Seismic behaviour of cable-stayed bridges under multi-component random ground motion

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Abstract

A frequency domain spectral analysis is presented for the seismic analysis of cable-stayed bridges for the multi-component stationary random ground motion incident at an angle with the longitudinal axis of the bridge. The ground motion is represented by its power spectral density function and a spatial correlation function. The analysis duly takes into account the spatial variation of ground motions between the supports, the modal correlation between different modes of vibration and the quasi-static excitation. Using the proposed method of analysis, an extensive parametric study is conducted to investigate the behaviour of the cable-stayed bridge under the seismic excitation. The parameters include the spatial correlation of ground motion, the angle of incidence of the earthquake, the ratio between the three components of the earthquake, the number and nature of modes considered in the analysis, the inertia ratio between the tower and the deck, and the nature of the power spectral density function of the ground motion. Elsevier Science Ltd. All rights reserved.

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1. Introduction

The open competitive design situation that existed in Germany after the second world war has indicated that cable-stayed bridges are an economical solution for moderately long span bridges. The distribution of supported cables along the span deck as well as the axial compression which is produced by these cables make the dynamic behaviour of these kinds of bridges different from those of suspension bridges. In the literature, the reported work on the subject include both linear and non-linear dynamic analysis based upon either finite element or lumped mass modelling. Although the non-linear analysis represents a more realistic dynamic behaviour of the bridge, a linear analysis is found to be economical and justified in many of the cases without losing the accuracy to a great extent.

The previous work on the dynamic behaviour of cable-stayed bridges considered either a free vibration problem [1,2] or a forced vibration problem due to seis-mic excitations [3–7] and/or vehicular movement [8]. Kajita and Cheung [9] used the finite element method to find the natural frequencies and mode shapes of the vertical (transverse) and torsional vibration of a two-plane cable-stayed bridge, with the deck assumed to be a uniform thick plate. Morris [10] utilized the lumped mass approach for the linear and non-linear dynamic responses of the cable-stayed bridges due to sinusoidal load applied at a node. From the analysis of two different types of cable-stayed bridges, he concluded that a linear dynamic analysis could suitably describe their structural behaviour. Fleming and Egeseli [3] also assumed a lumped mass model of cable-stayed bridges for finding the responses due to three different types of loading namely vertical and horizontal earthquake excitations, wind induced force and a single constant moving force. Their conclusion was that the structure could be assumed to behave as a linear system during the application of dynamic loads, starting from the deformed state under dead loads; although there might be significant non-linear behaviour during the static application of the dead loads. Nazmy and Abdel Ghaffar [5,6] investigated the non-linear earthquake response of cable-stayed bridges by lumped mass idealization of the bridge and showed

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that upto moderately long span bridges, a linear dynamic analysis would be adequate. They [7] also carried out a linear dynamic analysis for moderately long span cable-stayed bridge to investigate the seismic response of the cable-stayed bridges to both uniform and multi-support excitations using the time domain analysis.

So far as the effect of spatially varying ground motion on the response of bridges is concerned, considerable interest has been shown by various researches. Harichandran [11,12] had shown that the assumption of fully coherent support motions may be over-conservative for some bridges and under-conservative for others. Zerva [13] showed that the effect of the spatial correlation of ground motion mainly depends upon the dynamic characteristics of the structure. Soliman and Datta [14] showed that the inclusion of the spatial correlation of the ground motion in the seismic analysis of bridges lead to changes in the response by varying degrees depending upon the type of the power spectral density function of the ground motion used.

Despite the previous researches on the dynamic response of cable-stayed bridges, the seismic behaviour of the cable-stayed bridges subjected to random ground motion is not thoroughly investigated. The research is still continuing.

Herein, a frequency domain spectral analysis for obtaining the response of cable-stayed bridges to partially correlated stationary random ground motion is presented. A continuum approach, along with a matrix formulation, is used for finding the seismic response of the bridge. The response analysis duly considers the effect of differential support movements, the angle of incidence of the earthquake and the modal correlation between different modes of vibration. Using the proposed approach, the responses of a cable-stayed bridge are obtained under a set of parametric variations. The parameters include the tower-deck inertia ratio, the angle of incidence of earthquake, the ratio between the three principal components of ground motion, the number and nature of mode shapes being considered, the nature of the power spectral density function (psdf) of the ground motion, and the spatial correlation function.

