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The importance of distribution types on finite element analyses of foundation settlement

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A B S T R A C T

We investigate the effects of using different types of statistical distributions (lognormal, gamma, and beta) to characterize the variability of Young's modulus of soils in random finite element analyses of shallow foundation settlement. We use a two-dimensional linear elastic, plane-strain, finite element model with a rigid footing founded on elastic soil. Poisson's ratio of the soil is considered constant, and Young's modulus is characterized using random fields with extreme values of the scale of fluctuation. We perform an extensive sensitivity analysis to compare the distributions of computed settlements when different types of statistical distributions of Young's modulus, different coefficients of variation of Young's modulus, and different scales of fluctuation of the random field of Young's modulus are considered. A large number of realizations are employed in the Monte Carlo simulations to investigate the influence of the tails of the statistical distributions under study. Results indicate the type of distribution considered for characterization of the random field of Young's modulus can have a significant impact on computed settlement results. In particular, considering different types of distributions of Young's modulus can lead to more than 600% differences on computed mean settlements for cases with high coefficient of variation and large scale of fluctuation of Young's modulus. The effect of considering different types of distributions is reduced, but not completely eliminated, for smaller coefficients of variation of Young's modulus (because the differences between distributions decrease) and for small values of the scale of fluctuation of Young's modulus (because of an identified "averaging effect").

1. Introduction

Uncertainty is a central aspect of geotechnical engineering [1]. The sources of uncertainty for the estimation of soil properties include inherent soil variability, measurement error, and uncertainty in models to infer soil properties from in situ measurements [2,3]. Furthermore, when the response or the failure mode of a geotechnical structure is of interest, additional uncertainties exist due to the mechanics of the problem and to the calculation models employed [4].

Uncertainties due to inherent soil variability and to measurement error have been traditionally addressed using the theory of random fields [5]. The estimation of ranges of variability of random fields of soil properties at different scales has received wide attention [2,6,7], and several correlation models for characterization of spatial variability of soil properties have been proposed as well [4,8,9]. The estimation of the spatial correlation structure of soil formations has been mainly performed using a maximum likelihood approach [3,8], although Bayesian [4] or bootstrap re-sampling methods [10] have also been proposed. Given practical difficulties with the characterization of spatial variability in real applications, however, some authors perform sensitivity analyses to identify the most unfavorable scale of fluctuation [11–14], which can then be (conservatively) employed in subsequent analysis. (The scale of fluctuation, θ_E, represents a distance such that the values of the random field are effectively uncorrelated for points in the soil mass which are further apart than θ_E [15].)

Random fields have also been employed to study the effects of stochastic soil properties on a wide variety of geotechnical problems. For instance, there has been research to study the effects of spatial variability of soil properties on liquefaction [16–19]; on the performance of retaining walls [20]; on the stability of slopes [21–26]; on the mechanics of fluid transport and seepage in porous media [27–37]; on stochastic soil dynamics [38,39]; and on the stress–strain response of geotechnical materials [40], among other topics.

The effects of stochastic soil properties on foundation design have also received wide attention. Griffiths and Fenton [41] revised

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Prandtl's theory for the ultimate bearing capacity of shallow foundations considering the variability of undrained shear strength; Griffiths et al. [12] discussed the effect of the variability of undrained shear strength on the bearing capacity of a rough rigid footing; and Fenton and Griffiths [11] studied the influence of spatial variability and cross-correlation of friction and cohesion of soils on bearing capacity. More recently, Popescu et al. [42] presented a sensitivity analysis to study different aspects of soil's shear strength randomness on bearing capacity; whereas Niandou and Breysse [14] studied how the response of a piled raft can be influenced by horizontal soil variability. Studies of soil-structure interaction considering the effects of soil heterogeneity have been presented as well [43,44].

Settlements of shallow foundations on spatially random soil have been studied with the stochastic finite element method [45–47] and with the stochastic integral formulation method [48]. Analytical solutions have also been employed to compute differential settlements of structures [49], although the main trend has been toward the use of the finite element method in combination with Monte Carlo simulation. In that sense, for instance, several authors have studied the sensitivity of settlement results to changes in the mean, variance, and scale of fluctuation of geotechnical properties, see, e.g. [15,50,51]. Two-dimensional variability has been considered in most cases, but the variability of soil properties in three-dimensions has also been considered in recent studies [52]. The random finite element method has also been employed to develop a load a resistance factor approach to foundation settlement [13]. Another line of research to study the effects of uncertainties on foundation settlement has been based on the use of field observations of actual foundation settlements. In that sense, for instance, Sivakugan and Johnson [53] showed that the ratio of predicted to actual settlements of shallow foundations in granular soils follows a beta distribution, and they produced design charts (see [54]) to compute the probability of actual settlements being above specified limits, in which they considered settlements predicted with four design methods that are commonly employed in practice.

The research cited above illustrates the importance of considering the variability and uncertainty of soil properties for geotechnical design. In particular, it demonstrates that the stochastic nature of the soil can significantly reduce the reliability of geotechnical designs; it also allows one to model the occurrence of failure modes that cannot be identified under the assumption of homogeneous soil. In foundation design, for instance, the consideration of the stochastic nature of the soil can lead to a reduction of the bearing capacity or to an increase of the foundation settlement; it also allows the identification of non-symmetric failure modes and of differential settlements in numerical models. Such responses could not be reproduced in a deterministic analysis with non-variable soil properties.

