Unified Power Flow Controller (UPFC) Based Damping Controllers for Damping Low Frequency Oscillations in a Power System

(Ms) N Tambey, *Non-member*
Prof M L Kothari, *Member*

This paper presents a systematic approach for designing Unified Power Flow Controller (UPFC) based damping controllers for damping low frequency oscillations in a power system. Detailed investigations have been carried out considering four alternative UPFC based damping controllers. The investigations reveal that the damping controllers based on UPFC control parameters δ_E and δ_B provide robust performance to variations in system loading and equivalent reactance Xe.

Keywords : UPFC; Damping controller; Low frequency oscillations; FACTS

NOTATIONS

C_{dc}	: dc link capacitance
D	: damping constant
H	: inertia constant (M = 2H)
Ka	: AVR gain
K_{dc}	: gain of damping controller
m_B	: modulation index of series converter
m_E	: modulation index of shunt converter
P_e	: electrical power of the generator
P_m	: mechanical power input to the generator
Ta	: time constant of AVR
T_{do}	: d-axis open circuit time-constant of generator
T_1, T_2	: time constants of phase compensator
V_b	: infinite bus voltage
V_{dc}	: voltage at dc link
V_t	: terminal voltage of the generator
X_B	: reactance of boosting transformer (BT)
X_{Bv}	: reactance of the transmission line
X_d	: direct axis steady-state synchronous reactance of generator
X'_d	: direct axis transient synchronous reactance of generator
X_E	: reactance of excitation transformer (ET)
Xe	: equivalent reactance of the system
X_q	: quadrature axis steady-state synchronous reactance of generator
X_{tE}	: reactance of transformer
δ_B	: phase angle of series converter voltage
δ_E	: phase angle of shunt converter voltage
ω_n	: natural frequency of oscillation (rad/sec)

(Ms) N Tambey and Prof M L Kothari are with the Department of Electrical Engineering, Indian Institute of Technology, New Delhi.

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INTRODUCTION

The power transfer in an integrated power system is constrained by transient stability, voltage stability and small signal stability. These constraints limit a full utilization of available transmission corridors. Flexible ac transmission system (FACTS) is the technology that provides the needed corrections of the transmission functionality in order to fully utilize the existing transmission facilities and hence, minimizing the gap between the stability limit and thermal limit.

Unified power flow controller (UPFC) is one of the FACTS devices, which can control power system parameters such as terminal voltage, line impedance and phase angle. Therefore, it can be used not only for power flow control, but also for power system stabilizing control.

Recently researchers have presented dynamic models of UPFC in order to design suitable controllers for power flow, voltage and damping controls[1–6]. Wang[6–8], has presented a modified linearised Heffron-Phillips model of a power system installed with UPFC. He has addressed the basic issues pertaining to the design of UPFC damping controller, *ie*, selection of robust operating condition for designing damping controller; and the choice of parameters of UPFC (such as m_B, m_E, δ_B and δ_E) to be modulated for achieving desired damping.

Wang has not presented a systematic approach for designing the damping controllers. Further, no effort seems to have been made to identify the most suitable UPFC control parameter, in order to arrive at a robust damping controller.

In view of the above the main objectives of the research work presented in the paper are,

- To present a systematic approach for designing UPFC based damping controllers.
- To examine the relative effectiveness of modulating alternative UPFC control parameters (*ie*, m_B, m_E, δ_B and δ_E), for damping power system oscillations.

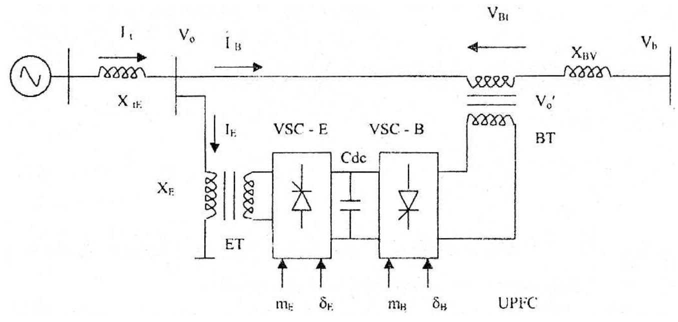

Figure 1 UPFC installed in a SMIB system

- To investigate the performance of the alternative damping controllers, following wide variations in loading conditions and system parameters in order to select the most effective damping controller.

SYSTEM INVESTIGATED

A single-machine-infinite-bus (SMIB) system installed with UPFC is considered (Figure 1). The static excitation system model type IEEE-ST1A has been considered. The UPFC considered here is assumed to be based on pulse width modulation (PWM) converters. The nominal loading condition and system parameters are given in Appendix-1.

UNIFIED POWER FLOW CONTROLLER

Unified power flow controller (UPFC) is a combination of static synchronous compensator (STATCOM) and a static synchronous series compensator (SSSC) which are coupled via a common dc link, to allow bi-directional flow of real power between the series output terminals of the SSSC and the shunt output terminals of the STATCOM and are controlled to provide concurrent real and reactive series line compensation without an external electric energy source. The UPFC, by means of angularly unconstrained series voltage injection, is able to control, concurrently or selectively, the transmission line voltage, impedance and angle or alternatively, the real and reactive power flow in the line. The UPFC may also provide independently controllable shunt reactive compensation.

Viewing the operation of the UPFC from the standpoint of conventional power transmission based on reactive shunt compensation, series compensation and phase shifting, the UPFC can fulfil all these functions and thereby meet multiple control objectives by adding the injected voltage V_{Bt} with appropriate amplitude and phase angle, to the terminal voltage V_0.