2. Seismic excitation

The seismic excitation is considered as a three component stationary random process. The components of the ground motion along an arbitrary set of orthogonal directions will be usually statistically correlated. However, as observed by Penzien and Watable [15], the three components of ground motion along a set of principal axes are uncorrelated. These components, directed along the principal axes, are usually such that the major principal axis is directed towards the expected epicenter, the moderate principal axis is directed perpendicular to it

(horizontally) and the minor principal axis is directed vertically. In this study, the three components of the ground motion are assumed to be directed along the principal axes. Each component is assumed to be a stationary random and partially correlated process with zero mean characterized by a psdf. The psdf of ground acceleration in each direction is defined by

$$S_{\ddot{u}_g\ddot{u}_g}(\omega) = |H_1(i\omega)|^2 \, |H_2(i\omega)|^2 S_0 \qquad (1)$$

in which S_0 is the spectrum of the white-noise bed rock acceleration; $|H_1(i\omega)|^2$ and $|H_2(i\omega)|^2$ are the transfer functions of the first and the second filters representing the dynamic characteristics of the soil layers above the bedrock, where

$$\left. \begin{array}{l} |H_1(i\omega)|^2 = \dfrac{1 + (2\zeta_g\omega/\omega_g)^2}{[1 - (\omega/\omega_g)^2]^2 + (2\zeta_g\omega/\omega_g)^2} \\[3mm] |H_2(i\omega)|^2 = \dfrac{(\omega/\omega_f)^4}{[1 - (\omega/\omega_f)^2]^2 + (2\zeta_f\omega/\omega_f)^2} \end{array} \right\} \qquad (2)$$

in which ω_g, ζ_g are the resonant frequency and damping ratio of the first filter, and ω_f, ζ_f are those of the second filter.

The cross spectrum between the random ground motion at two stations i,j along the bridge is described by that given by Hindy and Novak [16] as:

$$S_{\ddot{u}_g\ddot{u}_g}(r_{ij},\omega) = S_{\ddot{u}_g\ddot{u}_g}(\omega) \, r_{ij}(\omega) \qquad (3)$$

in which $S_{\ddot{u}_g\ddot{u}_g}(\omega)$ is the local spectrum of ground acceleration as given in equation (1), which is assumed to be the same for all supports and $r_{ij}(\omega)$ is the function of correlation function between two excitation points represented by

$$r_{ij}(\omega) = \mathrm{Exp} \, (-cL \, \omega/2\pi v_s) \qquad (4)$$

in which L is the separation distance between stations i and j measured in the direction of wave propagation; c is a constant depending upon the distance from the epicenter and the inhomogeneity of the medium; v_s is the shear wave velocity of the soil; and ω is the frequency (rad/s) of the ground motion. Apart from the correlation function given by equation (4), the cases of fully correlated and uncorrelated ground motions at the supports are considered in the study.

For one-sided spectrum it is well-known that

$$\sigma_{\ddot{u}_g}^2 = S_0\left[\int_0^\infty |H_1(i\omega)|^2 * |H_2(i\omega)|^2 \mathrm{d}\omega\right] \qquad (5)$$

where $\sigma_{\ddot{u}_g}^2$ is the variance of ground acceleration. Thus, by defining the filter characteristics ω_g, ζ_g, ω_f, ζ_f and specifying a standard deviation of the ground acceler-

ation $\sigma_{\ddot{u}_g}$, the psdf of the ground acceleration can be completely defined.

The psdfs $S_{u_g u_g}(\omega)$ and $S_{\dot{u}_g \dot{u}_g}(\omega)$ of the ground displacement and velocity are related to $S_{\ddot{u}_g \ddot{u}_g}(\omega)$ by

$$S_{u_g u_g}(\omega) = S_{\ddot{u}_g \ddot{u}_g}(\omega)/\omega^4$$

$$S_{\dot{u}_g \dot{u}_g}(\omega) = S_{\ddot{u}_g \ddot{u}_g}(\omega)/\omega^2 \qquad (6)$$

3. Assumptions

Following assumptions are made for the formulation of the problem:

(i) the bridge deck (girder) and the tower are assumed to be axially rigid;

(ii) the bridge deck is assumed to be a continuous beam; the beam does not transmit any moment to the tower through the girder–tower connection;

(iii) towers are assumed fixed at the locations of the pier or well foundation;

(iv) cables are assumed straight under high initial tensions due to dead load and capable of supporting negative force increment during vibration without losing its straight configuration.

