Variable Neighborhood Search heuristic for the Inventory Routing Problem in fuel delivery

Dražen Popović [a,*], Milorad Vidović [a], Gordana Radivojević [b]

[a] University of Belgrade, Faculty of Transport and Traffic Engineering, Department of Logistics, Serbia
[b] University of Belgrade, Mihajlo Pupin Institute, Serbia

ARTICLE INFO

ABSTRACT

In this paper we observe the extension of the vehicle routing problem (VRP) in fuel delivery that includes petrol stations inventory management and which can be classified as the Inventory Routing Problem (IRP) in fuel delivery. The objective of the IRP is to minimize the total cost of vehicle routing and inventory management. We developed a Variable Neighborhood Search (VNS) heuristic for solving a multi-product multi-period IRP in fuel delivery with multi-compartment homogeneous vehicles, and deterministic consumption that varies with each petrol station and each fuel type. The stochastic VNS heuristic is compared to a Mixed Integer Linear Programming (MILP) model and the deterministic "compartment transfer" (CT) heuristic. For three different scale problems, with different vehicle types, the developed VNS heuristic outperforms the deterministic CT heuristic. Also, for the smallest scale problem instances, the developed VNS was capable of obtaining the near optimal and optimal solutions (the MILP model was able to solve only the smallest scale problem instances).

1. Introduction

The transportation and inventories management have a decisive influence on the effectiveness of the distribution process. Although this fact is well known, modeling approaches to distribution process optimization usually consider inventory control and transportation independently, neglecting their mutual impact. However, the inter-relationship between the inventory allocation and vehicle routing has recently motivated some authors to model these two activities simultaneously by solving the Inventory Routing Problem (IRP). The objective of the IRP is to minimize the total cost of vehicle routing and inventory management. Regardless of the type and characteristics of the IRP an optimal solution for real life problems is so far unreachable due to the problem complexity which is related to simultaneous resolution of the routing problem and the allocation of deliveries over an observed planning horizon. The IRP assumes application of the VMI concept where suppliers determine an order quantity and the time of delivery. The VMI concept enables the supplier to better utilize the vehicles, but on the other hand it shifts the responsibility of inventory management

from clients to the supplier. There are many industries, including the petrochemical industry where the VMI concept is applied, and that can draw benefit from the integrated approach to the IRP (Campbell & Savelsbergh, 2004). Recently, Bersani, Minciardi, and Sacile (2010) discussed the VMI concept in distribution of petrol products to service stations.

The vehicle routing problem (VRP) in fuel delivery is a well known research area (Avella, Boccia, & Sforza, 2004; Boctor, Renaud, & Cornillier, 2011; Brown, Ellis, Graves, & Ronen, 1987; Brown & Graves, 1981; Bruggen, Gruson, & Salomon, 1995; Cornillier, Boctor, Laporte, & Renaud, 2007, 2008; Fallahi, Prins, & Calvo, 2008; Mendoza, Castanier, Gueret, Medaglia, & Velasco, 2010; Uzar & Catay, 2012) where the main objective is to minimize the transportation costs incurred by the delivery of petroleum products to a set of clients, usually trough the use of multi-compartment vehicles. In this paper we observe the extension of the VRP in fuel delivery that includes petrol stations inventory management. Hence, this problem can be classified as the IRP in fuel delivery. More precisely, we observe secondary distribution of different fuel types from one depot location to a set of petrol stations by a designated fleet of multi-compartment vehicles, and for which a single oil company has control over all of the managerial decisions over all of the resources. This enables the VMI concept, and therefore, the application of the IRP.

Bell et al. (1983) were among the first authors to observe the IRP. They considered distribution of liquefied industrial gases and

* Corresponding author. Address: Vojvode Stepe 305, 11000 Belgrade, Serbia. Tel.: +381 011 30 91 273; fax: +381 011 30 96 704.
  E-mail addresses: d.popovic@sf.bg.ac.rs (D. Popović), m.vidovic@sf.bg.ac.rs (M. Vidović), gordana.radivojevic@pupin.rs (G. Radivojević).

used the linear programming model and Lagrangian relaxation to obtain the delivery plan for short-term planning horizon. Recently research efforts on the IRP topic have been intensified (Coehlo, Cordeau, & Laporte, 2012; Li, Chen, & Chu, 2010; Li, Chu, & Chen, 2011; Liu & Chen, 2011; Liu & Chung, 2009; Moin, Salhi, & Aziz, 2011; Shen, Chu, & Chen, 2011; Stalhane et al. 2011; Yu, Chen, & Chu, 2008; Yu, Chu, Chen, & Chu, 2010; Zachariadis, Tarantilis, & Kiranoudis, 2009) where in all of them different heuristic approaches were developed for the purpose of solving the larger scale problems. However, there seems to be a lack of papers that considered the IRP with multi-compartment vehicles, with the exception of papers from the marine transport, for instance Siswanto, Essam, and Sarker (2011). Oppen, Lokketangen, and Desrosiers (2010) solved a multi-compartment vehicle routing and inventory problem in the Livestock Collection Problem (LCP) and Popovic, Bjelic, and Radivojevic (2011) presented a simulation approach to the analysis of the applicability of a deterministic IRP solution to real life stochastic fuel consumption with fuel distribution by multi-compartment vehicles. There are several papers from the IRP research area that have considered Variable Neighborhood Search (VNS) heuristic (Hemmelmayr, Doerner, Hartl, & Savelsbergh, 2009; Hemmelmayr, Doerner, Hartl, & Savelsbergh, 2010; Liu & Chen, 2012; Liu & Lee, 2011; Zao, Chen, & Zang, 2008), which was originally developed by Mladenovic and Hansen (1997).

For an insight in methods and application of VNS we recommend the paper by Hansen, Mladenovic, and Perez (2010). For a detailed review on the IRP we refer the reader to the papers of Moin and Salhi (2007) and Andersson, Hoff, Christiansen, Hasle, and Lokketangen (2010). Also, case studies from the Netherlands (Bruggen et al. 1995) and Hong Kong (Ng, Leung, Lam, & Pan, 2008) can give a detailed insight into the practical issues of the IRP in fuel delivery.