In this work we study the settlement of a rigid footing founded on spatially random soil and, in particular, we focus on the effects of selecting different types of statistical distributions to simulate

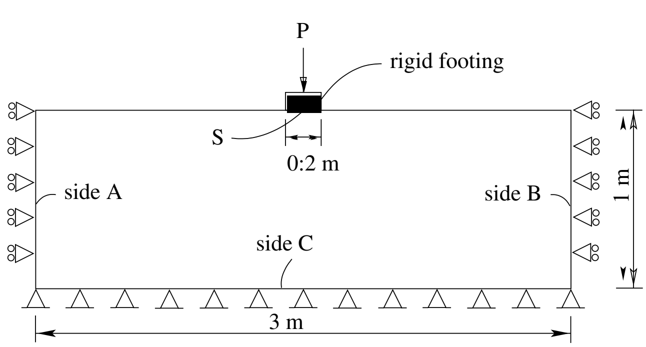


Fig. 1. Geometry of the model.

Table 1
Input parameters varied in the simulation analyses

Parameter	Values considered
Distribution type	{Lognormal, Gamma, Beta}
COV_E	{0.1, 0.5, 1.0}
$\theta_E \rightarrow$	{0.0, ∞}

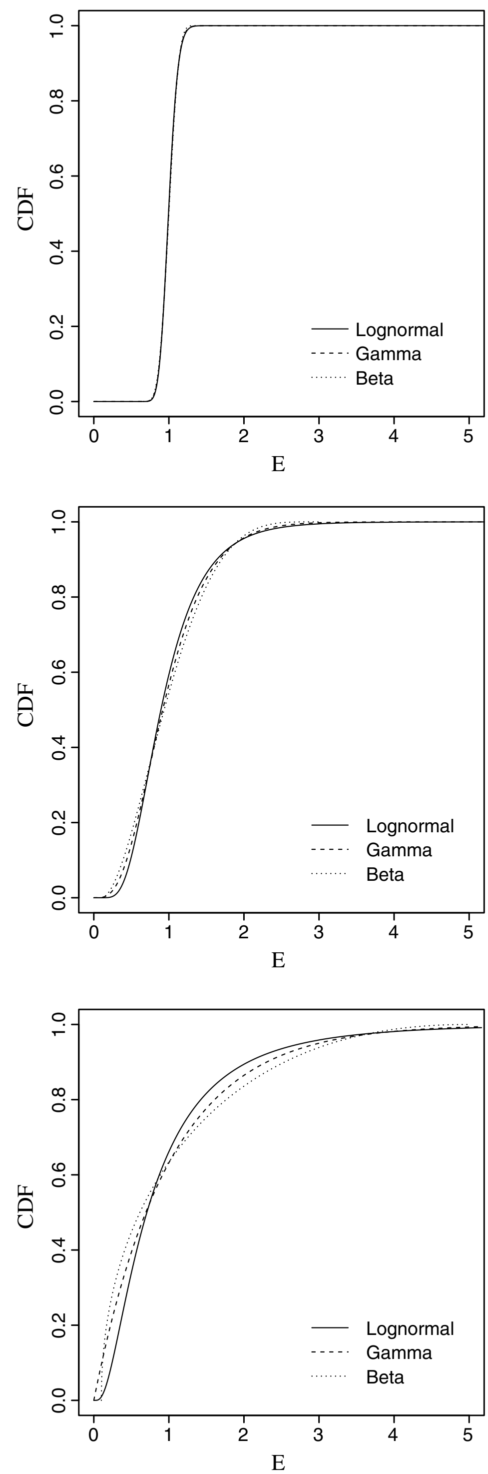


Fig. 2. Comparison between lognormal, gamma, and beta distributions employed to characterize variability of Young's modulus (*E* values scaled by $\times 10$ MPa).

the random field that represents the mechanical properties of the underlying soil. (For a study of the statistical distribution of actual foundation settlements in granular soils, see [53].) Accurate estimation of foundation settlement is an important problem of geotechnical engineering design, as buildings and supported elements can be damaged when settlements (in particular, differential settlements) exceed certain thresholds. For that reason, most design codes impose limitations on the amount of total settlements to reduce the likelihood of occurrence of serviceability limit states due to excessive differential settlements [51,52,55].

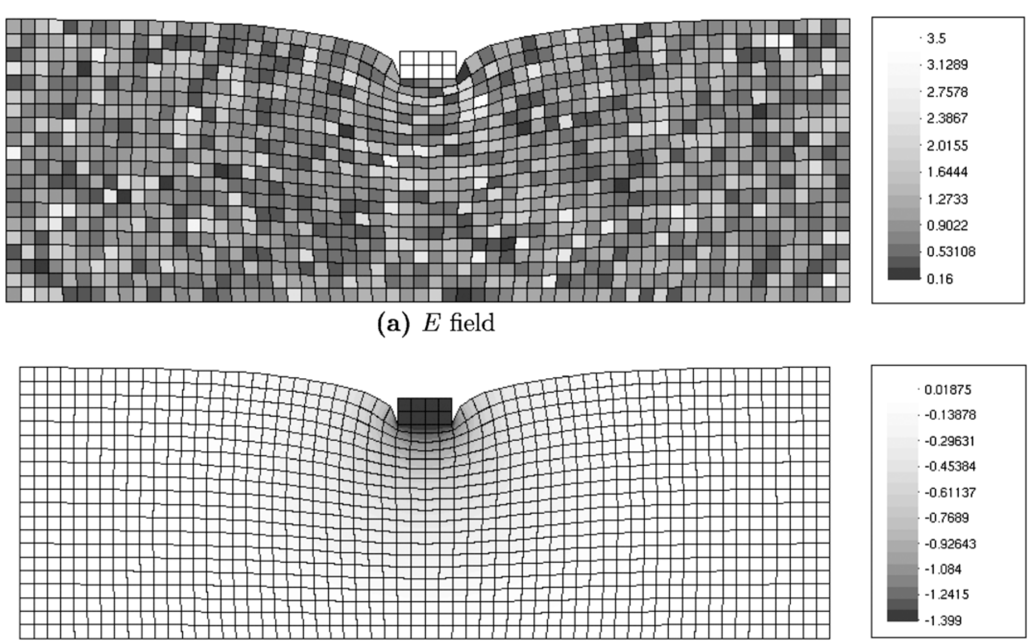
Total settlements are usually computed using elasticity theory in typical foundation design, and elastic properties of soils (i.e., Young's modulus, E; and Poisson's ratio, ν) are chosen to represent both the immediate and consolidation components of settlement [13,51]. Constant Poisson's ratio is usually considered due to difficulties with its characterization and because it has a secondary influence on settlement computations [15,51], although its variability has been also addressed using the beta distribution [50]. The lognormal distribution has been widely employed to model variations of the Young's modulus of soils, see, e.g. [15,50–52]. Arguments in favor of the lognormal distribution are that it has zero probability of generating negative values of Young's modulus and that it is mathematically convenient.