(v) an appropriate portion of the mass of the cables is included in the dynamic analysis of the bridge deck, and is assumed to be uniformly distributed over the idealized deck (in addition to the deck mass);

(vi) beam–column effect in the stiffness formulation of the beam is considered for the constant axial force in the beam; its fluctuation due to fluctuating tension in the cable is ignored for the stiffness calculation of the beam-column. Further, cable dynamics is ignored for the bridge deck vibration, i.e. the tension fluctuations in the cables are assumed as quasistatic, and not introduce any nonlinearity in the system.

4. Modelling of the bridge deck

The bridge deck is idealized as continuous beam over the outer abutments and the interior towers as shown in Fig. 1a,b and the effect of the cables is taken as vertical springs at the points of intersections between the cables and the bridge deck. Furthermore, the effect of the spring stiffness is taken as an additional vertical stiffness to the entire flexural stiffness of the bridge.

5. Vertical stiffness of the bridge due to cables

Referring to Fig. 2a, the fluctuation of tension in the ith cable at any instant of time t can be written as:

$$h_i(t) = K_i v(x_i,t) \sin \alpha_i + K_i \Delta_j(t) \cos \alpha_i \qquad (7)$$

where $K_i = E_c A_{c,i}/L_i$ is the stiffness of the ith cable; $v(x_i,t)$ is the displacement of the girder at time t at the joint of ith cable with the girder; $\Delta_j(t)$ is the horizontal sway of the tower at the jth tower–cable joint connecting the ith cable; α_i is the angle of inclination of the ith cable to the horizontal (measured clock-wise from the cable to the horizontal line as shown in Fig. 2a; $A_{c,i},L_i$ are the cross sectional area and the length of the ith cable and E_c is the equivalent modules of elasticity of the straight cables under dead loads.

The changes in tensions in the array of cables can be put in the following matrix form:

$$\{h\}_{Nc \times 1} = [A]_{Nc \times Nd} \{v\}_{Nd \times 1}$$

$$+ [B]_{Nc \times Nt} \{\Delta\}_{Nt \times 1} \qquad (8)$$

where Nc = number of cables (or pair of cables in case of a two-plane cable-stayed bridge); Nd is the number of unrestrained vertical d.o.f.s of the girder at the cable joints; Nt is the number of horizontal tower d.o.f.s at the cable-tower joints; $\{v\}$, $\{\Delta\}$ are the girder and the tower displacement vectors; $\{h\}$ is the vector of incremental cable tensions; $[A]$ and $[B]$ matrices are formed by proper positioning of the elements $K_i \sin \alpha_i$ and $K_i \cos \alpha_i$ as given by equation (7), respectively.

The deflection of the tower at the cable joints can be obtained by assuming that the tower behaves like a vertical beam fixed at the bottom end and restrained horizontally at the level of the bridge deck and subjected to the transverse forces $h_i(t)*\cos\alpha_i$, $(i = 1, Nc)$ at the cable tower joints as shown in Fig. 2b and are given by:

$$\{\Delta\} = [C] \{h\} \qquad (9)$$

where the elements of the matrix $[C]$ can be easily obtained from the deflection equations of a vertical beam fixed at the bottom and constrained horizontally at the deck level. Eliminating $\{\Delta\}$ from equations (8) and (9), the relation between the vectors of incremental cable tensions and girder deflections may be written as:

$$\{h\} = [[I] - [B][C]]^{-1} [A] \{v\} \qquad (10)$$

where $[I]$ is an unit matrix of order Nc.

Premultiplying both sides of equation (10) by a diagonal matrix $[D]$ of order N_c, where the diagonals consist of the terms of $\sin \alpha_i (i = 1$ to $N_c)$, equation (10) can be written as