In this paper we developed a Variable Neighborhood Search (VNS) heuristic for solving a multi-product multi-period IRP in fuel delivery with multi-compartment homogeneous vehicles, and deterministic consumption that varies over each petrol station and each fuel type. The local search and the shaking procedure (as two central procedures of the VNS) are based on three neighborhoods that are derived by the following changes of the delivery plan: relocation of individual compartments; relocation of all compartments for the observed station's fuel type; and relocation of all compartments for the observed station. The VNS heuristic is compared to a Mixed Integer Linear Programming (MILP) model and the deterministic "compartment transfer" (CT) heuristic; both models were developed by Vidovic, Popovic, and Ratkovic (2011).

This paper is organized as follows: The mathematical formulation is given in Section 2. Section 3 presents a description of the VNS heuristic. The computational results are presented in Section 4. Finally, conclusions are given in Section 5, together with directions for further research.

2. Mathematical formulation for the IRP in fuel delivery

The model assumptions are as follows:

- The delivery quantities of J fuel types for a given set of I petrol stations must be determined for each day within the planning horizon T;
- Fuel is transported by fleet of homogeneous multi-compartment vehicles of unlimited size. The total number of compartments is denoted as K. Only full compartments are delivered to petrol stations;

- Every petrol station i has a constant consumption q_{ij} of each fuel type j, while the intensity of the consumption varies over different stations and different fuel types;
- Petrol stations are equipped with underground tanks of known capacity Q_{ij} (one for each fuel type).
- Stations can be served only once a day (the observed time period);
- It is not allowed that inventory levels in petrol stations for any fuel type fall below their defined fuel consumption q_{ij};
- The total inventory costs are assumed to be dependent on the sum of the average stock levels in each day of the planning horizon, whereas transport costs depend on a vehicle's travel distance;
- One vehicle can visit up to three stations per route. This constraint is a consequence of the vehicle compartments structure and the total number of different fuel types (Cornillier et al., 2007; Cornillier et al., 2008).

The proposed mathematical formulation can be represented as the MILP model. The objective of the proposed MILP model is to minimize the sum of total inventory (IC) and routing costs (RC). Two types of binary decision variables are used for achieving this objective. The first type of binary decision variable x_{ijtk} defines the delivery quantities of all of the fuel types for all of the petrol stations over the entire planning horizon.

$$x_{ijtk} = \begin{cases} 1 & \text{— if petrol station } i \text{ is supplied with fuel type } j \\ & \text{in time period } t \text{ with } k \text{ compartments} \\ 0 & \text{— otherwise} \end{cases}$$

The second type of binary decision variable includes y_{pqwt}, y_{pqt}, and y_{pt} which are used to define the existence of routes with three, two, and one petrol stations, respectively, during each time period t. Only unique variables are used in the model. For example, variable y_{111t} is not used because the direct delivery for station "1" is represented with y_{1t}. Additionally, variable y_{123t} represents all routes visiting stations "1", "2", and "3" (length of this route is determined as the minimum length of all possible visiting orders). Travel costs c_r are calculated by multiplying minimum length and unit costs of the traveled distance for given route.

$$y_{pqwt} = \begin{cases} 1 & \text{— if petrol stations } p, q, \text{ and } w \text{ are supplied in} \\ & \text{the same route in time period } t \\ 0 & \text{— otherwise} \end{cases}$$

$$y_{pqt} = \begin{cases} 1 & \text{— if petrol stations } p \text{ and } q \text{ are supplied in} \\ & \text{the same route in time period } t \\ 0 & \text{— otherwise} \end{cases}$$

$$y_{pt} = \begin{cases} 1 & \text{— if petrol station } p \text{ is supplied} \\ & \text{with direct delivery in time period } t \\ 0 & \text{— otherwise} \end{cases}$$

Because of the mutual dependency between the delivery quantity variables and the routing variables, we have introduced an additional binary variable, H_{it}, that defines whether station i is being served in time period t. The purpose of this variable is to allow only those routes that visit petrol stations to be actually served in an observed time period t.

$$H_{it} = \begin{cases} 1 & \text{— if petrol station } i \text{ is supplied} \\ & \text{in time period } t \\ 0 & \text{— otherwise} \end{cases}$$

Indices:

i, p, q, w	petrol stations ($i, p, q, w \in \{1, 2, \ldots, I\},$)
j	fuel types ($j \in \{1, 2, \ldots, J\}$)
t, z	time period or day in the planning horizon T ($t, z \in \{1, 2, \ldots, T\}$)
k	number of compartments ($k \in \{1, 2, \ldots, K\}$)
d_k	delivery quantities ($k \in \{1, 2, \ldots, K\}$) that correspond to the total amount of delivered fuel

Parameters:

S_{ij}^0	stock level of fuel type j at station i at the beginning of the planning horizon
q_{ij}	consumption of the fuel type j at station i
c_{inv}	inventory carrying costs per day
c_r	transportation costs per unit of traveled distance
Q_{ij}	capacity of the underground reservoir for the fuel type j at station i
r_{pqw}	minimum length of a route that includes petrol stations p, q, and w
r_{pq}	minimum length of a route that includes petrol stations p and w
r_p	minimum length of a route that includes only petrol station p

Objective function:

Minimize $\rightarrow IC + RC$ (1)

$$IC = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}\left(\left(S_{ij}^0 - t \cdot q_{ij} + \frac{q_{ij}}{2}\right) + \sum_{z=1}^{t}\sum_{k=1}^{K}x_{ijzk} \cdot d_k\right) \cdot c_{inv} \quad (2)$$

$$RC = \sum_{t=1}^{T}\sum_{p=1}^{I}\left(y_{pt} \cdot r_p + \sum_{q=p+1}^{I}\left(y_{pqt} \cdot r_{pq} + \sum_{w=q+1}^{I}y_{pqwt} \cdot r_{pqw}\right)\right) \cdot c_r \quad (3)$$

The inventory segment (2) of the objective function is looking to minimize the total inventory costs for the fuel stored in all of the petrol stations during the observed planning horizon. Those costs are based on the average daily inventory levels at the petrol stations. The routing segment (3) of the objective function is looking to minimize the total travel distance of all routes that are used for delivery during the observed planning horizon. The routing costs are calculated as a sum of all travel costs incurred by deliveries in the observed planning horizon.