MODIFIED HEFFRON-PHILLIPS SMALL PERTURBATION TRANSFER FUNCTION MODEL OF A SMIB SYSTEM INCLUDING UPFC

Figure 2 shows the small perturbation transfer function block diagram of a machine-infinite bus system including UPFC relating the pertinent variables of electric torque, speed, angle, terminal voltage, field voltage, flux linkages, UPFC control parameters, and dc link voltage. This model has been obtained[6, 7] by modifying the basic Heffron- Phillips model[9] including UPFC. This linear model has been developed by linearising the nonlinear differential equations around a nominal operating point. The twenty-eight constants of the model depend on the system parameters and the operating condition (Appendix - 2).

In Figure 2, $[\Delta u]$ is the column vector while $[K_{pu}]$, $[K_{qu}]$, $[K_{vu}]$ and $[K_{cu}]$ are the row vectors as defined below,

$$[\Delta u] = [\Delta m_E \ \Delta \delta_E \ \Delta m_B \ \Delta \delta_B]^T$$

$$[Kpu] = [K_{pe} \ K_{p\delta e} \ K_{pb} \ K_{p\delta b}]$$

$$[K_{vu}] = [K_{ve} \ K_{v\delta e} \ K_{vb} \ K_{v\delta b}]$$

$$[K_{qu}] = [K_{qe} \ K_{q\delta e} \ K_{qb} \ K_{q\delta b}]$$

$$[K_{cu}] = [K_{ce} \ K_{c\delta e} \ K_{cb} \ K_{c\delta b}]$$

The significant control parameters of UPFC are,

1. m_B — modulating index of series inverter. By controlling m_B, the magnitude of series injected voltage can be controlled, thereby controlling the reactive power compensation.
2. δ_B — Phase angle of series inverter which when controlled results in the real power exchange.
3. m_E — modulating index of shunt inverter. By controlling m_E, the voltage at a bus where UPFC is installed, is controlled through reactive power compensation.
4. δ_E — Phase angle of the shunt inverter, which regulates the dc voltage at dc link.

ANALYSIS

Computation of Constants of the Model

The initial d-q axes voltage and current components and torque angle needed for computing K - constants for the nominal operating condition are computed and are as follows :

$Q = 0.1670$ pu	$E_{bdo} = 0.7331$ pu
$e_{do} = 0.3999$ pu	$E_{bqo} = 0.6801$ pu
$e_{qo} = 0.9166$ pu	$i_{do} = 0.4729$ pu
$\delta_o = 47.1304°$	$i_{qo} = 0.6665$ pu

The K - constants of the model computed for nominal operating condition and system parameters are

$K_1 = 0.3561$	$K_{pb} = 0.6667$	$K_{p\delta e} = 1.9315$
$K_2 = 0.4567$	$K_{qb} = 0.6118$	$K_{q\delta e} = -0.0404$
$K_3 = 1.6250$	$K_{vb} = -0.1097$	$K_{v\delta e} = 0.1128$
$K_4 = 0.09164$	$K_{pe} = 1.4821$	$K_{cb} = 0.1763$
$K_5 = -0.0027$	$K_{qe} = 2.4918$	$K_{ce} = 0.0018$
$K_6 = 0.0834$	$K_{ve} = -0.5125$	$K_{c\delta b} = -0.041$
$K_7 = 0.1371$	$K_{p\delta b} = 0.0924$	$K_{c\delta e} = 0.4987$
$K_8 = 0.0226$	$K_{q\delta b} = -0.0050$	$K_{pd} = 0.0323$
$K_9 = -0.0007$	$K_{v\delta b} = 0.0061$	$K_{qd} = 0.0524$
		$K_{vd} = -0.0107$