However, the use of the lognormal distribution to model the random field of Young's modulus in finite element computations of foundation settlement has been generally accepted (but not always) without further investigating whether it is the most adequate type of distribution to model the variability of soil's properties in each particular case, or what would be the effects of employing a different type of distribution. In that sense, for instance, Popescu et al. [42] illustrated the importance of this aspect of uncertainty characterization in bearing capacity analyses, and they showed that the coefficient of variation and the marginal probability distribution of the soil's shear strength "are the two most important parameters in reducing the bearing capacity (in an average sense) and producing substantial differential settlements in heterogeneous soils". Similarly, Zhou et al. [56] studied the effects of uncertainties on the design of prefabricated vertical drains, and they found that the type of statistical distribution employed to model the coefficient of radial consolidation (they considered the lognormal, gamma, and Weibull distributions) has a "considerable influence" on the consolidation results.

As we will show, the selection of a specific type of statistical distribution to model the random field of Young's modulus of the soil can also have a significant impact on the computed foundation settlement results. To illustrate such a point, we compare settlements computed using simulations with finite elements in which different types of statistical distributions are employed to characterize the variability of Young's modulus. In particular, we compare settlements computed in cases in which the Young's modulus of the soil is modeled using the lognormal distribution (defined by its first two moments; i.e., mean and standard deviation) with settlements computed cases in which the Young's modulus of the soil is modeled using the gamma and the beta distributions (with the same mean and standard deviation). The gamma and beta distributions have been selected for this study because they have the same proposed advantages of the lognormal distribution and, in particular, they also have zero probability of producing negative values of Young's modulus.

2. Finite element model

We use a two-dimensional linear elastic finite element model with a plane strain formulation to compute the settlement of a shallow rigid footing on spatially random soil. The geometry of the model is presented in Fig. 1. The finite element model has a mesh with 1208 elements (1200 for the soil and 8 for the footing) and 1291 nodes. The total number of degrees of freedom is 2420. Nodes located along sides A and B (see Fig. 1) have their horizontal displacement constrained, whereas nodes along side C have all their displacements constrained to simulate a rigid underlying stratum. Finite element computations are performed using the general purpose research finite element IRIS [57]. Some aspects



(b) Displacements

Fig. 3. Example realization of random field of Young's modulus of the soil (for $\theta_E \to 0$) and the corresponding field of computed displacements: (a) E field and (b) displacements.

of IRIS were specifically optimized for this research to allow a large number of analyses in a reasonable time.

The variability of the Young's modulus of the soil is characterized using an isotropic two-dimensional random field. This could be a limitation, as most soils have a stronger correlation structure in the horizontal direction than in the vertical direction; following Fenton and Griffiths [52], however, the use of anisotropic random fields is left as a refinement for site-specific studies. This model has the additional limitation that it does not consider the variability of soil properties in the out-of-plane direction; in other words, it considers that the scale of fluctuation in this direction is infinite, so that the Young's modulus of the soil in this direction is constant for each particular realization [15]. The consequences of considering a two-dimensional model, however, have been found to be not very significant [52] and, at the same time, considering a two-dimensional model allows us to perform more extensive simulations that explore the effects of slight differences at the tails of the statistical distributions that model Young's modulus.

We apply a vertical point load of value 10 kN on top of the rigid footing. The material properties of the soil are a constant Poisson's coefficient, $\nu = 0.25$, and a random Young's modulus E. Different types of statistical distribution are employed to characterize the variability of Young's modulus, as well as different coefficients of variation and scales of fluctuation (see Section 3). To model the footing as a rigid foundation, we select a Young's modulus for the footing which is orders of magnitude higher than the Young's modulus of the soil. The output variable of the model is the settlement of the central point of the footing. (We consider settlements to be positive for downward movement.) Such settlement corresponds with the vertical displacement of point S in Fig. 1.

3. Monte Carlo simulation

We perform an extensive Monte Carlo simulation analysis using the finite element model presented in Section 2. In particular, we consider three types of statistical distributions (lognormal, gamma, and beta) to model the variability of Young's modulus of the soil below the footing. To ease the comparison of results, simulations are performed using the same mean and standard deviation for all distributions. The mean Young's modulus of the soil is $\mu_E = 10$ MPa in all cases, and standard deviations considered are $\sigma_E = \{0.1, 0.5, 1.0\} \times 10$ MPa. That is, the coefficients of variation of Young's modulus are $COV_E = \{0.1, 0.5, 1.0\}$. These values of COV_E are thought to be representative of ranges of variability that can be expected for typical soils; for instance, Phoon and Kulhawy [2] propose values of COV_E (due to inherent variability only) in the range of 15–65%, although they also present data for one case (see Fig. A1 in their paper) in which COV_E due to inherent variability is more than 90%.