Subject to:

$$S_{ij}^0 + \sum_{t=1}^{z}\sum_{k=1}^{K}x_{ijtk} \cdot d_k - \sum_{t=1}^{z-1}q_{ij} \leqslant Q_{ij} \quad \forall i \in I \quad \forall j \in J \quad \forall z \in T \quad (4)$$

$$S_{ij}^0 + \sum_{t=1}^{z}\sum_{k=1}^{K}x_{ijtk} \cdot d_k - \sum_{t=1}^{z}q_{ij} \geqslant q_{ij} \quad \forall i \in I \quad \forall j \in J \quad \forall z \in T \quad (5)$$

$$y_{pt} \leqslant H_{pt} \quad \forall t \in T \quad \forall p \in I \quad (6.1)$$

$$2 \cdot y_{pqt} \leqslant H_{pt} + H_{qt} \quad \forall t \in T \quad \forall (p, q) \in I^2 \text{ for all } p < q \quad (6.2)$$

$$3 \cdot y_{pqwt} \leqslant H_{pt} + H_{qt} + H_{wt} \quad \forall t \in T \quad \forall (p, q, w) \in I^3 \text{ for all } p < q < w \quad (6.3)$$

$$H_{it} \leqslant \sum_{j=1}^{J}\sum_{k=1}^{K}x_{ijtk} \quad \forall t \in T \quad \forall i \in I \quad (7)$$

$$H_{it} \geqslant \frac{1}{J \cdot K} \cdot \sum_{j=1}^{J}\sum_{k=1}^{K}x_{ijtk} \quad \forall t \in T \quad \forall i \in I \quad (8)$$

$$\sum_{p=1}^{I}\left(y_{pt} + \sum_{q=p+1}^{I}\left(2 \cdot y_{pqt} + \sum_{w=q+1}^{I}3 \cdot y_{pqwt}\right)\right) = \sum_{i=1}^{I}H_{it} \quad \forall t \in T \quad (9)$$

$$y_{it} + \sum_{q=i+1}^{I}\left(y_{iqt} + \sum_{w=q+1}^{I}y_{iqwt}\right) + \sum_{p=1}^{I}\sum_{i=p+1}^{I}\left(y_{pit} + \sum_{w=i+1}^{I}y_{piwt}\right)$$
$$+ \sum_{p=1}^{I}\sum_{q=p+1}^{I}\sum_{i=q+1}^{I}y_{pqit} \leqslant 1 \quad \forall t \in T \quad \forall i \in I \quad (10)$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K}x_{ijtk} \cdot k \leqslant K \quad \forall t \in T \quad \forall i \in I \quad (11)$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K}(x_{pjtk} + x_{qjtk}) \cdot k \leqslant K \cdot (2 - y_{pqt}) \quad \forall t \in T \quad \forall (p, q) \in I^2$$
$$\text{for all } p < q \quad (12)$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K}(x_{pjtk} + x_{qjtk} + x_{wjtk}) \cdot k \leqslant K \cdot (3 - 2 \cdot y_{pqwt}) \quad \forall t \in T$$
$$\forall (p, q, e) \in I^3 \text{ for all } p < q < w \quad (13)$$

$$H_{it}, x_{ijtk}, y_{pqwt}, y_{pqt}, y_{pt} \in \{0, 1\} \quad \forall j \in J \quad \forall t \in T \quad \forall k \in K$$
$$\forall i \in I \quad \forall (p, q, w) \in I^3 \text{ for all } p < q < w \quad (14)$$

Constraints (4) limit the maximum quantity of a fuel type up to the reservoir capacity in each of the observed time periods, and constraints (5) define the minimum quantity of a fuel type in a station's reservoirs that can meet the demand in the observed time period. Constraints (6.1)–(6.3) define that a route can be performed if and only if all of the petrol stations included in the route are to be supplied on that day. Inequalities (7) and (8) define petrol stations that need to be served. A petrol station needs to be served if at least one compartment of at least one fuel type should be delivered. Constraints (9) assure that the number of stations served by all of the routes is equal to the number of all of the stations that need to be served. Constraints (10) limits the number of routes visiting petrol station i in time period t to only one. Constraints (11) prohibit multiple direct deliveries (vehicles serving only one station in a single route) to one petrol station during the same day. If constraints (11) are omitted, then different fuel types can be delivered to the same station, each in a quantity lower than the total vehicle capacity. However, when the total number of compartments for all of the fuel types delivered to the same station is greater than the total vehicle capacity, then the station is visited more than once in the same time period. Constraints (12) and (13) restrict the delivery quantity in each route serving more than one station to a maximum of K compartments in each day within the planning horizon. For example, if one station has a delivery of two compartments, then this station can be served either with direct delivery or together with the station whose delivery does not exceed the remainder of the vehicle's capacity. Constraints (14) define the binary nature of the variables.

3. Variable Neighborhood Search heuristic

The VNS algorithm, as a new metaheuristic concept, was developed by Mladenovic and Hansen (1997) with the basic idea of a systematic change of a neighborhood within a local search algorithm. This implies that several neighborhoods are used to search

for the solution improvement. Each succeeding neighborhood covers a larger search space. Shaking procedure is responsible for obtaining a new starting point for a local search, within each neighborhood. When a better solution is found, the current best solution is updated and the VNS algorithm starts its search from the first neighborhood. Different stopping criterions can be used: computation time, number of iterations, etc. The steps of the basic VNS algorithm are presented in Fig. 1.

In this paper, we use the VNS heuristic for solving the IRP in fuel delivery, in which the Randomized Variable Neighborhood Descent (RVND) method is used for local search procedure (step b in Fig. 1). In the RVND, neighborhoods are randomly changed opposed to the Variable Neighborhood Descent (VND) method where the change of neighborhoods is performed in a deterministic way. In the following three subsections we describe the procedures for obtaining the initial solution, shaking procedure, and the RVND local search procedure.

