

Trajectory Tracking Control of Nonlinear Full Actuated Ship with Disturbances

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Abstract—Based on backstepping technique, this paper presents a design of any reference trajectory tracking controller for marine surface vessels under unknown time-variant environmental disturbances. The mathematical model of nonlinear surface ship movement includes Coriolis and centripetal matrix and nonlinear damp term. The observer is constructed for providing an estimation of unknown disturbances. It is proved that the designed ship trajectory tracking control law can make the resulting closed loop trajectory tracking system of ship be the globally uniformly ultimately bounded and achieve the object of trajectory tracking, while guarantee the global uniform boundedness of all signals of system. The simulation results on a model ship show that the control force and control torque are reasonable, and the transient state and steady-state performances are satisfactory. The effectiveness of the designed control law is verified.

Keywords—ship trajectory tracking control; disturbance observer; backstepping; nonlinear;

I. INTRODUCTION

Controlling of surface marine vessels to track a given trajectory has been a representative control problem for marine applications and it has attracted considerable attention from the control community for many years[1]. It is of great significance for navigation in safety, energy-saving and emission-reduction to implement trajectory tracking of ship. The conventional course autopilot can't meet the requirements for ship steering because track bias is not directly controlled. Along with the development of science and technology, especially the appearance of high accuracy DGPS, it becomes possible to solve the problem of trajectory tracking of ship. On the other hand, the ship motion dynamics are characterized by large inertia, large delay and strong nonlinear, and the ship during navigation is easily influenced by environment such as wind, waves and currents, all of which make the ship trajectory tracking control more complicated. Therefore, the research on the problem of the ship trajectory tracking control has received a lot of attention both in theory and in practice.

A ship trajectory tracking controller was designed to make the system be local asymptotical stable combining LQG method with the feed forward control in [2]. In [3], the fuzzy logic control combined with PID control achieved the semi-global exponential stability of the ship trajectory tracking error system. In [4], taking the advantage of the

neural networks of the model free adaptive, a novel ship trajectory tracking controller with self-learning and self-tuning functions was developed. The models used in all above references are the simplified linear form. Along with the development of the theory of control and the higher requirements for control performance, this is an inevitable trend to use more accurate nonlinear mathematical model to describe the dynamics characteristics of ship maneuvering. In recent years, a good number of nonlinear design techniques have been developed for the tracking control problem for under-actuated ship. Jiang proposed two global tracking control laws for underactuated vessels based on Lyapunov's direct method[5]. Do et al. extended Jiang's work and developed a robust control law to environmental disturbances such as wave, wind, and ocean currents for ships on a linear course[6]. In [7], Petterson and Nijmeijer provided a semi-global exponential stabilization of the tracking error for any desired trajectory using an integrator backstepping approach. In [8], furthermore, they improved the control law in [7] and attained the exponential stability of the closed-loop trajectory tracking system, which was validated by the experimental results with a small model ship. The approach in [8] was further improved to obtain a globally exponentially convergent control law in [9], which was tested with the same vessel and on the same trajectory as the one presented in [8]. [10] introduced a design of global smooth controllers that achieved the practical stabilization of arbitrary reference trajectories. For the full-actuated ships, based on passive theory, in [11] the output feedback controller was designed for the nonlinear ship motion mathematical model containing linear damping only, which achieved the semi-global exponentially stability of the closed loop system of ship trajectory tracking. Du et.al designed a nonlinear trajectory tracking control law for full-actuated ship including Coriolis and centripetal matrix and nonlinear damp term under constant disturbances based on backstepping design tool with integrator in [12]. However, the sea state during navigation of ships always changes. Therefore, assuming the disturbances induced by wind, waves and currents are unknown time-variant, this paper proposes a robust trajectory tracking controller of ship based on disturbance observer, backstepping technique and Lyapunov direct method. The disturbance observer is introduced in the controller design process to provide an estimation of unknown external disturbances.

The rest of this paper is organized as follows. In section II, the 3 DOF nonlinear mathematical model of a surface ship motion under disturbances due to wind, waves and ocean currents is presented. In Section III, the trajectory tracking control law of ship is derived based on backstepping design tool and a disturbance observer. The simulation results are provided to validate the designed controller in Section IV, followed by the conclusions in Section V.