Similarly, we consider two extreme values of the scale of fluctuation: $\theta_E \to 0.0$ and $\theta_E \to \infty$. This simplified model has the advantage that it removes the influence of the type of correlation structure considered (e.g., exponential, squared exponential, etc.) as, for any correlation structure, values of E at each element become statistically independent as θ_E approaches zero, whereas they are identical in every element of the finite element model (for a given realization) as θ_E tends to infinity [15]. (Considering only two such extreme values of the scale of fluctuation largely simplifies the process of random field modeling for assignment of E values to each element of the finite element mesh. The reason is that a random number generator for the selected statistical distribution, such as those widely available in standard mathematical packages, is the only tool needed for the task.) Table 1 lists the input parameters that are varied in the sensitivity analysis performed, while keeping constant the footing geometry, vertical load, and Poisson's ratio and mean Young's modulus of the soil (see Section 2).

Fig. 2 shows a comparison between the cumulative density functions (CDF) of the lognormal, gamma, and beta distributions employed for characterization of Young's modulus in the simulation analyses. The same mean value of Young's modulus μ_E is

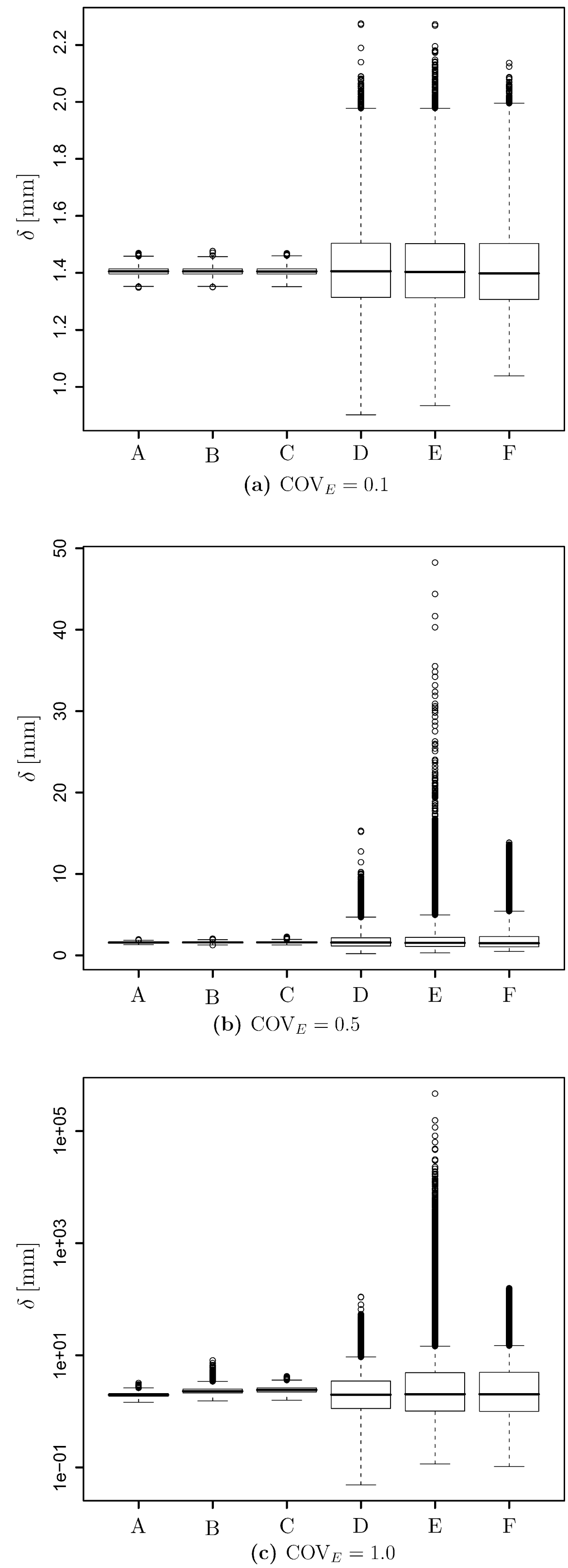


Fig. 4. Computed settlements considering different scales of variability and different distribution types (Key: A = Lognormal; $\theta_E \to 0$; B = Gamma, $\theta_E \to 0$; C = Beta, $\theta_E \to 0$; D = Lognormal, $\theta_E \to \infty$; E = Gamma, $\theta_E \to \infty$; F = Beta, $\theta_E \to \infty$).

considered in all cases. As observed, distributions are almost identical for small values of the coefficient of variation (i.e., when $COV_E = 0.1$), whereas they increase their differences as COV_E increases.

Monte Carlo simulations consist of a total of 240,000 realizations of the Young's modulus random field that are performed, as part of a comprehensive sensitivity analysis, for each combination of Young's modulus distribution type, coefficient of variation,