3.1. Initial solution

Usually, the initial solution can be obtained by some constructive heuristic or one can use a random feasible solution. We tested the VNS with three different initial solutions: solution obtained from the inventory model (solution from MILP model presented in this paper without the *RC* segment in the objective function); solution obtained with the CT heuristic; and solution obtained from the Reduced VNS where its initial solution is set as the solution from the inventory model (corresponds to the VNS without the local search step). The best results from the VNS heuristic were obtained with the initial solution derived from the CT heuristic.

A detailed description of the CT heuristic is given in Vidovic et al. (2011) which we will use for the comparison with the proposed VNS heuristic. In this paper we outline only the main features of the deterministic CT heuristic. The CT heuristic is based on the idea of solving the inventory model of the MILP model and iterative change of the delivery plan by transferring the deliveries to be realized one or a few days earlier (compartment transfers to days ahead would incur inventory levels below consumption intensity). To speed up the process of finding a better solution, the improvement steps are evaluated by the use of eligibility calculation and, on each day, only the *E* transfers that are the most eligible are observed. Eligibility estimation is based on the values of the three criteria: *possibility of moving the delivery*, *change in the number of "to be served" petrol stations*, and *spatial grouping of stations*. Additionally, in order to further improve the solution obtained from the CT heuristic we apply the deterministic VND local search.

For the route construction procedure we use the Sweep method (Gillet & Miller, 1974), in a somewhat modified form: routes are being constructed starting from the station that has the biggest "polar" gap to its nearest preceding station. This Sweep Big Gap method was proven to be effective for the multi-compartment vehicle routing, according to Derigs et al. (2010).

3.2. Shaking procedure

The main problem of the IRP in fuel delivery is to allocate the deliveries over the planning horizon so that the overall inventory and routing costs are minimal. Therefore, shaking procedure in the proposed VNS heuristic is based on the deliveries transfer between days of planning horizon. Inventory model without routing costs determines the total amount of compartments that needs to be delivered in the observed planning horizon. Shaking procedure has the task of changing only the time instants of each delivery. Delivery plan structure implies three basic types of shaking: shaking of individual compartments; shaking of all compartments of the same fuel type in the observed station and day; shaking of all compartments in the same station in the observed day. The fourth shaking procedure simultaneously applies all of these three basic shaking procedures. An example of shaking procedures is presented in Fig. 2.

Each shaking type can have neighborhoods with different transfer quantities T_q expressed in percentages of all possible transfers. We use an increasing list of these percentages to derive different neighborhoods of increasing size. For example, in the entire planning horizon there can be 200 compartments in total for delivery to 150 separate fuel types in 100 petrol stations. The 2% *of all possible transfers* neighborhood would incur 4 compartments transfer for the "compartment" shaking, 3 separate fuel types transfer for the "entire fuel type" shaking, and 2 petrol stations transfer for the "entire station" shaking. By the transfer depth we are referring to the number of days between the observed day and the day to which transfer is being executed. The transfer depth can randomly take value from one day to $T - 1$ for each transfer. The transfer can be made either to earlier or to later days with regard to the observed day. Each shaking move takes following steps: (1) randomly choose the day from where the transfer will be made; (2) randomly choose the transfer from this day; (3) randomly choose the transfer depth (excluding the day from where the transfer is made); (4) while chosen transfer incurs infeasible solution repeat steps 1–3; (5) execute the transfer and repeat steps 1–5 until the transfer of all required transfers from the current neighborhood. The feasibility of the solution must be respected in all search procedures where this feasibility is also defined by the constraints of the MILP model: minimum and maximum inventory level, vehicle capacity, single route per vehicle, single delivery in each day for a single station, and maximum three stops per route.

3.3. RVND local search

We use the RVND guided search with the first improvement approach. This means that the first occurrence of improvement in neighborhood search is accepted, and the search is repeated until no more improvements can be found. A randomized change of the neighborhoods gives more diversification to the local search. Instead of using the same order of neighborhoods, we randomly change this order after each shaking move. The RVND local search has two levels: the micro level (intra-period search) where routes

---

Initialization. Select the set of neighborhood structures N_k, $k = 1,...,k_{max}$, that will be used in the search; find an initial solution x; choose a stopping condition;

Repeat the following until the stopping condition is met:
   (1) Set $k \leftarrow 1$; (2) Until $k = k_{max}$, repeat the following steps:
      (a) *Shaking*. Generate a point x' at random from the kth neighborhood of x ($x' \in N_k(x)$);
      (b) *Local search*. Apply some local search method with x' as initial solution; denote with x'' the so obtained local optimum;
      (c) *Move or not*. If this local optimum is better than the incumbent, move there ($x \leftarrow x''$), and continue the search with N_1 ($k \leftarrow 1$);
         otherwise, set $k \leftarrow k + 1$;