II. PROBLEM STATEMENTS

The 3 DOF nonlinear motion equations of a ship can be expressed as [13][14]

$$\dot{\eta} = R(\psi)v \quad (1)$$

$$M\dot{v} + C(v)v + D(v)v = \tau + b \quad (2)$$

where $\eta = [x, y, \psi]^T$. x, y is the position variables of the ship and $\psi \in [0, 2\pi]$ is the yaw angle of the ship in the earth-fixed frame. $v = [u, v, r]^T$ is the ship velocity vector. u, v and r are respectively the velocities in the surge, sway and yaw of ship in the body-fixed frame. $\tau = [\tau_1, \tau_2, \tau_3]^T$ is the control input vector consisting of the surge force τ_1 , the sway force τ_2 and the yaw torque τ_3 . $b = [b_1, b_2, b_3]^T$ is the vector representing unknown and time-variant external disturbances due to waves, wind and ocean currents in the body-fixed frame. And it is assumed that $\|\dot{b}(t)\| \leq C_d$, where C_d is a nonnegative known constant. The rotation matrix $R(\psi)$, inertia matrix M , hydrodynamic Coriolis and centripetal matrix $C(v)$, and damping matrix $D(v)$ are given by

$$R(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$M = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{23} & m_{33} \end{bmatrix} \quad (4)$$

$$C(v) = \begin{bmatrix} 0 & 0 & -m_{22}v - m_{23}r \\ 0 & 0 & m_{11}u \\ m_{22}v + m_{23}r & -m_{11}u & 0 \end{bmatrix} \quad (5)$$

$$D(v) = D_L + D_{NL} = \begin{bmatrix} d_{11}(u) & 0 & 0 \\ 0 & d_{22}(v, r) & d_{23}(v, r) \\ 0 & d_{32}(v, r) & d_{33}(v, r) \end{bmatrix} \quad (6)$$

where

$$m_{11} = m - X_u, m_{22} = m - Y_v, m_{23} = m x_g - Y_r, m_{33} = I_z - N_r,$$

$$d_{11}(u) = -X_u - X_{u|u}|u|, d_{22}(v, r) = -Y_v - Y_{v|v}|v| - Y_{v|r}|r|, \\ d_{23}(v, r) = -Y_r - Y_{v|r}|v| - Y_{r|r}|r|, d_{32}(v, r) = -N_v - N_{v|v}|v| - N_{v|r}|r|, \\ d_{33}(v, r) = -N_r - N_{v|r}|v| - N_{r|r}|r|,$$

where m is the mass of the ship, I_z is the moment of inertia about the yaw rotation, and the other symbols are referred as the Society of Naval Architects and Marine Engineers (SNAME) notation in [15]. It is noted that M is a positive definite matrix.

The control objective in this paper is to design a feedback control law τ for the system (1)-(2) such that $\eta(t)$ tracks a reference trajectory $\eta_d(t)$, while the closed-loop system of ship trajectory tracking is globally uniformly ultimately bounded.

III. CONTROLLER DESIGN

In this section, a disturbance observer is constructed to provide an estimation of the time-variant external disturbances of the ship and the backstepping technique is introduced to design the controller for trajectory tracking.

A. Disturbance Observer Design

For the disturbance vector, the observer is constructed as follows [9]

$$\hat{b}(t) = \beta + K_0 M v \quad (7)$$

$$\dot{\beta} = -K_0 \beta - K_0 [-C(v)v - D(v)v + \tau + K_0 M v] \quad (8)$$

where K_0 is a positive definite symmetric design matrix and β is an intermediate auxiliary vector.

Define the observer estimation error as:

$$\tilde{b} = b - \hat{b} \quad (9)$$

Differentiating both sides of (9) results in

$$\begin{aligned} \dot{\tilde{b}} &= \dot{b} + K_0 M \dot{v} \\ &= -K_0 \beta - K_0 [-C(v)v - D(v)v + \tau + K_0 M v] \\ &\quad + K_0 M \cdot M^{-1} [-C(v)v - D(v)v + \tau + b] \\ &= -K_0 (\beta + K_0 M v - b) \\ &= K_0 [b - (\beta + K_0 M v)] \\ &= K_0 [b(t) - \hat{b}] \end{aligned} \quad (10)$$

Then

$$\begin{aligned} \dot{\tilde{b}} &= \dot{b} - \dot{\hat{b}} \\ &= \dot{b} - K_0 [b(t) - \hat{b}] \\ &= \dot{b} - K_0 \tilde{b} \end{aligned} \quad (11)$$