$$\{h_{v,b}\} = [K_{c,b}] \{v\} \qquad (11)$$

where $\{h_{v,b}\}$ is the vector containing the vertical components of incremental cable tensions, and $[K_{c,b}] = [D]$
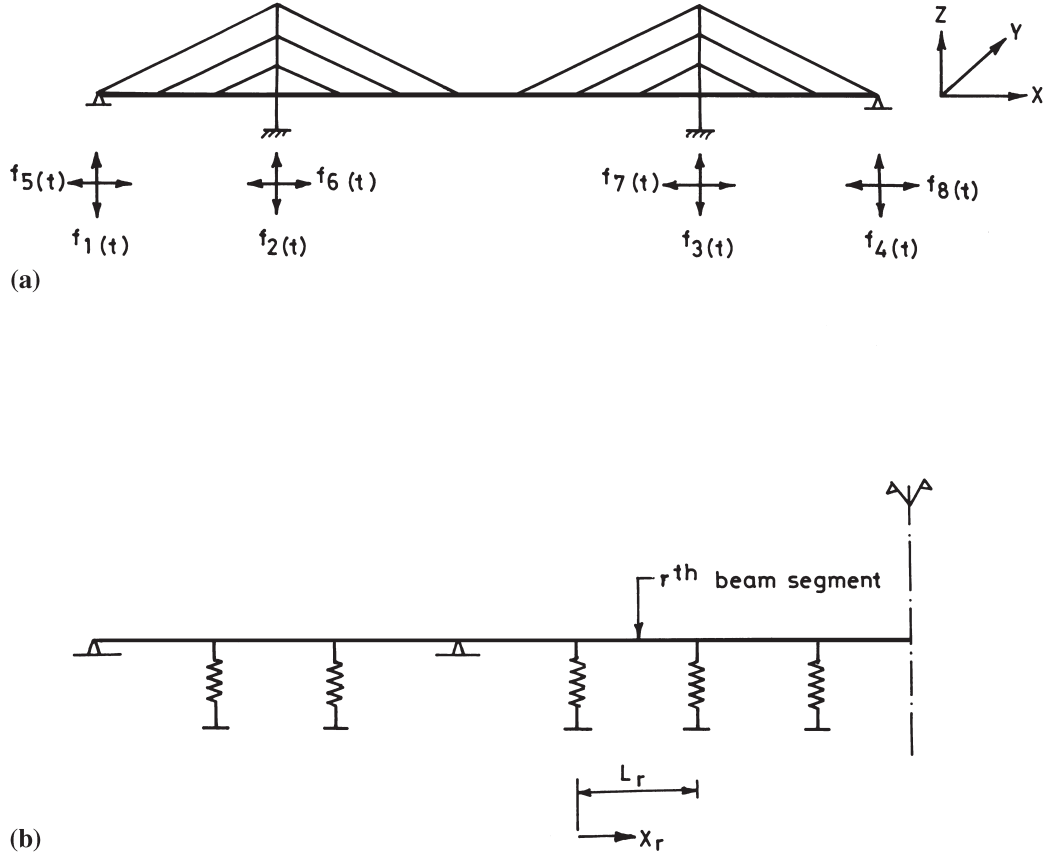
Fig. 1. Problem identification: *a* layout of the bridge under multi-component of support excitations; *b* idealization of the bridge deck.
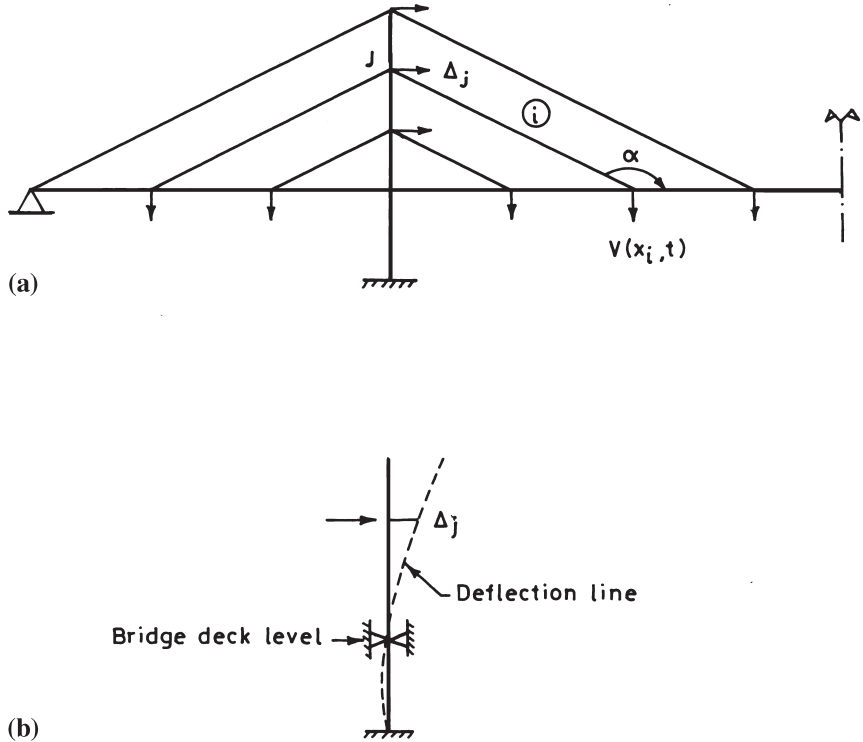

Fig. 2. Tower–deck displacement relationship: *a* displacement due to the fluctuation of the *i*th cable; *b* main system and the displacement of the tower.