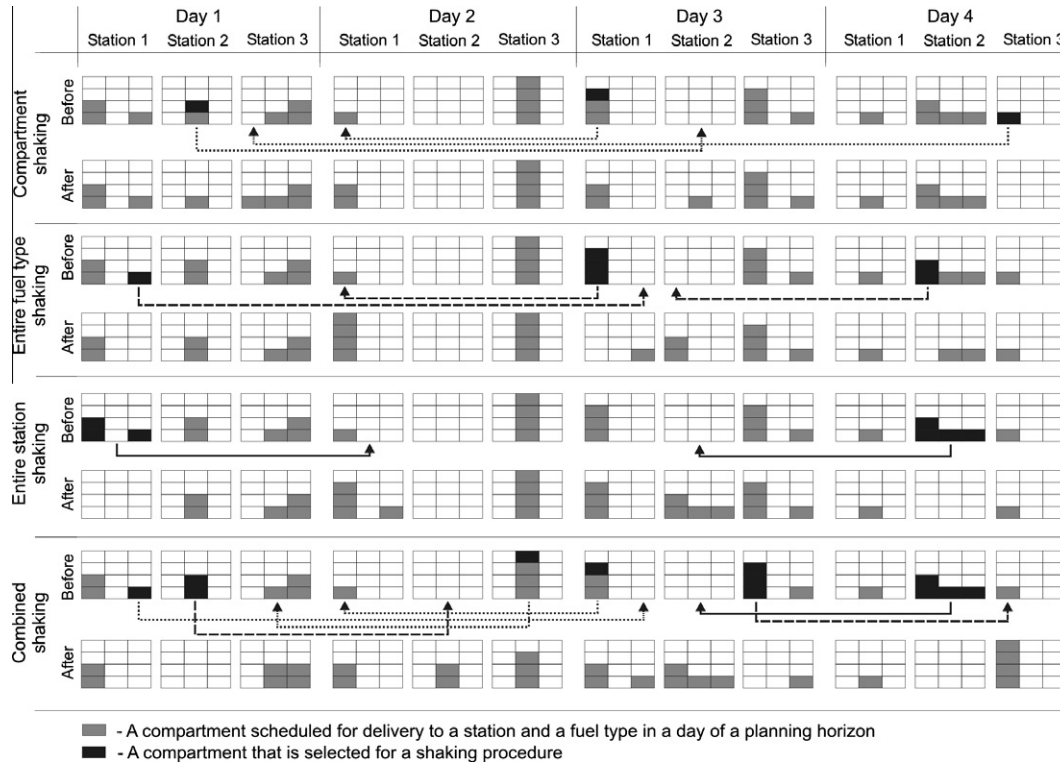**Fig. 1.** Steps of the VNS (Hansen & Mladenovic, 2001).

Fig. 2. An example of four shaking methods of the delivery plan used in the VNS.
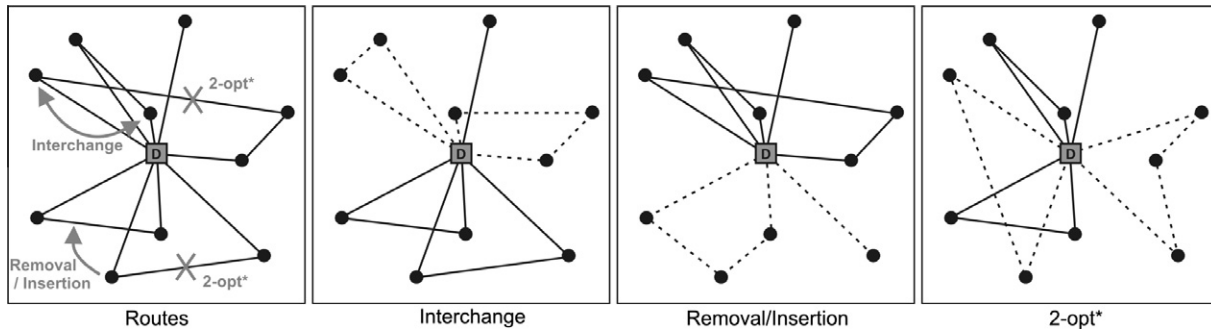

Fig. 3. An example of three methods for the neighborhoods construction in the intra-period VND search; used for routes improvement in each day of planning horizon.

improvement is carried out in each day of planning horizon with the aim of minimizing the vehicles traveled distance; and the macro level (inter-period search) where we try to minimize the total inventory and routing costs by the reallocation of deliveries over the planning horizon.

For the local intra-period search, we used three neighborhood structures to improve the routing of deliveries on each day of the planning horizon: (1) the interchange of a single station between two routes for the same day; (2) the removal of a single station from one route and its insertion into another route for the same day; and (3) 2-opt*, the removal of two arcs (one from each route) and finding the best possible routes reconnections (Potvin & Rousseau, 1995). An example of three methods for neighborhoods construction in the intra-period search is presented in Fig. 3. The local intra-period search procedure for the improvement of vehicle routing is given in Algorithm 1.

Algorithm 1 RVND intra-period search procedure

solution ← Sweep_Big_Gap()
while *improvement*:
  improvement ← False
  Intra_neigh_set = ['Stations_interchange', 'Removal/insertion', '2-opt*']
  Intra_neigh_set ← Randomize_order(*Intra_neigh_set*)
  for N in *Neigh_set*:
    if N = 'Stations_interchange':
      for each pair of routes in *all_routes*:
      for all feasible swaps of a pair of stations from two routes:
          if interchange incurs shorter routes:
            improvement ← True
            update *The_routes*
            break to while

```
if N = "Removal/insertion":
    for route_from in all_routes:
        for station in route_from:
            for route_to in all_routes/route_from:
                remove station from route_from and insert in
                route_to
                if solution is feasible and incurs shorter routes:
                    improvement ← True
                    update The_routes
                    break to while
if N = '2-opt*':
    for route_1 in all_routes:
        for route_2 in all_routes/route_1:
            for arc_1 in route_1:
                for arc_2 in route_2:
                    delete arc_1 and arc_2
                    make all feasible reconnections
                    if there exists reconnection with shorter
                    routes:
                        improvement ← True
                        update The_routes
                        break to while
```

For the inter-period search we use the same logic as in the shaking procedure. The differences lie in the deterministic approach of evaluating all possible solutions for the given value of the maximum transfer depth within three neighborhoods: the transfer of individual compartments; the transfer of compartments from entire fuel type; the transfer of compartments from entire station. The inter-period search is given in Algorithm 2.

Algorithm 2 RVND inter-period search procedure

---

while *improvement*:
  improvement ← False
  Inter_neigh_set = ['station compartments','fuel type compartments', 'individual compartment']
  Inter_neigh_set ← Randomize_order(*Inter_neigh_set*)
  for *N* in *Neigh_set*:
    depth = 1
    for *day_from* in *planning_horizon*:
    for *day_to* in [*day_from*-*depth*: *day_from*+*depth*] / *day_from*:
      'Move or not procedure'
      for Q_N in *The_Delivery_Plan*[*day_from*]: *
        Temp_solution ← *The_Delivery_Plan*
        remove Q_N from *day_from* in *Temp_solution*
        insert Q_N in *day_to* in *Temp_solution*
        if *Temp_solution* is feasible and incurs lower total costs than *The_Delivery_Plan*:**
          improvement ← True
          The_Delivery_Plan ← *Temp_solution*
          break to while
        temp ← *day_from*; *day_from* ← *day_to*;
        day_to ← *temp*; repeat **'Move or not procedure'**
        if *depth* < *maximal_depth*:
          depth ← *depth* +1

---

*The Q_N denotes minimal unit of given neighborhood N (a compartment, all compartments for observed fuel type, all compartments for observed petrol station).
**The calculation of the total costs for the *Temp_solution* requires routes construction and improvement by intra-period search procedure for days with the changes of the delivery quantities

---

The flowchart of the proposed VNS heuristic is presented in Fig. 4. The stopping criterion becomes active in the case when the last neighborhood in the last shaking procedure does not incur an improvement of the current best solution.