Considering the following Lyapunov function candidate

$$V_e = \frac{1}{2} \tilde{b}^T \tilde{b} \quad (12)$$

The time derivative of V_e along the solution of (11) is

$$\begin{aligned} \dot{V}_e &= \tilde{b}^T \dot{\tilde{b}} \\ &= \tilde{b}^T (-K_0 \tilde{b} + \dot{b}) \\ &= -\tilde{b}^T K_0 \tilde{b} + \tilde{b}^T \dot{b} \\ &\leq -\lambda_{\min}(K_0) \tilde{b}^T \tilde{b} + \varepsilon \tilde{b}^T \tilde{b} + (1/4\varepsilon) \dot{b}^T \dot{b} \\ &\leq -2[\lambda_{\min}(K_0) - \varepsilon] V_e + (1/4\varepsilon) C_d^2 \end{aligned} \quad (13)$$

where $\lambda_{\min}(K_0)$ is the minimum eigenvalue of the matrix K_0 ; ε is a positive constant such that

$$\lambda_{\min}(K_0) - \varepsilon > 0 \quad (14)$$

And we have used $\tilde{b}^T \dot{b} \leq \varepsilon \tilde{b}^T \tilde{b} + \frac{1}{4\varepsilon} \dot{b}^T \dot{b}$. It is seen from the equation (13) that V_e exponentially converges to a ball centered at the origin with the radius $R_v = C_d^2 / [8\varepsilon(\lambda_{\min}(K_0) - \varepsilon)]$. Furthermore, it is known from the definition of V_e that the disturbance observer error \tilde{b} exponentially converges to a ball centered at the origin with the radius $R_d = C_d / [2\sqrt{\varepsilon(\lambda_{\min}(K_0) - \varepsilon)}]$. Therefore the following theorem can be proved from the above facts.

Theorem 3.1: Assume that the solution of (7) and (8) exist for all $t \geq t_0 \geq 0$ where t_0 is the initial time, the disturbance observer (7)-(8) guarantees that the observer error \tilde{b} exponentially converges to a ball centered at the origin under the condition (14). The error can be made arbitrarily small by adjusting the matrix K_0 and ε .

B. Control Law Design

The control law design consists of two steps, using the backstepping technique and the disturbance observer in Part A, Section III.

In order to design the control law, firstly define error variables as follows:

$$\eta_e = \eta - \eta_d \quad (15)$$

$$\chi_e = v - \chi_1 \quad (16)$$

$$\tilde{b} = b - \hat{b} \quad (17)$$

where χ_1 is stabilization function vector. Then

$$\begin{aligned} \dot{\eta}_e &= \dot{\eta} - \dot{\eta}_d \\ &= R(\psi)v - \dot{\eta}_d \end{aligned}$$

$$= R(\psi)\chi_e + R(\psi)\chi_1 - \dot{\eta}_d \quad (18)$$

Step 1:

Considering the following Lyapunov function candidate:

$$V_1 = \frac{1}{2} \eta_e^T \eta_e \quad (19)$$

The time derivative of V_1 along the solution of the equation (18) is

$$\begin{aligned} \dot{V}_1 &= \eta_e^T \dot{\eta}_e \\ &= \eta_e^T [R(\psi)\chi_e + R(\psi)\chi_1 - \dot{\eta}_d] \\ &= \eta_e^T [R(\psi)\chi_1 - \dot{\eta}_d] + \eta_e^T R(\psi)\chi_e \end{aligned} \quad (20)$$

We choose the stabilization function vector

$$\chi_1 = R^{-1}(\psi) [-C_1 \eta_e + \dot{\eta}_d] \quad (21)$$

where C_1 is a positive definite symmetric design parameter matrix.

According to (16), we have

$$\dot{V}_1 = -\eta_e^T C_1 \eta_e + \eta_e^T R(\psi)\chi_e \quad (22)$$

Step 2:

From (16) and (21), we have

$$\begin{aligned} \dot{\chi}_e &= \dot{v} - \dot{\chi}_1 \\ &= M^{-1} [-C(v)v - D(v)v + \tau + b - M\dot{\chi}_1] \end{aligned} \quad (23)$$

Considering the Lyapunov function candidate:

$$V_2 = V_1 + \frac{1}{2} \chi_e^T M \chi_e + \frac{1}{2} \tilde{b}^T \tilde{b} \quad (24)$$