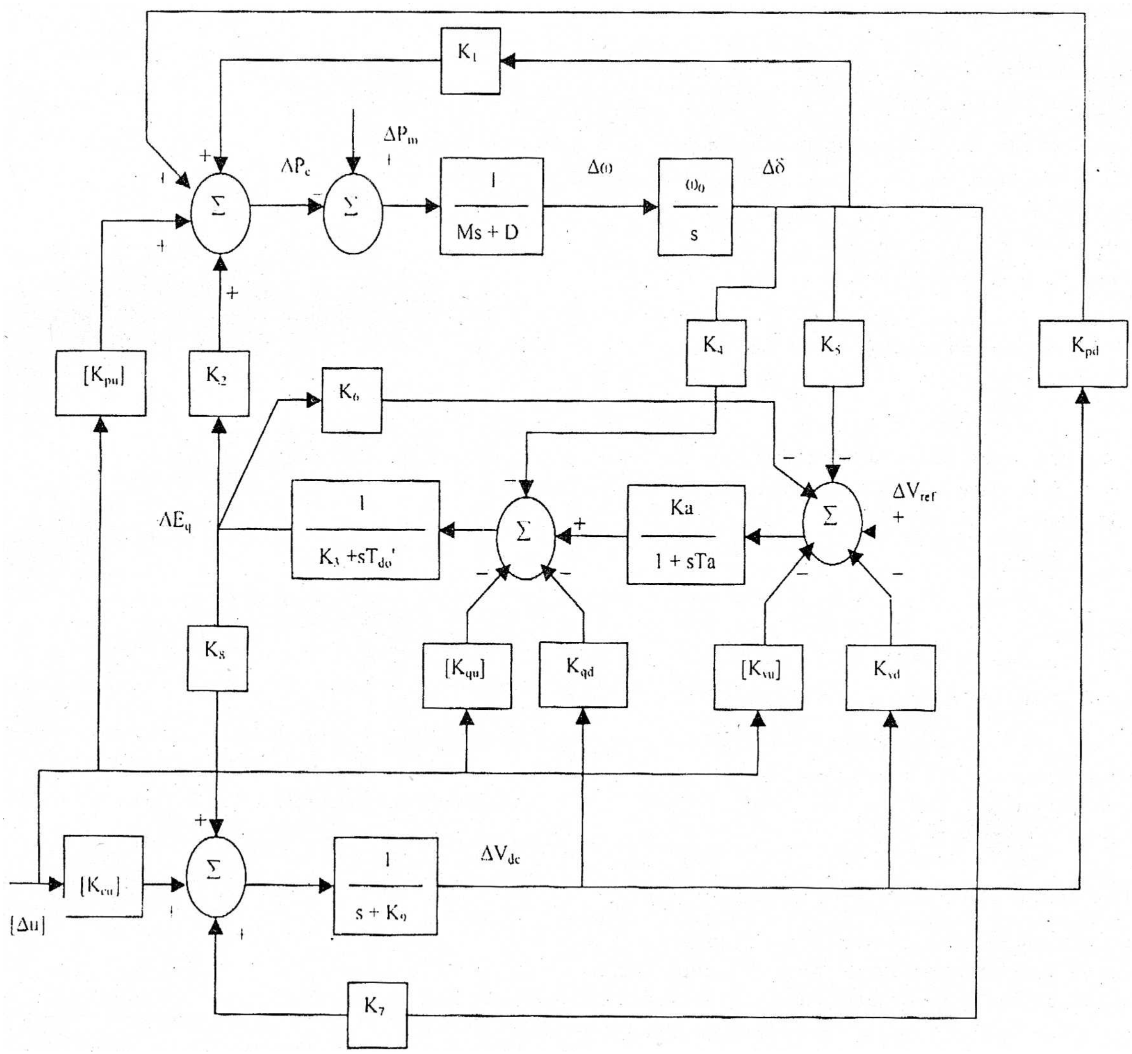

Figure 2 Modified Heffron-Phillips model of SMIB system with UPFC

For this operating condition, the eigen-values of the system are obtained (Table 1) and it is clearly seen that the system is unstable.

Design of Damping Controllers

The damping controllers are designed to produce an electrical torque in phase with the speed deviation. The four control parameters of the UPFC (*ie*, m_B, m_E, δ_B and δ_E) can be modulated in order to produce the damping torque. The speed deviation $\Delta\omega$ is considered as the input to the damping controllers. The four alternative UPFC based damping controllers are examined in the present work.

Damping controller based on UPFC control parameter m_B shall henceforth by denoted as damping controller (m_B). Similarly damping controllers based on m_E, δ_B and δ_E shall henceforth be denoted as damping controller (m_E), damping controller (δ_B), and damping controller (δ_E), respectively.

Table 1 Eigen-values of the closed loop system

	Eigen-values	ω_n of Oscillatory Mode	ζ of the Oscillatory Modes
System without any damping controller	− 19.1186 0.0122 ± 4.0935i − 1.2026	4.09 rad/s	− 0.00297

The structure of UPFC based damping controller is shown in Figure 3. It consists of gain, signal washout and phase compensator blocks. The parameters of the damping controller are obtained using the phase compensation technique[9]. The detailed step-by-step procedure for computing the parameters of the damping controllers using phase compensation technique is given below :

1. Computation of natural frequency of oscillation ω_n from the mechanical loop.

$$\omega_n = \sqrt{\frac{K_1\,\omega_0}{M}}$$

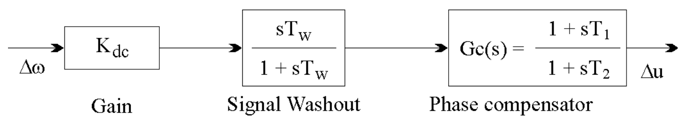

Figure 3 Structure of UPFC based damping controller

2. Computation of ∠GEPA (Phase lag between Δu and ΔP_e) at $s = j\,\omega\, n$. Let it be γ.

3. Design of phase lead/lag compensator Gc:

   The phase lead/lag compensator Gc is designed to provide the required degree of phase compensation. For 100% phase compensation,

   $$\angle Gc\,(j\,\omega_n) + \angle \text{GEPA}\,(j\,\omega_n) = 0$$

   Assuming one lead-lag network, $T_1 = a\ T_2$, the transfer function of the phase compensator becomes,

   $$Gc\,(s) = \frac{1 + saT_2}{1 + sT_2}$$

   Since the phase angle compensated by the lead-lag network is equal to $-\gamma$, the parameters a and T_2 are computed as,

   $$a = \frac{1 + \sin\gamma}{1 - \sin\gamma}$$

   $$T_2 = \frac{1}{\omega_n\sqrt{a}}$$

4. Computation of optimum gain K_{dc}.

   The required gain setting K_{dc} for the desired value of damping ratio $\zeta = 0.5$ is obtained as,

   $$K_{dc} = \frac{2\,\zeta\,\omega_n M}{|Gc\,(s)|\;|\text{GEPA}\,(s)|}$$

Where $|Gc\,(s)|$ and $|\text{GEPA}\,(s)|$ are evaluated at $s = j\,\omega_n$.

The signal washout is the high pass filter that prevents steady changes in the speed from modifying the UPFC input parameter. The value of the washout time constant T_w should be high enough to allow signals associated with oscillations in rotor speed to pass unchanged. From the viewpoint of the washout function, the value of T_w is not critical and may be in the range of 1s to 20s. T_w equal to 10s is chosen in the present studies.