4. Computational results

In order to evaluate the solutions obtained from the proposed VNS heuristic, the CT heuristic, and the MILP model we have generated 10 instances of the smallest scale problem (the problem set P1) that has 10 petrol stations and a 3 day planning horizon. The larger scale problems are not suitable for the MILP model considering the computational time (for example, the instance 1 with $3 \times 6t$ vehicle required more than 20 h of CPU to obtain the optimal solution). Heuristic models were tested on following two larger scale problems (10 instances for each problem): the problem set P2 that has 15 petrol stations and a 4 day planning horizon; and the problem set P3 that has 20 petrol stations and a 5 day planning horizon. The heuristic parameters have following values: the T_q takes value from the set $(3, 6, 9, 12, 15, 25, 35, 50, 75, 100)$ (in total 10 neighborhoods for each shaking type); the maximum transfer depth for inter-period local search is 2 days; the eligibility E is 3, 4, and 5 for the problem sets P1, P2, and P3 respectively.

Instances for all three problem sets have the following parameters. There are three fuel types ($J = 3$). Fuel consumption q_{ij} takes values from the interval [1.0, 2.0] with the probability of 0.4; from the interval [2.0, 3.0] with the probability of 0.4; and from the interval [3.0, 6.0] with the probability of 0.2 (fuel consumptions are rounded to the first decimal place). The spatial coordinates of the petrol stations are randomly generated in a square $[-50, 50]$ km and the location of the depot is in the center of that square (coordinates $(0, 0)$). Routing costs c_r are 2€ per traveled kilometer and daily inventory carrying costs c_{inv} are 1€ per ton of average daily inventory level. Stock levels of fuels S_{ij}^0 at the beginning of the observed time period are generated randomly between 6 and 12t. All reservoir capacities Q_{ij} can hold up to 40t of fuel. Additionally, we wanted to test the quality of models with three different vehicle types: vehicles with 3 compartments of 6t; vehicles with 4 compartments of 6t; and vehicles with 5 compartments of 6t. Because of the stochastic nature of the VNS we solved each instance of the P1 in 50 iterations, each instance of the P2 in 20 iterations and each instance of the P3 in 10 iterations.

The MILP model was implemented through the CPLEX 12 on a desktop PC with a 2.0 GHz Dual Core processor with 2 GB of RAM memory. All of the input data needed for the model implementation as well as for the heuristics were implemented in Python 2.6.

The comparison of the results obtained from optimal and heuristic models for the P1 instances are presented in Table 1. The solution from the CT heuristic is improved with the VND local search (for the intra-tour search neighborhood the order is the stations interchange, the removal/insertion, and the 2-opt*; for the inter-period search neighborhood the order is the "station compartments" transfer, the "fuel type compartments" transfer, and the "individual compartment" transfer). From Table 1 we can see that the stochastic VNS heuristics incurs in average 0.271% higher total costs than MILP model, while the deterministic CT heuristic incurs 2.451% higher total costs than MILP model. Both heuristic models give the lowest error in the case with the $3 \times 6t$ vehicle type, and the highest error in the case with the $4 \times 6t$ vehicle type.

The comparison of results obtained from stochastic VNS heuristic and deterministic CT heuristic for the P2 and P3 instances are presented in Table 2. Because P2 and P3 instances could not be solved in reasonable time by the MILP model, the error stands for difference between the average total cost obtained by VNS and the cost obtained by CT heuristic. VNS heuristics gives in aver-
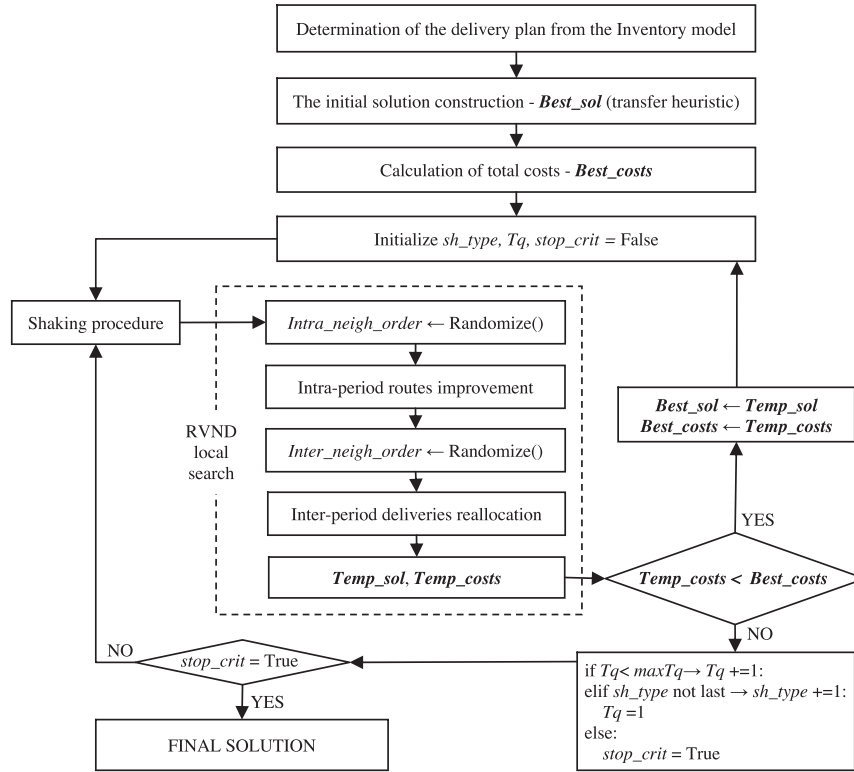
Fig. 4. The flowchart of the VNS.