In terms of Equations (11), (22) and (23), the time derivative of V_2 is

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \chi_e^T M \dot{\chi}_e + \tilde{b}^T \dot{\tilde{b}} \\ &= -\eta_e^T C_1 \eta_e + \eta_e^T R(\psi)\chi_e + \chi_e^T M \cdot M^{-1} [-C(v)v - D(v)v \\ &\quad + \tau + b - M\dot{\chi}_1] + \tilde{b}^T (\dot{b} - K_0 \tilde{b}) \\ &= -\eta_e^T C_1 \eta_e + \chi_e^T R^T(\psi)\eta_e + \chi_e^T [-C(v)v - D(v)v + \tau + b \\ &\quad - M\dot{\chi}_1] - \tilde{b}^T K_0 \tilde{b} + \tilde{b}^T \dot{b} \\ &= -\eta_e^T C_1 \eta_e + \chi_e^T [R^T(\psi)\eta_e - C(v)v - D(v)v + \tau + b \\ &\quad - M\dot{\chi}_1] - \tilde{b}^T K_0 \tilde{b} + \tilde{b}^T \dot{b} \end{aligned} \quad (25)$$

The control input vector is designed as

$$\begin{aligned} \tau &= C(v)v + D(v)v + M\dot{\chi}_1 - R^T(\psi)\eta_e - C_2 \chi_e - \hat{b} \\ &= -(MS^T R^T C_1 + R^T + C_2 R^T C_1)(\eta - \eta_d) + MR^T \dot{\eta}_d \\ &\quad + (MS^T R^T + MR^T C_1 + C_2 R^T) \dot{\eta}_d \\ &\quad + (C + D - MR^T C_1 R - C_2 - K_0 M)v - \beta \end{aligned} \quad (26)$$

where C_1, C_2 is a positive definite symmetric design

$$\text{parameter matrix, and } S = \begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Substituting (26) into (25) results in:

$$\begin{aligned} \dot{V}_2 &= -\eta_e^T C_1 \eta_e + \chi_e^T [R(\psi)\eta - C(v)v - D(v)v + C(v)v \\ &\quad + D(v)v + M\dot{\chi}_1 - R(\psi)\eta_e - C_2\chi_e - \hat{b} + b - M\dot{\chi}_1] \\ &\quad - \tilde{b}^T K_0 \tilde{b} + \tilde{b}^T \dot{b} \\ &= -\eta_e^T C_1 \eta_e + \chi_e^T (-C_2\chi_e + b - \hat{b}) - \tilde{b}^T K_0 \tilde{b} + \tilde{b}^T \dot{b} \\ &= -\eta_e^T C_1 \eta_e - \chi_e^T C_2 \chi_e + \chi_e^T \tilde{b} - \tilde{b}^T K_0 \tilde{b} + \tilde{b}^T \dot{b} \\ &= -\eta_e^T C_1 \eta_e - \chi_e^T K_1 M \chi_e + \chi_e^T \tilde{b} - \tilde{b}^T K_0 \tilde{b} + \tilde{b}^T \dot{b} \end{aligned} \quad (27)$$

where $K_1 = C_2 M^{-1}$. Considering the following complete square inequalities:

$$\chi_e^T \tilde{b} \leq \varepsilon_1 \chi_e^T \chi_e + \frac{1}{4\varepsilon_1} \tilde{b}^T \tilde{b} \quad (28)$$

$$\tilde{b}^T \dot{b} \leq \varepsilon \tilde{b}^T \tilde{b} + \frac{1}{4\varepsilon} \dot{b}^T \dot{b} \quad (29)$$

$$-\chi_e^T K_1 M \chi_e \leq -\lambda_{\min}(K_1) \chi_e^T M \chi_e \quad (30)$$

the Equation (27) can be rewritten as

$$\begin{aligned} \dot{V}_2 &\leq -\lambda_{\min}(C_1) \eta_e^T \eta_e - \lambda_{\min}(K_1) \chi_e^T M \chi_e + \varepsilon_1 \chi_e^T \chi_e + \frac{1}{4\varepsilon_1} \tilde{b}^T \tilde{b} \\ &\quad - \lambda_{\min}(K_0) \tilde{b}^T \tilde{b} + \varepsilon \tilde{b}^T \tilde{b} + \frac{1}{4\varepsilon} \dot{b}^T \dot{b} \\ &\leq -\lambda_{\min}(C_1) \eta_e^T \eta_e - \lambda_{\min}(K_1) \chi_e^T M \chi_e + \frac{\varepsilon_1}{\lambda_{\min}(M)} \chi_e^T M \chi_e \\ &\quad + \frac{1}{4\varepsilon_1} \tilde{b}^T \tilde{b} - \lambda_{\min}(K_0) \tilde{b}^T \tilde{b} + \varepsilon \tilde{b}^T \tilde{b} + \frac{1}{4\varepsilon} \dot{b}^T \dot{b} \\ &\leq -2\min \left\{ \lambda_{\min}(C_1), \lambda_{\min}(K_1) - \frac{\varepsilon_1}{\lambda_{\min}(M)}, \lambda_{\min}(K_0) - \frac{1}{4\varepsilon_1} - \varepsilon \right\} V_2 \\ &\quad + \frac{1}{4\varepsilon} C_d^2 \end{aligned} \quad (31)$$

