-specific CRR–V\textsubscript{s1} curves are likely to be located. Therefore, for practical use, by measuring the field V\textsubscript{s} in a specified area and using the proposed chart, for a given earthquake, the investigated area can be classified into three different zones: 1 – liquefiable zone, 2 – non-liquefiable zone, and 3 – zone with liquefaction susceptibility. The soil-specific CRR–V\textsubscript{s1} correlation will not be required in zones with liquefaction potential and zones with no liquefaction potential. But in the suspected to liquefaction potential zones, if the accurate assessment of liquefaction potential is desired, the soil-specific CRR–V\textsubscript{s1} correlation will be required; otherwise, as a conservative approach, this zone may be considered to have liquefaction potential.

References