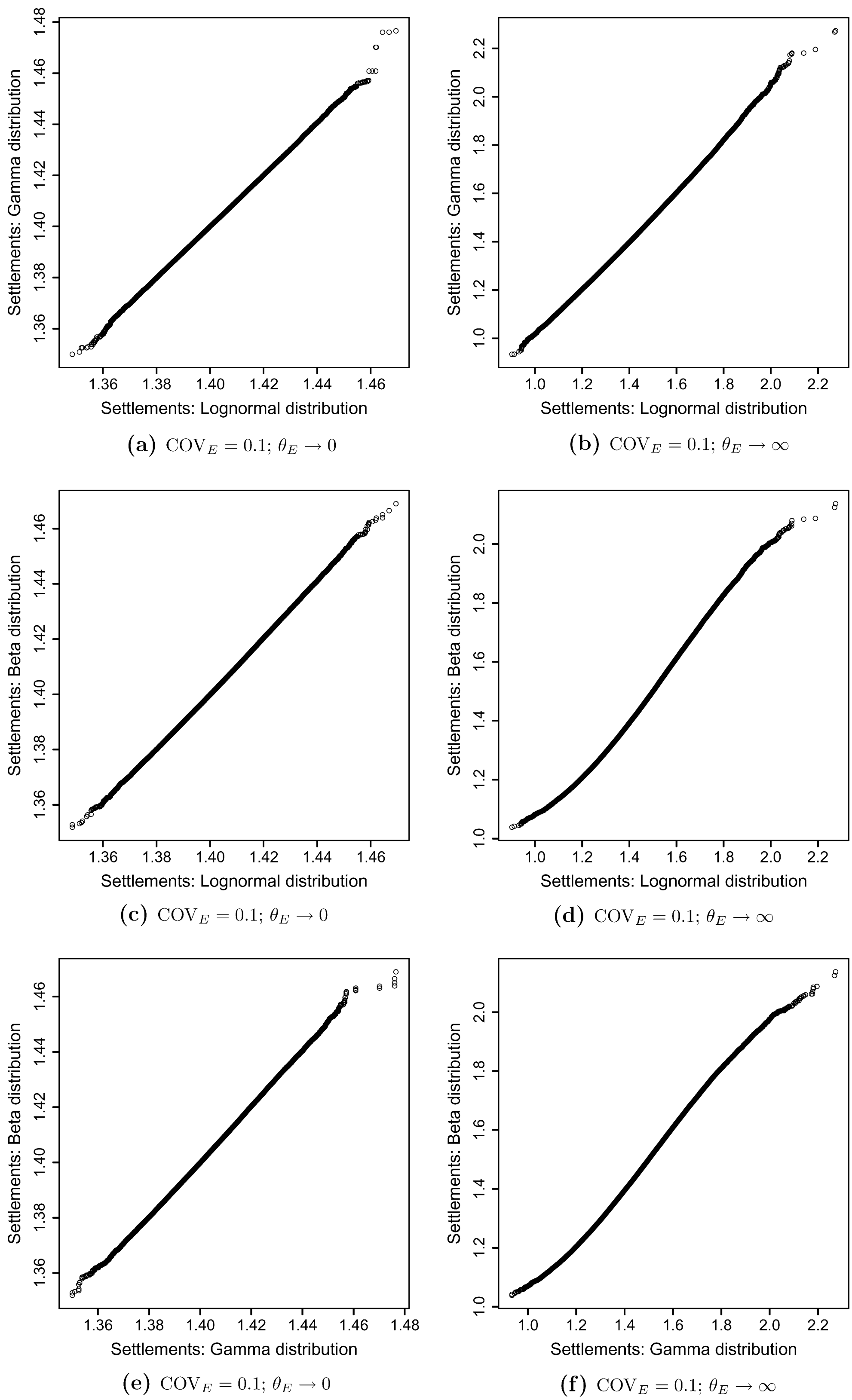


Fig. 5. Comparison between distributions of computed settlement for lognormal and gamma distributed Poisson's modulus.

COV_E, and scale of fluctuation, θ_E (see Table 1). Note that the number of realizations in each simulation is significantly higher than those considered in other analyses of random foundation settlements reported in the literature. Such large number of simulations allows us to account for the effects that the tails of the statistical distributions (i.e., those values with low probability of being sampled) have on the computed settlement results.

In the case of $\theta_E \rightarrow 0.0$, foundation settlements are computed, for each realization of the random field, using the finite element model presented in Section 1. In the case of realizations with