Table 1
The results for the P1 instances.

Vehicle	Inst.	MILP model		VNS heuristic				CT heuristic		
		Cost	CPU (s)	Avg. cost	Min. cost	Avg. Error (%)	Avg. CPU (s)	Cost	Error (%)	CPU (s)
3 × 6t	1	3500.9	111.2	3506.8	3500.9	0.167	9.0	3507.8	0.195	1.1
	2	3495.3	62.9	3508.7	3495.3	0.383	14.8	3551.0	1.593	1.1
	3	3047.4	5.5	3052.0	3047.4	0.151	6.3	3113.1	2.158	0.9
	4	2951.5	46.7	2952.1	2951.5	0.020	11.1	3076.3	4.231	1.0
	5	2827.1	4.1	2827.1	2827.1	0.000	5.4	2827.1	0.000	0.9
	6	3490.8	27.0	3496.6	3490.8	0.166	11.9	3516.9	0.747	1.0
	7	4564.4	27.8	4572.8	4564.4	0.183	13.4	4661.6	2.129	1.0
	8	3068.9	5.8	3068.9	3068.9	0.000	7.6	3086.8	0.583	0.9
	9	2958.5	260.8	2965.1	2958.5	0.223	13.3	3081.7	4.162	0.9
	10	3105.8	1041.2	3105.8	3105.8	0.000	8.9	3105.8	0.000	1.0
	Avg.	*3301.1*	*159.3*	*3305.6*	*2827.1*	*0.129*	*10.2*	*3352.8*	*1.580*	*1.0*
4 × 6t	1	2996.0	143.1	3007.4	2996.0	0.380	11.4	3153.6	5.259	1.1
	2	3049.9	403.7	3068.8	3052.7	0.620	25.2	3127.7	2.553	1.2
	3	2666.4	35.0	2684.0	2672.6	0.660	12.1	2702.0	1.336	1.0
	4	2563.1	150.8	2575.9	2563.1	0.498	12.6	2658.9	3.736	1.1
	5	2462.3	16.5	2490.5	2470.0	1.144	12.0	2536.1	2.997	1.1
	6	3087.3	200.8	3091.4	3087.3	0.134	17.8	3150.1	2.036	1.1
	7	3705.6	928.6	3711.0	3705.6	0.145	19.3	3975.8	7.292	1.2
	8	2732.2	68.4	2732.2	2732.2	0.000	15.1	2736.9	0.171	1.3
	9	2611.2	2774.6	2616.3	2611.2	0.193	18.6	2636.4	0.964	1.1
	10	2656.5	565.8	2664.0	2656.5	0.281	16.7	2779.9	4.644	1.1
	Avg.	*2853.1*	*528.7*	*2864.1*	*2470.0*	*0.406*	*16.1*	*2945.7*	*3.099*	*1.1*
5 × 6t	1	2654.9	113.3	2654.9	2654.9	0.000	11.4	2684.9	1.130	1.2
	2	2767.1	1023.0	2773.6	2767.1	0.234	27.1	2937.1	6.144	1.3
	3	2376.4	40.2	2396.3	2376.4	0.840	17.0	2414.2	1.592	1.3
	4	2267.4	111.8	2267.4	2267.4	0.000	15.5	2267.4	0.000	1.3
	5	2164.2	9.2	2165.0	2164.2	0.037	13.5	2293.3	5.967	1.1
	6	2593.0	24.4	2593.0	2593.0	0.000	15.6	2599.0	0.231	1.2
	7	3402.5	2768.3	3434.5	3402.5	0.942	30.4	3454.8	1.537	1.5
	8	2468.4	49.3	2468.4	2468.4	0.000	23.3	2641.4	7.008	1.2
	9	2304.2	617.9	2304.6	2304.2	0.019	17.4	2335.5	1.360	1.2
	10	2327.9	255.7	2344.5	2327.9	0.713	14.5	2369.2	1.775	1.1
	Avg.	*2532.6*	*501.3*	*2540.2*	*2164.2*	*0.279*	*18.6*	*2599.7*	*2.674*	*1.2*
Total avg.		2895.6	396.4	2903.3	2164.2	0.271	14.9	2966.1	2.451	1.1

Table 2
The results for the P2 and P3 instances.