Figure 4 shows the transfer function of the system relating the electrical component of the power (ΔP_e) produced by the damping controller (m_B).

The time constants of the phase compensator are chosen so that the phase lag/lead of the system is fully compensated. For the nominal operating condition, the natural frequency of oscillation ω_n = 4.0974 rad/sec. The transfer function relating ΔP_e and Δm_B is denoted as GEPA. For the nominal operating condition, phase angle of GEPA *ie*, ∠GEPA = 9.057° lagging. The magnitude of GEPA *ie*, |GEPA| = 0.6798. To compensate the phase lag, the time constants of the lead compensator are obtained as T_1 = 0.2860 s and T_2 = 0.2082 s.

Following the same procedure, the phase angle to be compensated by the other three damping controllers are computed and are given in Table 2.

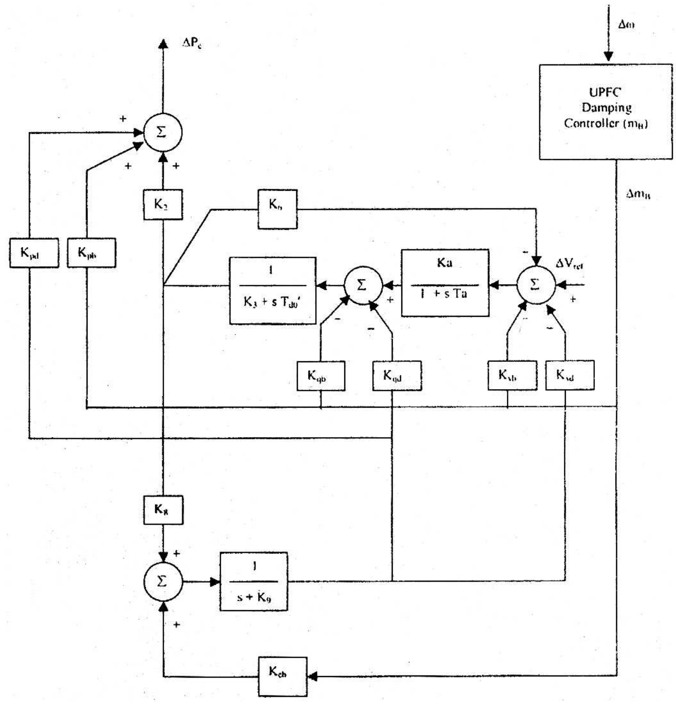

Figure 4 Transfer function of the system relating component of electrical power (ΔP_e) produced by damping controller (m_B)

Table 2 Gain and phase angle of the transfer function GEPA

GEPA	|GEPA|	∠GEPA
$\Delta P_e/\Delta m_E$	1.5891	− 18.3805°
$\Delta P_e/\Delta\delta_E$	1.9251	3.4836°
$\Delta P_e/\Delta m_B$	0.6789	− 9.0527°
$\Delta P_e/\Delta\delta_B$	0.0923	4.2571°

Table 3 Parameters of the UPFC based damping controllers

	K_{dc}	T_1, s	T_2, s
Damping controller (m_E)	14.8813	0.3383	0.1761
Damping controller (δ_E)	18.0960	0.2296	0.2516
Damping controller (m_B)	41.1419	0.2860	0.2082
Damping controller (δ_B)	382.4410	0.2266	0.2694

The critical examination of Table 2 reveals that the phase angle of the system *ie*, ∠GEPA, is lagging for control parameter m_B and m_E. However, it is leading for δ_B and δ_E. Hence the phase compensator for the damping controller (m_B) and damping controller (m_E) is a lead compensator while for damping controller (δ_B) and damping controller (δ_E) is a lag compensator. The gain settings (K_{dc}) of the controllers are computed assuming a damping ratio ξ = 0.5. Table 3 shows the parameters (gain and time constants) of the four alternative damping controllers. Table 3 clearly shows that gain settings of the damping controller (m_E) and damping controller (δ_E) doesn't differ much. However, the gain settings of damping controller (δ_B) is much higher as compared to the damping controller (m_B).