$[[I] - [B][C]]^{-1}[A]$ is the stiffness matrix of the bridge contributed by the cables in transverse vibration.

6. Equation of motion

The equation of motion for the relative vertical vibration $y(x_r,t)$ of any segment r of the idealized deck with constant axial force N_r, neglecting the shear deformation and rotary inertia, is given by

$$E_d I_r \frac{\partial^4 y(x_r,t)}{\partial x_r^4} + N_r \frac{\partial^2 y(x_r,t)}{\partial x_r^2} + C_r \frac{\partial y(x_r,t)}{\partial t}$$

$$+ \frac{W_r}{g} \frac{\partial^2 y(x_r,t)}{\partial t^2} = P(x_r,t) \qquad (12)$$

where $r = 1,2,\ldots,N_b$ and

$$P(x_r,t) = -\frac{W_r}{g} \sum_{j=1}^{8} Q_j(x_r)\ddot{f}_j(t)$$

in which $E_d I_r$, W_r, g, E_d, N_r are the flexural rigidity, load per unit length, acceleration due to gravity, the modules of elasticity of the deck material and the axial force given to the beam segment r due to cables, respectively. $p(x_r,t)$ is defined as the applied load due to seismic excitations at different support degrees-of-freedom. $\ddot{f}_j(t)$, $j = 1,2,\ldots,8$ are the accelerations at the different support degrees-of-freedom and $Q_j(x_r)$ is the vertical displacement of the rth segment of the bridge deck due to a unit displacement given at the jth degree of freedom of the supports. $Q_j(x_r)$ is obtained by solving the entire bridge (i.e. deck, towers and cables) considering no moment transfer between the deck and the tower by a separate analysis using stiffness approach.

7. Mode shapes and frequencies

The expression for nth mode shape (undamped) for vertical vibration of the rth segment of the bridge deck is given by:

$$\phi_n(x_r) = A_{nr} \cos\beta_{nr}x_r + B_{nr} \sin\beta_{nr}x_r$$

$$+ C_{nr} \cosh\gamma_{nr}x_r + D_{nr}\sinh\gamma_{nr}x_r \qquad (13)$$

where A_{nr}, etc., are integration constants expressed in terms of nth natural frequency of vertical vibration ω_{bn} and

$$\beta_{nr} = \sqrt{\frac{N_r}{2E_d I_r}(Z_{nr} + 1)} \; ; \; \gamma_{nr} = \sqrt{\frac{N_r}{2E_d I_r}(Z_{nr} - 1)}$$

where

$$Z_{nr} = \sqrt{1 + \frac{4E_d I_r W_r/g}{N_r^2}} \; \omega_{bn}^2$$

The origin for the rth segment is fixed at the left end as shown in Fig. 1b.

Utilizing equation (13), a relation between end displacements (vertical deflection and slope) and end forces (shear forces and bending moments) for the rth segment may be written as:

$$\{F\}_r = [K]_r \{x\}_r \qquad (14)$$

where $\{F\}_r$ and $\{x\}_r$ are the end force and end displacement vectors and $[K]_r$ is the flexural dynamic stiffness of the rth beam segment. The integration constants A_{nr}, B_{nr},...etc. are related to the end displacements as

$$\{C\}_r = [T]_r \{x\}_r \qquad (15)$$

where $\{C\}_r$ is the vector of integration constants containing A_{nr}, etc., and $[T]_r$ is the integration constants matrix. The sign conventions used in the dynamic stiffness formulation are shown in Fig. 3. Both $[K]_r$ and $[T]_r$ are given by Chatterjee and Datta [8].