where

$$\lambda_{\min}(K_1) - \frac{\varepsilon_1}{\lambda_{\min}(M)} > 0 \quad (32)$$

$$\lambda_{\min}(K_0) - \frac{1}{4\varepsilon_1} - \varepsilon > 0 \quad (33)$$

Notate

$$\mu = \min \left\{ \lambda_{\min}(C_1), \lambda_{\min}(K_1) - \frac{\varepsilon_1}{\lambda_{\min}(M)}, \lambda_{\min}(K_0) - \frac{1}{4\varepsilon_1} - \varepsilon \right\} \quad (34)$$

$$\sigma = \frac{C_d^2}{4\varepsilon} \quad (35)$$

Substituting (34) and (35) into the equation of (31) gives

$$0 \leq V_2(t) \leq \frac{\sigma}{2\mu} + \left[V_2(0) - \frac{\sigma}{2\mu} \right] e^{-2\mu t} \quad (36)$$

Therefore,

$$\lim_{t \rightarrow \infty} V_2(t) \leq \frac{\sigma}{2\mu} \quad (37)$$

It is seen from Equation (37) that $V_2(t)$ is ultimately bounded. Furthermore, all signals of the closed-loop system of ship trajectory tracking are globally uniformly ultimately bounded in light of Equations (15)-(17), (21) and (24). The above analysis proves the following theorem.

Theorem 3.2 Applied to the 3 DOF nonlinear motion mathematical model of ships with unknown time-variant disturbances, the control input vector τ described by (26) together with the disturbance observer (7) and (8) guarantees that the arbitrary reference trajectories tracking error of ship achieves to any prescribed accuracy by appropriately choosing the design parameter matrix C_1, C_2 and K_0 while keeps the global uniform ultimate boundedness of all the signals of the closed-loop system of ship trajectory tracking under the conditions of (32) and (33).

IV. SIMULATION

In this section, the numerical simulation on CyberShip II is carried out to demonstrate the effectiveness of the proposed control scheme. CyberShip II is a 1:70 scale replica of a supply ship of the Marine Cybernetics Laboratory (MCLab) in Norwegian University of Science and Technology. The ship has the length of 1.255m, mass of 23.8kg and other parameters of the ship are given in [16] in detail.

In the simulation, the reference trajectory is

$$\begin{aligned} x_d &= 4\sin(0.02t) \\ y_d &= 2.5(1-\cos(0.02t)) \\ \psi_d &= 0.02t \end{aligned} \quad (38)$$

which is an ellipse. The disturbance vector $b = [b_1, b_2, b_3]^T = [1.3+2.0\sin(0.02t)+1.5\sin(0.1t), -0.9+2.0\sin(0.02t-\pi/6)+1.5\sin(0.3t), -\sin(0.09t+\pi/3)-3\sin(0.01t)-\sin(0.01t)]^T$.

Initial conditions are selected as $[x(0), y(0), \psi(0), u(0), v(0), r(0)] = [1m, 1m, \pi/4 \text{ rad}, 0m/s, 0m/s, 0rad/s]$ and $\hat{b} = [0, 0, 0]^T$. And the design parameters are taken as $C_1 = \text{diag}[0.05, 0.05, 0.05]$, $C_2 = \text{diag}[120, 120, 120]$, $K_0 = \text{diag}[2, 2, 2]$. Simulation results are depicted in Fig.1-4.

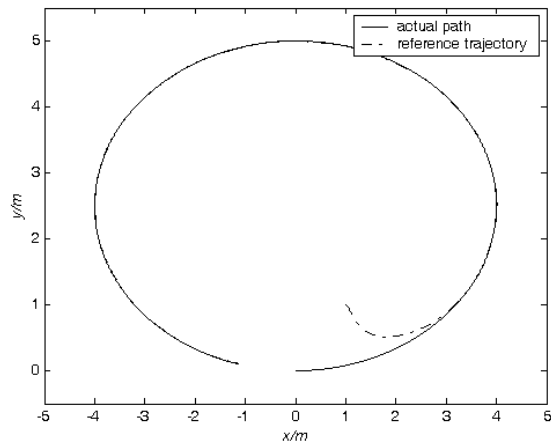


Fig.1. The actual trajectory and reference trajectory in xy-plane.