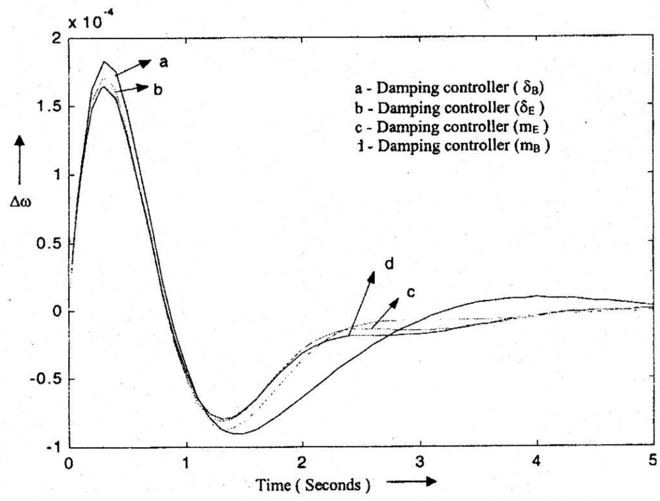

Figure 5 Dynamic responses for Δω with four alternative damping controllers

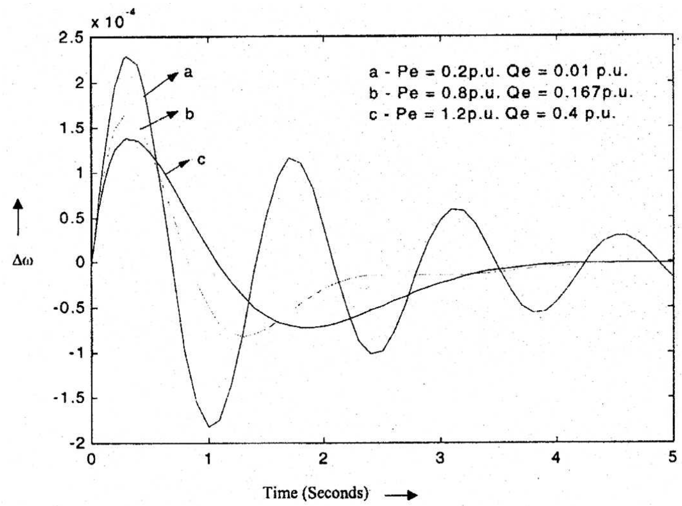

Figure 6 Dynamic responses for Δω with damping controller (m_B) for different loading conditions

Dynamic Performance of the System with Damping Controllers

Figure 5 shows the dynamic responses for $\Delta\,\omega$ obtained considering a step load perturbation $\Delta\,P_m\,=\,0.01$ pu with the following four alternative damping controllers :

1. Damping controller (m_B) : K_{dc} = 41.1419, T_1 = 0.2860 s, T_2 = 0.2082 s
2. Damping controller (δ_B) : K_{dc} = 382.4410, T_1 = 0.2266 s, T_2 = 0.2594 s
3. Damping controller (m_E) : K_{dc} = 14.8813, T_1 = 0.3383s, T_2 = 0.1761 s
4. Damping controller (δ_E) : K_{dc} = 18.0960, T_1 = 0.2296 s, T_2 = 0.2516 s

Figure 5 clearly shows that the dynamic responses of the system obtained with the four alternative damping controllers are virtually identical. At this stage it can be inferred that any of the UPFC based damping controllers provide satisfactory dynamic performance at the nominal operating condition.

Effect of Variation of Loading Condition and System Parameters on the Dynamic Performance of the System

In any power system, the operating load varies over a wide range. It is extremely important to investigate the effect of variation of the loading condition on the dynamic performance of the system.

In order to examine the robustness of the damping controllers to wide variation in the loading condition, loading of the system is varied over a wide range ($P_e\,=\,0.1$ pu to P_e = 1.2 pu) and the dynamic responses are obtained for each of the loading condition considering parameters of the damping controllers computed at nominal operating condition for the step load perturbation in mechanical power (*ie*, $\Delta\,P_m\,=\,0.01$ pu).

Figures 6 and 7 show the dynamic responses of $\Delta\,\omega$ with nominal optimum damping controller (m_B) and damping controller (m_E) at different loading conditions. It is clearly seen that the dynamic performance at light load condition deteriorates significantly as compared to that obtained at the nominal loading.

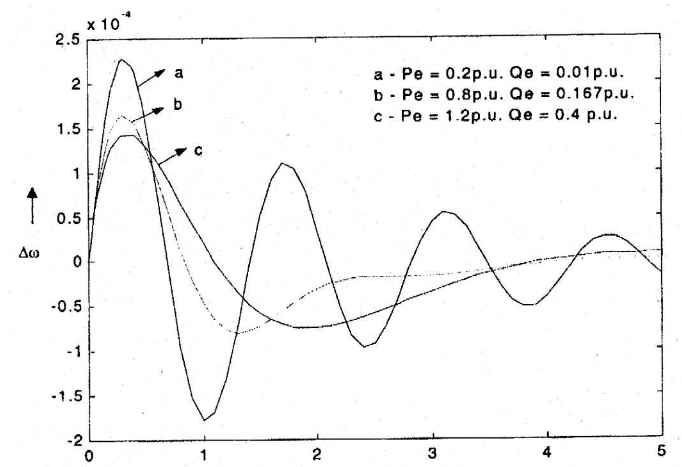

Figure 7 Dynamic responses for Δω with damping controller (m_E) for different loading conditions

Figures 8 and 9 show the dynamic responses of $\Delta\,\omega$ with nominal optimum damping controller (δ_B) and damping controller (δ_E), respectively. It is clearly seen that the responses are hardly affected in terms of settling time following wide variations in loading condition.

From the above studies, it can be concluded that the damping controller (δ_B) and damping controller (δ_E) exhibit robust dynamic performance as compared to that obtained with damping controller (m_B) or damping controller (m_E).

In view of the above, the performance of damping controller (δ_B) and damping controller (δ_E) are further studied with variation in equivalent reactance, *Xe* of the system. Figures 10 and 11 show the dynamic performance of the system with damping controller (δ_B) and damping controller (δ_E), respectively, for wide variation in *Xe*. Examining Figures 10 and 11, it can be inferred that damping controller (δ_B) and damping controller (δ_E) are quite robust to variations in *Xe* also.