Assembling the stiffness $[K]_r$ for each element (r) and adding the vertical stiffness due to cables, the overall stiffness of the bridge $[K]$ is obtained. The condition for the free vibration of the bridge deck may then be written as

$$[K] \{U\} = \{0\} \qquad (16a)$$

where $\{U\}$ is the unknown end displacement vector for the beam corresponding to the dynamic degrees-of-freedom (see Fig. 3). Using equation (16a) leads to

$$\det [K] = 0 \qquad (16b)$$

Using Regula–Falsi approach the natural frequencies for the system are determined from the solution of equation (16b). Once the natural frequencies are obtained, mode shapes can be known through the use of equations (16a), (16b) and (13).

8. Modal analysis

The modal analysis for the relative vertical displacement $y(x_r,t)$ for any point in the rth deck segment is given as:

$$y(x_r,t) = \sum_{n=1}^{\infty} \phi_n(x_r)q_n(t) \; r = 1,2,\ldots,N_b \qquad (17)$$

in which $\phi_n(x_r)$ is the nth mode shape of the rth beam segment of the bridge deck and $q_n(t)$ is the nth gen-
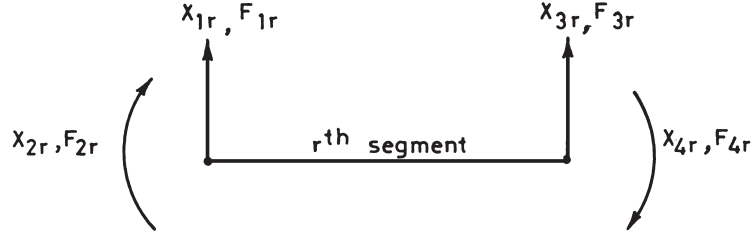
Fig. 3. Sign conventions used in the dynamic stiffness formulation.

eralized coordinate. Substituting equation (17) into equation (12), multiplying by $\phi_m(x_i)$, integrating w.r.t. L_r and using the orthogonality of the mode shapes leads to

$$\ddot{q}_n(t) + 2\zeta_n\omega_n[\dot{q}^{b4}]_n(t) + \omega_n^2 q_n(t) = \bar{P}_n(t) \quad n = 1,\dots,M$$

(18)

in which ζ_n and ω_n are the damping ratio and the natural frequency of the nth vertical mode; M is the number of modes considered and $\bar{P}_n(t)$ is the generalized force given as

$$\bar{P}_n(t) = \sum_{j=1}^{8} R_{jn}(x_r)\ddot{f}_j(t)$$

(19)

where R_{jn} is the modal participation factor given by

$$R_{jn} = -\frac{\displaystyle\sum_{r=1}^{N_b} \frac{W_r}{g} \int_0^{L_r} Q_j(x_r)\phi_n(x_r)\mathrm{d}x_r}{\displaystyle\sum_{r=1}^{N_b} \frac{W_r}{g} \int_0^{L_r} \phi_n^2(x_r)\mathrm{d}x_r}$$

in which $Q_j(x_r)$ is the vertical displacement in the rth beam segment of the bridge deck due to unit displacement given in the jth direction of support movement.

Equation (19) can be put in the following matrix form

$$\bar{P}_n(t) = [G_n]\{f\}$$

(20)

in which $[G_n] = \{G_{1n},\dots,G_{8n}\}$; $\{f\}^{\mathrm{T}} = \{\ddot{f}_1(t),\dots,\ddot{f}_8(t)\}$

where $[G_n]$ is the generalized force coefficients at the nth mode and can be obtained by equation (19).

9. Spectral analysis

9.1. Evaluation of the relative displacement

Applying the principles of modal spectral analysis, the cross power spectral density function between two generalized co-ordinates $q_n(\omega)$ and $q_m(\omega)$ is given by

$$S_{q_n q_m}(\omega) = H_n^*(\omega)H_m(\omega)S_{\bar{P}_n\bar{P}_m}$$

(21)

in which $H_n(\omega)$ is the nth modal complex frequency response function given by

$$H_n(\omega) = [(\omega_n^2 - \omega^2) + i(2\zeta_n\omega_n\omega)]^{-1}$$

(22)

$H_n^*(\omega)$ denotes the complex conjugate of $H_n(\omega)$. $S_{\bar{P}_n\bar{P}_m}$ can be written in the matrix form as:

$$S_{\bar{P}_n\bar{P}_m} = [G_n][S_{ff}][G_m]^{\mathrm{T}}$$

(23)

$[S_{ff}]$ is the psdf matrix for the ground motion inputs (of size 8×8) which are the support accelerations, i.e. $\ddot{f}_1(t)$, $\ddot{f}_2(t)$, $\ddot{f}_3(t)$, $\ddot{f}_4(t)$, $\ddot{f}_5(t)$, $\ddot{f}_6(t)$, $\ddot{f}_7(t)$, $\ddot{f}_8(t)$.