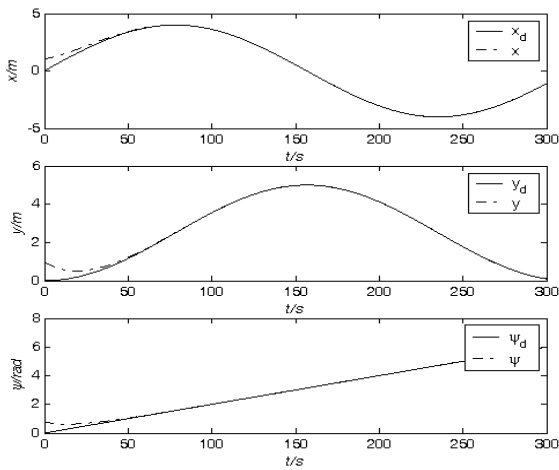


Fig.2. The curves of the desired and actual position and yaw angle.

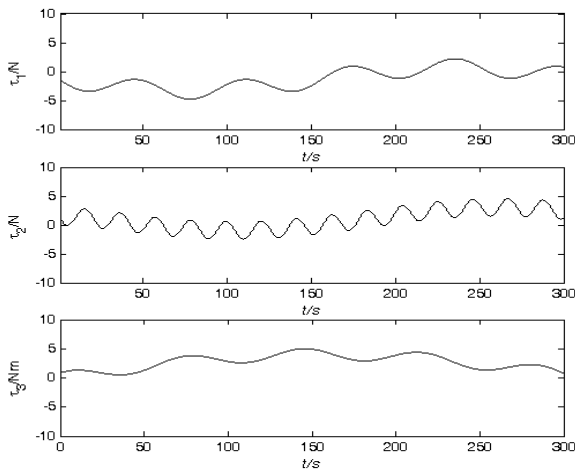


Fig.3. The curves of the control inputs.

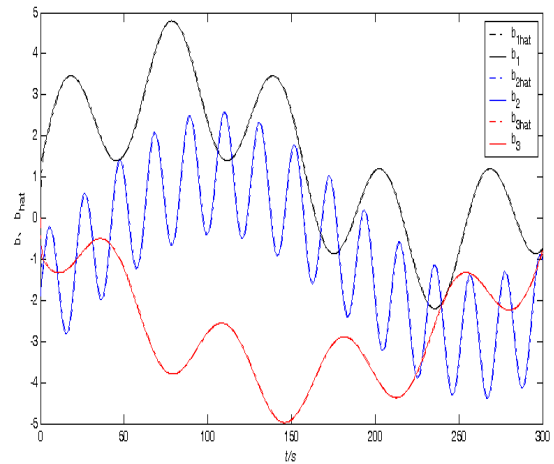


Fig.4. The curves of external disturbances and their estimation.

Figure 1 shows that the proposed controller is able to force the ship to converge to the reference trajectory under the unknown disturbances. And the desired and actual positions and yaw angles are presented in Fig.2 respectively, which indicates that the actual ship position x, y and the actual yawing angle ψ can reach the desired trajectory $\eta_d = [x_d, y_d, \psi_d]^T$ in around 50s and track it at a high precision. The corresponding control inputs are presented in Fig.3, which shows that the control force and control torque are smooth and reasonable. Finally, the external disturbances b and their estimate value \hat{b} are represented in Fig. 4. From Fig. 4, it is obvious that the disturbance observer provides the exponentially convergent estimation of unknown time-variant external disturbances.

V. CONCLUSION

In this paper, a robust trajectory tracking control law is presented for ship steering under unknown time-variant disturbances due to wind waves, and ocean currents. The considered nonlinear ship surface movement mathematical model includes both Coriolis and centripetal matrix and nonlinear damp term. The designed control strategy is based on disturbance observer, backstepping technique, Lyapunov direct method. The observer is constructed to provide an estimation of unknown external disturbances. It is proved that the resulting closed-loop system of ship trajectory tracking error is globally uniformly ultimately bounded. Furthermore, numerical simulations on a model ship CyberShip II of MCLab in Norwegian University of Science and Technology show that the control force and control torque are reasonable, and the transient state performance and stable performance are satisfactory, which validates the proposed scheme.

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