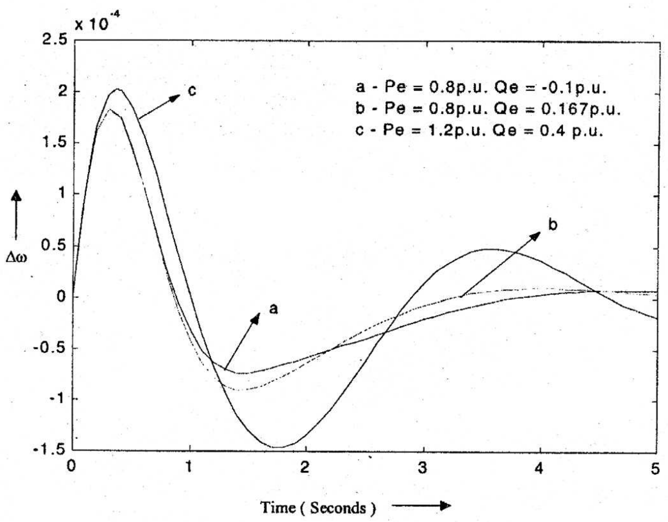

Figure 8 Dynamic responses for Δω with damping controller (δ_B) for different loading conditions

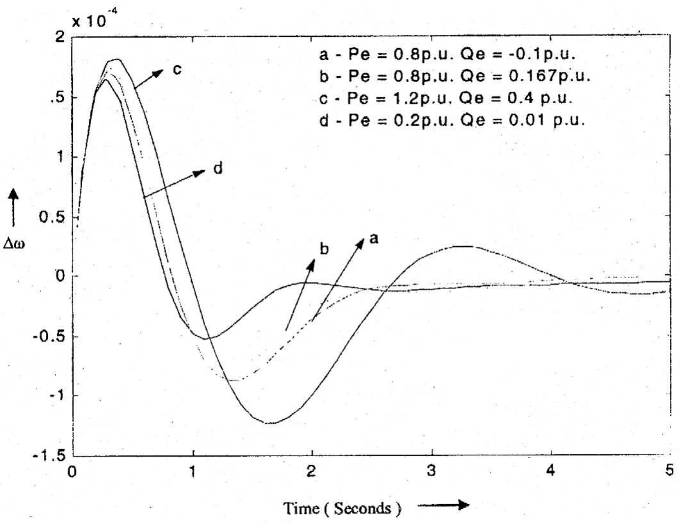

Figure 9 Dynamic responses for Δω with damping controller (δ_E) for different loading conditions

It may thus be concluded that damping controller (δ_B) and damping controller (δ_E) are quite robust to wide variation in loading condition and system parameters. The reason for the superior performance of damping controller (δ_B) and damping controller (δ_E) may be attributed to the fact that modulation of δ_B and δ_E results in exchange of real power.

CONCLUSIONS

The significant contributions of the research work presented in this paper are as follows :

- A systematic approach for designing UPFC based controllers for damping power system oscillations has been presented.
- The performance of the four alternative damping controllers, (*ie*, damping controller (m_E), damping controller (δ_E), damping controller (m_B), and damping controller (δ_B) has been examined considering wide variation in the loading conditions and line reactance *Xe*.

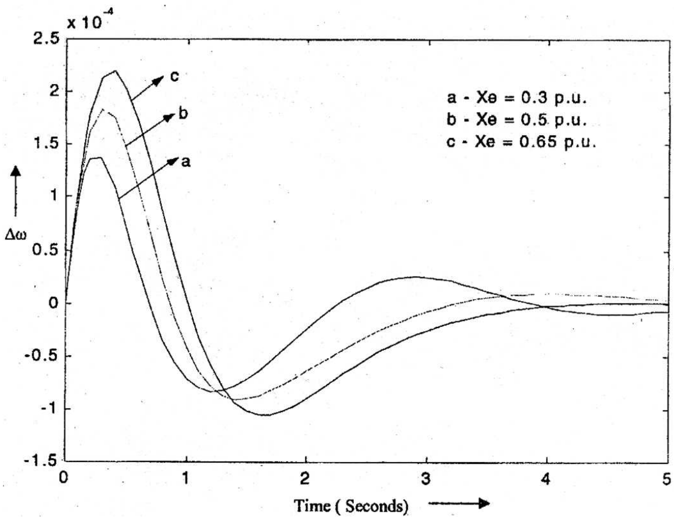

Figure 10 Dynamic responses for Δω with damping controller (δ_B) for different values of Xe

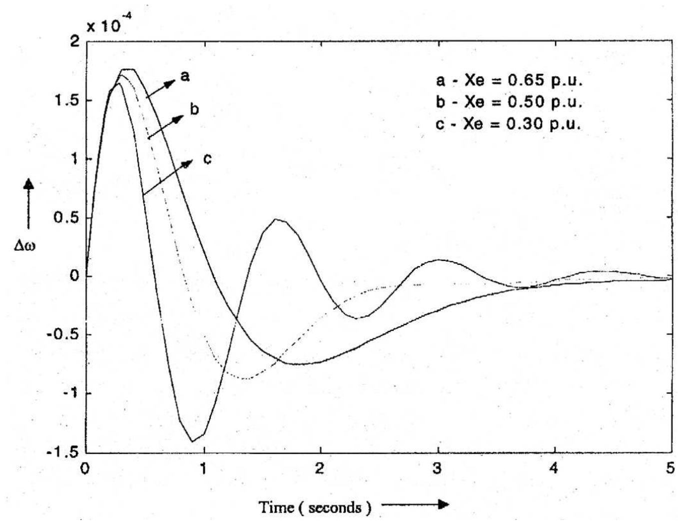

Figure 11 Dynamic responses for Δω with damping controller (δ_E) for different values of Xe

- Investigations reveal that the damping controller (δ_E) and damping controller (δ_B) provide robust performance to wide variation in loading conditions and line reactance *Xe*. It may thus be recommended that the damping controllers based on UPFC control parameters δ_E and δ_B may be preferred over the damping controllers based on control parameters m_B or m_E.

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APPENDIX 1

The nominal parameters and the operating condition of the system are given below.