Any element of the matrix $[S_{ff}]$ may be written in the form $r_{ij}(\omega)$ $S_{\ddot{u}_g\ddot{u}_g}(\omega)$ where $r_{ij}(\omega)$ is the correlation function between the ith and jth support; $S_{\ddot{u}_g\ddot{u}_g}(\omega)$ is the psdf of ground acceleration $(\ddot{u}_g = \ddot{f}_j, j = 1,2,\dots,8)$.

Thus, $[S_{ff}]$ may be assembled in the form

$$[S_{ff}] = [R]\, S_{\ddot{u}_g\ddot{u}_g}(\omega)$$

(24)

where $[R]$ is a matrix of size 8×8. Using the expression given in equation (21), the elements of the matrix $[S_{qq}]$ may be formed which has the dimension of $M \times M$. Since the relative displacement $y(x_r,t)$ is given by

$$y(x_r,t) = [\phi(x_r)]_{(1\times M)}\{q\}_{(M\times 1)}$$

(25)

the psdf of the response $y(x_r,t)$ is given by

$$S_{yy}(x_r,\omega) = [\phi(x_r)]\,[S_{qq}]\,[\phi(x_r)]^{\mathrm{T}}$$

(26)

9.2. Evaluation of the quasi-static displacement

The quasi-static component of the vertical displacement at any point in the rth deck segment at time (t) is given as:

$$Q(x_r,t) = [Q]\{\bar{f}\}$$

(27)

where $[Q] = \{Q_1(x_r)\ \ Q_2(x_r),\dots,Q_8(x_r)\}$; $\{\bar{f}\}^{\mathrm{T}} = \{f_1(t)\ f_2(t),\dots,f_8(t)\}$

$Q_j(x_r)$ is the vertical displacement at any point in the rth beam segment of the bridge deck due to unit movement of the jth support d.o.f. The psdf of the quasi-static displacement at any point in the rth deck segment is given by

$$S_{QQ}(x_r,\omega) = [Q]\,[S_{\bar{f}\bar{f}}]\,[Q]^{\mathrm{T}} \tag{28}$$

where $[S_{\bar{f}\bar{f}}]$ is the psdf matrix for the ground displacements at the support d.o.f.s and can be readily obtained from the matrix $[S_{ff}]$.

9.3. Evaluation of the total displacement

The total displacement at any point of the rth beam segment of the bridge deck at any time (t) can be written as

$$v(x_r,t) = y(x_r,t) + Q(x_r,t) \tag{29}$$

the psdf of the total vertical displacement can be expressed as

$$S_{vv}(x_r,\omega) = S_{yy}(x_r,\omega) + S_{QQ}(x_r,\omega)$$
$$+ S_{yQ}(x_r,\omega) + S_{Qy}(x_r,\omega) \tag{30}$$

where $S_{yQ}(x_r,\omega)$, $S_{Qy}(x_r,\omega)$ are the cross power spectral density functions between the relative and quasi-static displacements. Using equations (20), (25) and (27), the expression for $S_{yQ}(x_r,\omega)$ may be obtained as:

$$S_{yQ}(x_r,\omega) = \{\phi_n(x_r)\}^{\mathrm{T}}_{(1 \times M)} * \operatorname*{diag}_{(M \times M)}\,[H_n(\omega)]$$
$$* \underset{(M \times 8)}{[G]} * \underset{(8 \times 8)}{[S_{\bar{f}f}]} * \{Q_j(x_r)\}_{(8 \times 1)}$$
$$n = 1,2,\ldots,M;\; j = 1,2,\ldots,8;\; r = 1,2,\ldots,N_{\mathrm{b}} \tag{31}$$

where $[S_{\bar{f}f}]$ is the cross power spectral density matrix of the random vectors $\{f\}$ and $\{\bar{f}\}$, i.e. the support accelerations and displacements. $S_{Qy}(x_r,\omega)$ is the complex conjugate of $S_{yQ}(x_r,\omega)$.