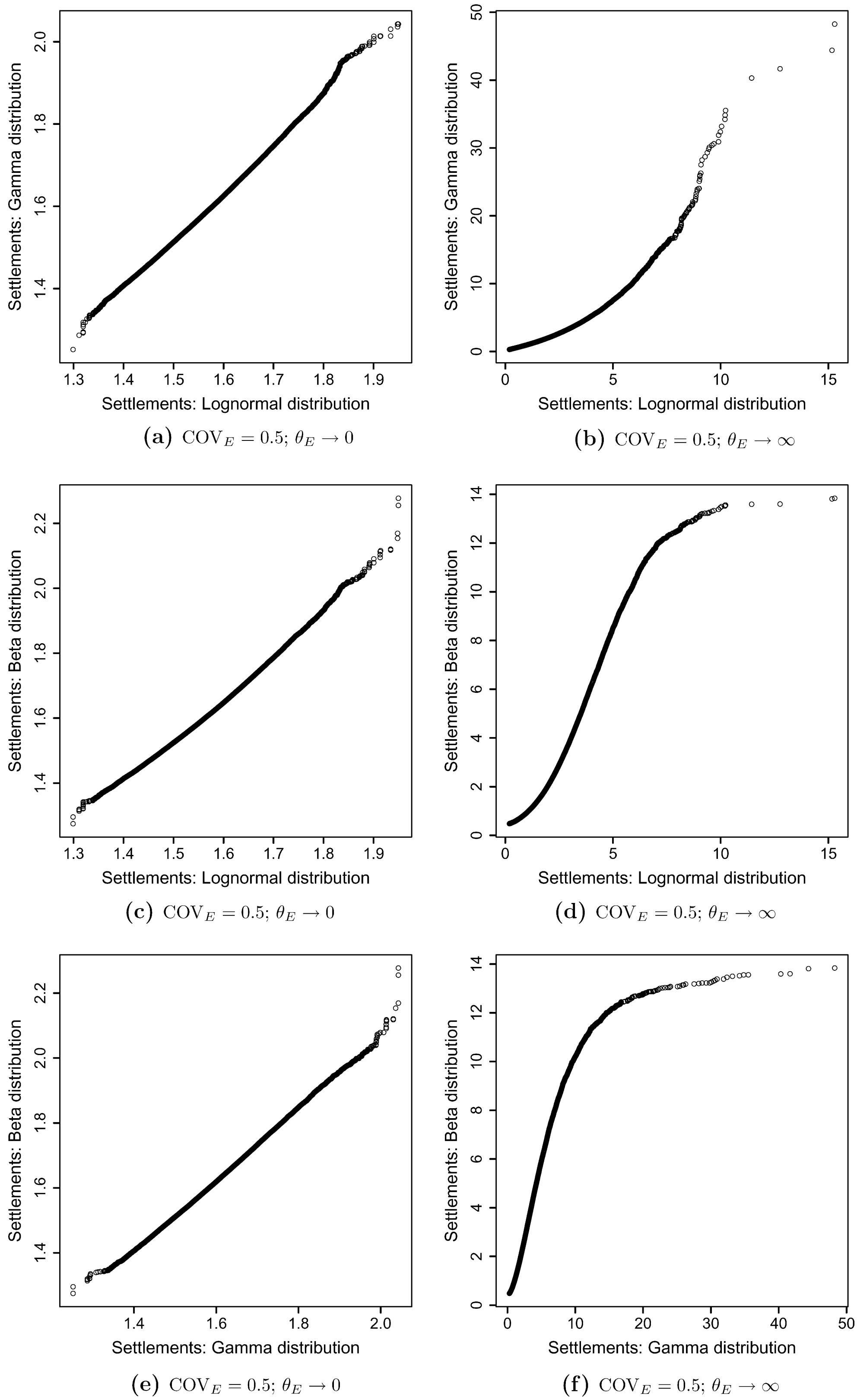
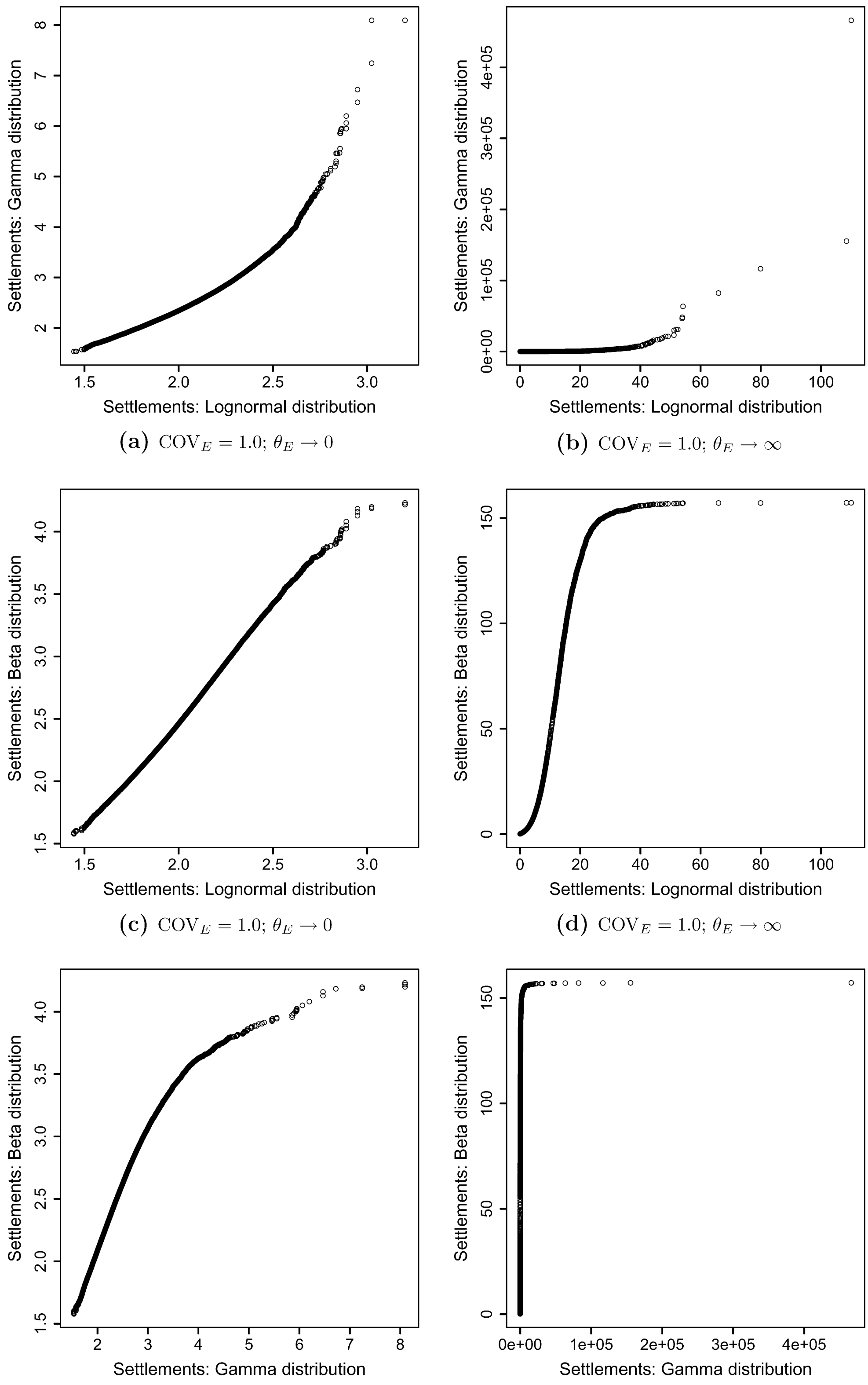


Fig. 6. Comparison between distributions of computed settlement for lognormal and gamma distributed Poisson's modulus.

constant Young's modulus for the entire domain (i.e., $\theta_E \to \infty$), however, there is no need to perform finite element computations for each realization and settlements can be easily scaled from a single finite element computation, as follows [51,52]:

$$\delta = \frac{\delta_{det}\mu_E}{E}, \tag{1}$$

where δ_{det} is the settlement result of a single (deterministic) finite element calculation using $E = \mu_E$ for all elements of the finite element model.