Generator	: $M = 2H = 8.0$ MJ/MVA	D = 0.0	$T_{do} = 5.044$ s
	$X_d = 1.0$ pu	$X_q = 0.6$ pu	$X'_d = 0.3$ pu
Excitation system	: $K_a = 100$		$T_a = 0.01$ s
Transformer	: $X_{tE} = 0.1$ pu		$X_E = X_B = 0.1$ pu
Transmission line	: $X_{Bv} = 0.3$ pu		$Xe = X_{Bv} + X_B + X_{tE} = 0.5$ pu
Operating condition	: $P_e = 0.8$ pu		$V_t = 1.0$ pu
	$V_b = 1.0$ pu		$f = 60$ Hz
UPFC parameters	: $m_E = 0.4013$		$m_B = 0.0789$
	$\delta_E = -85.3478°$		$\delta_B = -78.2174°$
Parameters of dc link	: $V_{dc} = 2$ pu		$C_{dc} = 1$ pu

APPENDIX 2

Computation of constants of the model

The constants of the modified Heffron-Phillips model are computed from the expressions given below

$$K_1 = (V_{td} - I_{tq} X'_d)\,(X_{dE} - X_{dt})\,V_b \sin\delta / X_d\textstyle\sum + (X_q I_{td} + V_{tq}) \times (X_{qt} - X_{qE})\,V_b \cos\delta / X_q \textstyle\sum$$

$$K_2 = -(X_{BB} + X_E)\,V_{td}/X_d\textstyle\sum X_d + (X_{BB} + X_E)\,X'_d\,I_{tq}/X_d\textstyle\sum$$

$$K_3 = 1 + (X'_d - X_d)\,(X_{BB} + X_E)/X_d\textstyle\sum$$

$$K_4 = -(X'_d - X_d)\,(X_{dE} - X_{dt})\,V_b \sin\delta/X_d\textstyle\sum$$

$$K_5 = (V_{td}/V_t)\,X_q\,(X_{qt} - X_{qE})\,V_b \cos\delta/X_q\textstyle\sum - (V_{tq}/V_t)\,X'_d\,(X_{dE} - X_{dt})\,V_b\sin\delta/X_d\textstyle\sum$$

$$K_6 = (V_{tq}/V_t)\,(X_d\textstyle\sum + X'_d\,(X_{BB} + X_E))/X_d\textstyle\sum$$

$$K_7 = (3/4\,C_{dc})\left\{V_b \sin\delta\,(m_E\cos\delta_E X_{dE} - m_B \cos\delta_B X_{dt})/ \cdot X_d\textstyle\sum + V_b\cos\delta\,(m_B\sin\delta_B X_{qt} - m_E\sin\delta_E X_{qE})\right\}$$

$$K_8 = -(3/4\,C_{dc})\,(m_B\cos\delta_B X_E + m_E\cos\delta_E X_{BB})/X_d\textstyle\sum$$

$$K_9 = (3/4\,C_{dc})\Big\{m_B\sin\delta_B\,(m_B\cos\delta_B X_{dt} - m_E\cos\delta_E X_{dE})/2\,X_d\textstyle\sum + m_E\sin\delta_E\,(m_E\cos\delta_E X_{Bd} - m_B\cos\delta_B X_{dt})/2\,X_d\textstyle\sum + m_B\cos\delta_B\,(m_B\sin\delta_B X_{qt} - m_E\sin\delta_E X_{qE})/ 2\,xq\textstyle\sum + m_E\cos\delta_E\,(-m_B\sin\delta_B X_{qE} + m_E\sin\delta_E X_{Bq})/2\,X_q\textstyle\sum\Big\}$$

$$K_{pe} = (V_{td} - I_{tq} X'_d)\,(X_{Bd} - X_{dE})\,V_{dc}\sin\delta_E/2\,X_d\textstyle\sum + (X_q I_{td} + V_{tq})\,(X_{Bq} - X_{qE})\,V_{dc}\cos\delta_E/2\,X_q\textstyle\sum$$

$$K_{p\delta e} = (V_{td} - I_{tq} X'_d)\,(X_{Bd} - X_{dE})\,V_{dc}\,m_E\cos\delta_E/2\,X_d\textstyle\sum + (X_q I_{td} + V_{tq})\,(-X_{Bq} + X_{qE})\,V_{dc}\,m_E \sin\delta E/2\,X_q\textstyle\sum$$

$$K_{pb} = (V_{td} - I_{tq} X'_d)\,(X_{dt} - X_{dE})\,V_{dc}\sin\delta_B/2\,X_d\textstyle\sum + (X_q I_{td} + V_{tq})\,(X_{qt} - X_{qE})\,V_{dc}\cos\delta_B/2\,X_q\textstyle\sum$$

$$K_{p\delta b} = (V_{td} - I_{tq} X'_d)\,(X_{dE} + X_{dt})\,V_{dc}\,m_B\cos\delta_B/2\,X_d\textstyle\sum + (X_q I_{td} + V_{tq})\,(-X_{qt} + X_{qE})\,V_{dc}\,m_B\sin\delta_B/2\,X_q\textstyle\sum$$

$$K_{pd} = (V_{td} - I_{tq} X'_d)\left\{(X_{dt} - X_{dE})\,m_B\sin\delta_B/2\,X_d\textstyle\sum + (X_{Bd} - X_{dE})\,m_E\sin\delta_E/2\,X_d\textstyle\sum\right\} + (X_q I_{td} + V_{tq})\left\{(X_{qt} - X_{qE})\,m_B\cos\delta_B/2\,X_q\textstyle\sum + (X_{Bq} - X_{qE})\,m_E\cos\delta_E/2\,X_q\textstyle\sum\right\}$$