9.4. Evaluation of the bending moment

Using equations (25) and (27), and differentiating the expression for $v(x_i,t)$ twice with respect to x, the following expression for the bending moment can be obtained

$$E_{\mathrm{d}}I_r\,\frac{\partial^2 v}{\partial x^2} = \sum_{n=1}^{M} E_{\mathrm{d}}I_r\,\frac{\mathrm{d}^2\phi(x_r)}{\mathrm{d}x^2}$$
$$q_n(t) + \sum_{j=1}^{8} E_{\mathrm{d}}I_r\,\frac{\mathrm{d}^2 Q_j(x_r)}{\mathrm{d}x^2}\,f_j(t) \tag{32}$$

Similar expressions can be obtained for the psdf of the bending moment at any point in the rth beam segment

of the bridge deck as those derived for the total displacement by replacing $\phi(x_r)$ and $Q(x_r)$ by $E_{\mathrm{d}}I_r\,\mathrm{d}^2\phi(x_r)/\mathrm{d}x^2$ and $E_{\mathrm{d}}I_r\,\mathrm{d}^2 Q(x_r)/\mathrm{d}x^2$, respectively. $E_{\mathrm{d}}I_r\,\mathrm{d}^2 Q(x_r)/\mathrm{d}x^2$ is obtained from the quasi-static analysis of the entire bridge using the stiffness approach as mentioned before.

10. Parametric study

A double plane symmetrical cable-stayed bridge (as considered by Morris [10], Fig. 4) is considered for the parametric study with the following data: $E_{\mathrm{d}} = E_{\mathrm{t}} = 2.0683 \times 10^{11}$ N/m^2; $I_{\mathrm{d}} = 1.131$ m^4; distributed mass of the bridge over half width deck is 9.016×10^3 kg/m; areas and initial tensions for the cables (1–6) are 0.04, 0.016, 0.016, 0.016, 0.016, 0.04 m^2 and 15.5×10^6, 5.9×10^6, 5.9×10^6, 4.3×10^6, 5.9×10^6, 15.5×10^6 N, respectively.

In addition, the following data are assumed for the analysis of the problem, $E_{\mathrm{c}} = E_{\mathrm{d}}$; $\zeta = 0.02$ for all modes; the connections between the cables and the towers are assumed to be of hinged type and the tower-deck inertia ratio is taken as four unless mentioned otherwise; the ratio between three components of ground motion ($\sigma_x{:}\sigma_y{:}\sigma_z$) is taken as 1.0:1.0:1.0 and the major principal axis of the ground motion is directed with the longitudinal direction of the bridge, unless mentioned otherwise.

The random ground motion is assumed to be a homogeneous stochastic process which is represented by Clough and Penzien double filter psdf given by equation (1) with two sets of filter coefficients representing the soft and firm soils, respectively. For the soft soil, the coefficients are $\omega_{\mathrm{g}} = 6.2832$ rad/s; $\omega_{\mathrm{f}} = 0.62832$ rad/sec; $\zeta_{\mathrm{g}} = \zeta_{\mathrm{f}} = 0.4$, while those for the firm soil are $\omega_{\mathrm{g}} = 15.708$ rad/s; $\omega_{\mathrm{f}} = 1.5708$ rad/s; $\zeta_{\mathrm{g}} = \zeta_{\mathrm{f}} = 0.6$. Another set of filter coefficients is also considered in the study namely, $\omega_{\mathrm{g}} = 31.416$ rad/s; $\omega_{\mathrm{f}} = 3.1416$ rad/s; $\zeta_{\mathrm{g}} = \zeta_{\mathrm{f}} = 0.8$. This represents the ground motion in a very firm soil medium. The three psdfs corresponding to the three sets of filter coefficients are shown in Fig. 5. The spatial correlation function used in the parametric study is given by equation (4) in which the value of c is taken as 2.0 and v_{s} is taken as 70, 330, 550 m/s for the first, second and third psdfs, respectively. The r.m.s. of ground acceleration is taken as $\sigma_{\mathrm{u}_g} = 0.61$ m/s^2. The first 10 frequencies and the corresponding nature of the mode shapes are given in Table 3.

10.1. Effect of mode shapes on the response

The effect of the number of modes on the response quantities of interest is shown in Table 1. It is seen from the table that the first six modes practically govern the overall response for displacement, whereas practically 10 modes are required to compute the bending moments correctly. There is practically no change in the responses

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