(a) $\mathrm{COV}_E = 1.0;\ \theta_E \to 0$ (b) $\mathrm{COV}_E = 1.0;\ \theta_E \to \infty$

(c) $\mathrm{COV}_E = 1.0;\ \theta_E \to 0$ (d) $\mathrm{COV}_E = 1.0;\ \theta_E \to \infty$

(e) $\mathrm{COV}_E = 1.0;\ \theta_E \to 0$ (f) $\mathrm{COV}_E = 1.0;\ \theta_E \to \infty$

Fig. 7. Comparison between distributions of computed settlement for lognormal and gamma distributed Poisson's modulus.

Fig. 3(a) shows an example realization of the simulated random field of Young's modulus for the case of $\theta_E \to 0$. The corresponding displacement field is presented in Fig. 3(b).

4. Computed results

Fig. 4 shows box-and-whisker plots of the distributions of settlements computed with the Monte Carlo simulation described in Section 3. The horizontal lines inside the boxes indicate the statistical median settlement (i.e., half of the settlements are higher, and half lower, than this value), whereas extremes of the boxes indicate the first and third quartiles of the computed settlement distribution. Whiskers in Fig. 4 extend to points that are within 2.5 times the inter-quartile range, and computed settlements that are outsize that range are explicitly indicated using point symbols. (Note also that settlements in Fig. 4(c) are presented in a logarithmic scale.)

As expected, Fig. 4 shows that the variability of computed settlements increases as the variability of Young's modulus of the soil increases (i.e., as COV_E increases). Results also show that the variability of computed settlements is significantly higher for large scales of fluctuation (i.e., when $\theta_E \to \infty$). In other words, smaller scales of fluctuation tend to produce similar results for different realizations of the random field. (To model this "averaging effect", it has been suggested that the geometric average is the best way to characterize the equivalent deformability of the soil mass around the foundation [51,52].) For large scales of fluctuation, however, this averaging effect is not present, and computed settlements are more sensitive to the type of statistical distribution considered. In that sense, for instance, Fig. 4(b) and (c) show that, specially for large values of θ_E and large values of COV_E, the type of distribution can have a significant influence on the distribution of computed settlements. This is due to the different tail-behavior of the lognormal, gamma, and beta distributions, even when they have the same mean and standard deviation. For instance, Fig. 2(c) shows that the gamma distribution has much higher probability of producing values in its left tail; such low values of Young's modulus are responsible for producing corresponding high values of settlement.

Figs. 5–7 present qq-plots that compare the distributions of computed settlements when the random field of Young's modulus is characterized with the lognormal, gamma, and beta distributions. (qq-plots are employed in statistics to assess if two samples come from the same distribution; if that is the case, points on the qq-plot should lie on a straight line at 45°.) Fig. 5 shows that, as expected, the type of statistical distribution considered for Young's modulus has a negligible effect on computed settlements when such distributions are very similar (i.e., when the variability of Young's modulus is small; $COV_E = 0.1$). The effect of the type of statistical distribution to characterize Young's modulus is significantly higher as COV_E increases and, for the same COV_E, for large values of the scale of fluctuation (i.e., $\theta_E \to \infty$). That is, the type of statistical distribution employed to model Young's modulus has a stronger effect on computed settlements as the distributions of Young's modulus are more different (i.e., as COV_E increases) and as the averaging effect is less significant (i.e., as $\theta_E \to \infty$).

The observation that the influence of the type of statistical distribution increases as COV_E increases agrees with the results obtained by Zhou et al. [56] when they studied the problem of consolidation with vertical drains. In particular, Zhou et al. [56] found that influence of the type of distribution considered "increases considerably when the COV [of the horizontal coefficient of consolidation] increases". Note, however, that they did not employ random fields in their analysis and, therefore, they do not provide information about the influence of the type of distribution when different values of the scale of fluctuation are considered to model the random field of soil properties.

Table 2 illustrates the variation of computed mean settlements, for different scales of fluctuation, as differences between statistical distributions of Young's modulus increase (i.e., as COV_E increases). Our results show that computed mean settlements tend to the deterministic value of settlement (in this case $\delta_{det} = 1.40$ mm) as COV_E approaches zero, with results more similar to δ_{det} as COV_E becomes smaller. In other words, the variability of Young's modulus of the soil tends to produce (in an average sense) larger settlements. At the same time, as we discussed above, our results also show that there is an "averaging effect" for small scales of fluctuation (i.e., $\theta_E \to 0$) that significantly reduces the effects of considering different types of statistical distributions of Young's modulus. Despite such averaging effect, however, the influence of the type of distribution can still be significant, and Table 2 shows that (for $COV_E = 1.0$) a random field of Young's modulus with gamma distribution can produce mean settlements that are up to 18% higher than settlements computed with a random field of Young's modulus with lognormal distribution (for the same mean and standard deviation). Similarly, mean settlements computed with the beta distribution can be up to 23% higher than settlements computed with the lognormal distribution.