$$K_{qe} = -(X'_d - X_d)\,(X_{Bd} - X_{dE})\,V_{dc}\sin\delta_E/2\,X_d\textstyle\sum$$

$$K_{q\delta e} = -(X'_d - X_d)\,(X_{Bd} - X_{dE})\,m_E\,V_{dc}\cos\delta_E/2\,X_d\textstyle\sum$$

$$K_{qb} = -(X'_d - X_d)\,(X_{dt} - X_{dE})\,V_{dc}\sin\delta_B/2\,X_d\textstyle\sum$$

$$K_{q\delta b} = -(X'_d - X_d)\,(X_{dE} + X_{dt})\,m_B\,V_{dc}\cos\delta_B/2\,X_d\textstyle\sum$$

$$K_{qd} = -(X'_d - X_d)\left\{(X_{Bd} - X_{dE})\,m_E\sin\delta_E/2\,X_d\textstyle\sum + (X_{dt} - X_{dE})\,m_B\sin\delta_B/2\,X_d\textstyle\sum\right\}$$

$$K_{ve} = (V_{td}/V_t)\,X_q\,(X_{Bq} - X_{qE})\,V_{dc}\cos\delta_E/2\,X_q\textstyle\sum - (V_{tq}/V_t)\,X'_d\,(X_{Bd} - X_{dE})\,V_{dc}\sin\delta_E/2\,X_d\textstyle\sum$$

$$K_{v\delta e} = (V_{td}/V_t)\,X_q\,(X_{qE} - X_{Bq})\,m_E\,V_{dc}\sin\delta_E/2\,X_q\textstyle\sum - (V_{tq}/V_t)\,X'_d\,(X_{Bd} - X_{dE})\,m_E\,V_{dc}\cos\delta_E/2\,X_{q2}$$

$$K_{vb} = (V_{td}/V_t)\,X_q\,(X_{qt} - X_{qE})\,V_{dc}\cos\delta_B/2\,X_q\textstyle\sum - (V_{tq}/V_t)\,X'_d\,(X_{dt} - X_{dE})\,V_{dc}\sin\delta_B/2\,X_d\textstyle\sum$$

$$K_{v\delta b} = (V_{td}/V_t)\,X_q\,(X_{qE} - X_{qt})\,m_B\,V_{dc}\sin\delta_B/2\,X_q\textstyle\sum - (V_{tq}/V_t)\,X'_d\,(X_{dE} + X_{dt})\,m_B\,V_{dc}\cos\delta_B/2\,X_d\textstyle\sum$$

$$K_{vd} = (V_{td}/V_t)\,X_q\left\{(X_{Bq} - X_{qE})\,m_E\cos\delta_E/2\,X_q\textstyle\sum + (X_{qt} - X_{qE})\,m_B\cos\delta_B/2\,X_q\textstyle\sum\right\} - (V_{tq}/V_t)\,X'_d\left\{(X_{Bd} - X_{dE})\,m_E\sin\delta_E/2\,X_d\textstyle\sum + (X_{dt} - X_{dE})\,m_B\sin\delta_B/2\,X_d\textstyle\sum\right\}$$

$$K_{ce} = (3/4\,C_{dc})\left\{V_{dc}\sin\delta_E\,(m_E\cos\delta_E X_{Bd} - m_B\cos\delta_B X_{dE})\right\}/ 2\,X_d\textstyle\sum + V_{dc}\cos\delta_E\,(m_E\sin\delta_E X_{Bq} - m_B\sin\delta_B X_{qE})/2\,X_q\textstyle\sum$$

$$K_{c\delta e} = (3\,m_E/4\,C_{dc})\,(\cos\delta_E I_{Eq} - \sin\delta_E I_{Ed}) + (3/4\,C_{dc})\left\{m_E V_{dc}\cos\delta_E\,(m_E\cos\delta_E X_{Bd} - m_B\cos\delta_B X_{dE})/2\,X_d\textstyle\sum + m_E V_{dc}\sin\delta_E\,(m_B\sin\delta_B X_{qE} - m_E\sin\delta_E X_{Bq})/2\,X_q\textstyle\sum\right\}$$

$$K_{cb} = (3/4\,C_{dc})\left\{V_{dc}\sin\delta_B\,(-m_E\cos\delta_E X_{dE} + m_B\cos\delta_B X_{dt})\right\}/ 2\,X_d\textstyle\sum + V_{dc}\cos\delta_B\,(m_B\sin\delta_B X_{qt} - m_E\sin\delta_E X_{qE})/2\,X_q\textstyle\sum$$

$$K_{c\delta b} = (3\,m_B/4\,C_{dc})\,(\cos\delta_B I_{Bq} - \sin\delta_B I_{Bd}) + (3/4\,C_{dc})\left\{m_B V_{dc}\cos\delta_B\,(m_E\cos\delta_E X_{dE} + m_B\cos\delta_B X_{dt})/2\,X_d\textstyle\sum + m_B V_{dc}\sin\delta_B\,(-m_B\sin\delta_B X_{qt} + m_E\sin\delta_E X_{qE})/2\,X_q\textstyle\sum\right\}$$

where,

$$X_{qt} = X_q + X_{tE} + X_E \quad X_{qE} = X_q + X_{tE} \quad X_{dt} = X_E + X'_d + X_{tE}$$

$$X_{dE} = X'_d + X_{TE} \quad X_{BB} = X_B + X_{Bv} \quad X_q\textstyle\sum = X_{qt} X_{BB} + X_E X_{qE}$$

$$X_d\textstyle\sum = -X_{dt} X_{BB} - X_{dE} X_E$$