Table 2 also shows that the mean settlement can be significantly higher when such "averaging effect" is not present (i.e., when $\theta_E \to \infty$), specially for cases with high variability of properties (i.e., high COV_E). In such cases, it is observed that the type of distribution has a critical effect on computed settlements and, in particular, it is also observed that both the gamma and beta distributions produce mean settlements that are significantly higher than those computed with the lognormal distribution. For instance, for $COV_E = 1.0$ and $\theta_E \to \infty$, the gamma distribution produces mean settlements that are more than 600% higher than mean settlements computed with the lognormal distribution; whereas the beta distribution produces mean settlements that are more than 200% higher than mean settlements computed with the lognormal distribution.

Table 3 illustrates the variation of the computed standard deviation of settlements as the difference between the statistical distributions of Young's modulus increases (i.e., as COV_E increases) when different scales of fluctuation are considered. As discussed

Table 2
Mean foundation settlements computed with different statistical distributions considering different ranges of variability of soil Young's modulus

Distributions	$\theta_E \to$	COV_E		
		0.1	0.5	1.0
Lognormal	0	1.405506	1.563167	1.982429
Gamma	0	1.405510	1.585055	2.337585
Beta	0	1.405586	1.604132	2.439610
Lognormal	∞	1.412745	1.749519	2.800917
Gamma	∞	1.413047	1.862464	17.537013
Beta	∞	1.413184	1.943004	6.451887

Table 3
Standard deviation of settlements computed with different statistical distributions considering different ranges of variability of soil Young's modulus

Distributions	$\theta_E \to$	COV_E		
		0.1	0.5	1.0
Lognormal	0	0.01321550	0.07091457	0.16461638
Gamma	0	0.01330829	0.08152655	0.31380578
Beta	0	0.01341906	0.09036828	0.30383636
Lognormal	∞	0.1414753	0.8747804	2.7978748
Gamma	∞	0.1428554	1.3026505	1082.7932422
Beta	∞	0.144897	1.423578	14.839530

above, the variability of computed settlements is observed to increase (for the same distribution) as COV_E increases. Such an increase is even more significant for large scales of fluctuation (i.e., when the averaging effect is not present; $\theta_E \to \infty$). Finally, note that the type of distribution also affects the variability of computed settlements significantly. In that sense, the results show that the standard deviation of settlements computed with the beta distribution can be about one order of magnitude higher than the standard deviation of settlements computed with the lognormal distribution; whereas the standard deviation of settlements computed with the gamma distribution can be up to several orders of magnitude higher than the standard deviation of settlements computed with the lognormal distribution.

5. Conclusions

We investigate the effects of using different types of statistical distributions (lognormal, gamma, and beta) to characterize the variability of Young's modulus of soils in random finite element analyses of shallow foundation settlement. To that end, we perform an extensive sensitivity analysis to compare the distributions of computed settlements when different types of statistical distributions of Young's modulus, *E*, different coefficients of variation, COV_E, and different scales of fluctuation of the random field, θ_E, are considered. A large number of realizations are employed in such Monte Carlo simulations to investigate the influence of the tails of the statistical distributions under study.

As expected, our results show that distributions of computed settlements are very similar when low values of the coefficient of variation (for the same mean value) are employed for all the distributions considered (i.e., when the distributions of Young's modulus are almost identical). Similarly, our results show that there is an "averaging effect" when the random field of Young's modulus has small scales of fluctuation (i.e., when values of soil deformability at different elements of the random field are independent or, equivalently, when $\theta_E \to 0.0$). Such "averaging effect" makes settlement results to be similar among different realizations of the random field and it reduces the effect of considering different types of statistical distributions of Young's modulus, although it does not eliminate it completely. In this case, for instance, our results suggest that mean settlements computed with the lognormal distribution tend to be lower than mean settlements computed with the gamma and beta distributions (for the same μ_E and COV_E). In particular, for $COV_E = 1.0$, our results show that a random field of Young's modulus with gamma distribution can produce settlements that are (in an average sense) up to 18% higher than settlements computed with the lognormal distribution. Similarly, settlements computed with the beta distribution can be up to 23% higher (in an average sense) than settlements computed with the lognormal distribution.

The influence of the type of distribution can be significantly higher when the scale of fluctuation increases to its other extreme (i.e., when $\theta_E \to \infty$) or, equivalently, when all the soil elements are assigned an identical (but random) value of Young's modulus for a given realization of the random field. In this case the "averaging effect" does not exist, and the influence of the tails of the distributions of Young's modulus of the soil is maximum. In this case, for instance, our results show that settlements computed with the lognormal distribution also tend to be lower than settlements computed with the gamma and beta distributions (for the same μ_E and COV_E). In particular, for the case of $COV_E = 1.0$, our results show that a random field of Young's modulus with gamma distribution can produce settlements that are (in an average sense) up to more than 600% higher than settlements computed with a random field of Young's modulus with lognormal distribution. Similarly, settlements computed with the beta distribution can be (in an average sense) more than 200% higher than settlements computed with the lognormal distribution.

Finally, the results of this research suggest that the identification of the type of statistical distribution that is more adequate to characterize the variability of soil's Young's modulus is an important aspect of modeling in the context of random finite element computations of shallow foundation settlements. The characterization of the range of variability of Young's modulus, COV_E, and of the scale of fluctuation of its random field, θ_E, also appear as crucial aspects of this problem of uncertainty characterization. Despite their importance, however, the level characterization of the uncertainty of soil properties (i.e., type of distribution, mean values and coefficients of variation, scale of fluctuation, etc.) in real geotechnical applications is usually limited. This suggests that our computational models have perhaps surpassed our capability to characterize uncertainties, emphasizing the importance of future research into this significant aspect of geotechnical engineering.

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