Vehicle	Inst.	P2						P3					
		VNS heuristic			CT heuristic			VNS heuristic			CT heuristic		
		Avg. cost	Min. cost	Avg. CPU (s)	Cost	Avg. Error (%)	CPU (s)	Avg. cost	Min. cost	Avg. CPU (s)	Cost	Avg. Error (%)	CPU (s)
3 × 6t	1	7005.3	7002.0	73.3	7055.9	0.723	2.7	12246.2	12199.8	641.9	12376.9	1.067	12.3
	2	6977.0	6959.9	59.6	6987.9	0.156	2.6	11715.0	11712.6	386.3	11715.3	0.002	9.1
	3	5292.4	5275.0	59.6	5326.0	0.636	2.4	9791.2	9755.4	763.2	10023.1	2.369	6.8
	4	5390.2	5377.4	65.0	5482.1	1.705	2.6	9858.3	9801.5	464.9	9908.0	0.504	10.5
	5	6774.0	6745.1	66.3	6813.9	0.588	2.6	10453.1	10346.1	669.8	10759.9	2.935	7.7
	6	6977.1	6967.1	49.2	7198.9	3.179	2.9	11672.5	11604.4	601.2	11888.2	1.848	10.0
	7	8019.4	8007.5	90.7	8098.3	0.984	3.1	13022.7	12953.8	794.8	13285.0	2.014	6.9
	8	6162.6	6158.5	39.5	6238.7	1.236	1.7	10343.5	10312.7	488.4	10451.6	1.045	8.9
	9	5950.8	5937.9	53.8	6047.9	1.632	2.4	10214.6	10173.8	491.7	10437.1	2.178	7.4
	10	6065.9	6055.0	61.3	6135.6	1.148	2.3	9867.2	9832.0	529.2	9900.3	0.335	9.7
	Avg.	*6461.5*	*6448.5*	*61.8*	*6538.5*	*1.199*	*2.5*	*10918.4*	*10869.2*	*583.1*	*11074.5*	*1.430*	*8.9*
4 × 6t	1	5943.5	5908.9	206.6	6110.5	2.810	3.5	10428.2	10340.6	1654.1	10892.5	4.453	48.2
	2	5994.9	5952.9	198.2	6140.6	2.430	7.9	10022.6	9978.9	1956.5	10324.8	3.015	34.7
	3	4618.7	4589.2	108.7	4696.0	1.673	3.3	8374.9	8293.9	1175.1	8466.8	1.097	16.7
	4	4687.2	4657.6	186.3	4809.4	2.606	5.0	8469.1	8432.1	1008.8	8715.2	2.906	21.8
	5	5716.4	5677.6	161.6	5937.0	3.859	3.7	8970.0	8905.1	1277.6	9244.7	3.062	18.4
	6	5980.6	5942.9	149.2	6127.1	2.450	3.6	10062.6	9996.5	1245.4	10232.4	1.687	27.9
	7	6837.4	6771.4	151.3	7036.0	2.904	4.1	10878.3	10779.6	1983.7	11205.6	3.009	23.0
	8	5233.9	5218.9	94.6	5295.7	1.182	3.8	8953.4	8909.2	1404.4	9089.6	1.521	61.5
	9	5169.1	5122.5	145.6	5469.5	5.811	4.7	8835.4	8749.2	1652.6	8989.5	1.744	32.4
	10	5189.1	5150.6	156.1	5554.8	7.047	3.7	8525.7	8451.6	1214.5	8758.2	2.728	22.5
	Avg.	*5537.1*	*5499.3*	*155.8*	*5717.7*	*3.277*	*4.3*	*9352.0*	*9283.7*	*1457.3*	*9591.9*	*2.522*	*30.7*
5 × 6t	1	5254.9	5166.9	356.1	5363.6	2.069	4.8	9233.2	9150.5	3657.0	9599.1	3.963	60.9
	2	5341.3	5313.6	333.4	5572.9	4.336	5.7	8937.2	8881.5	3598.2	9267.4	3.695	41.8
	3	4198.1	4173.0	228.5	4286.0	2.095	7.4	7516.6	7428.6	3177.0	7878.8	4.818	54.3
	4	4233.9	4208.9	263.2	4532.4	7.050	5.6	7612.9	7524.0	2774.5	8066.2	5.954	53.4
	5	5072.5	5037.4	271.2	5243.9	3.379	8.0	7981.6	7919.1	2844.2	8083.1	1.271	52.7
	6	5433.3	5397.3	225.4	5555.8	2.255	6.0	8885.8	8830.3	2408.8	9046.7	1.811	39.9
	7	5956.5	5924.6	336.1	6026.0	1.167	9.5	9628.0	9525.6	5658.6	9786.8	1.650	86.0
	8	4731.2	4691.4	265.7	5074.5	7.257	7.5	7956.6	7828.7	2712.7	8336.8	4.779	41.1
	9	4730.4	4687.6	215.4	4788.9	1.236	6.2	7917.3	7830.7	3064.1	8092.4	2.211	50.4
	10	4680.0	4647.3	247.6	4818.8	2.966	4.5	7563.1	7497.5	2016.1	7814.1	3.319	30.9
	Avg.	*4963.2*	*4924.8*	*274.3*	*5126.3*	*3.381*	*6.5*	*8323.2*	*8241.6*	*3191.1*	*8597.1*	*3.347*	*51.1*
Total avg.		5653.9	5624.2	164.0	5794.2	2.619	4.5	9531.2	9464.8	1743.8	9754.5	2.433	30.3

age 2.619% and 2.433% lower total costs than the CT heuristic, for the P2 and P3 respectively. Both heuristic models give lowest error in the case with the 3 × 6t vehicle type, and the highest error in the case with the 5 × 6t vehicle type. At the same time, total inventory and routing costs are decreasing and computational time is increasing with the increase of the number of vehicle's compartments (for all three problem sets).

5. Conclusions

In this paper we developed the VNS heuristic model for solving the multi-period multi-product IRP in fuel delivery with multi-compartment homogenous vehicles. This stochastic VNS heuristic is compared to deterministic CT heuristic and to the optimal MILP model. For the P1 problem instances, VNS heuristic was able to find optimal solution for 27 out of 30 instances. The stochastic VNS heuristic showed better results than the deterministic CT heuristic in 87 out of 90 instances for all three problem sets (for only three instances of P1 problem set both VNS and CT heuristics found optimal solution).

Future research should be focused on the heterogeneous vehicle fleet and stochastic nature of the IRP. The model extensions by considering these factors can enrich and bring the IRP model closer to the real life conditions.

References

Andersson, H., Hoff, A., Christiansen, M., Hasle, G., & Lokketangen, A. (2010). Industrial aspects and literature survey: Combined inventory management and routing. *Computers and Operations Research., 37*, 1515–1536.

Avella, P., Boccia, M., & Sforza, A. (2004). Solving a fuel delivery problem by heuristic and exact approaches. *European Journal of Operational Research, 152*, 170–179.

Bell, W. J., Dalberto, L. M., Fisher, M. L., Greenfield, A. J., Jaikumar, R., Kedia, P., et al. (1983). Improving the distribution of industrial gases with an on-line computerized routing and scheduling optimizer. *Interfaces, 13*(6), 4–23.

Bersani, C., Minciardi, R., & Sacile, R. (2010). Economic and risk implications in the distribution of petrol products to service stations under retailer managed and vendor managed inventories. *International Journal of Sustainable Transportation, 4*(3), 129–153.

Boctor, F. F., Renaud, J., & Cornillier, F. (2011). Trip packing in petrol stations replenishment. *Omega, 39*, 86–98.

Brown, G. G., Ellis, C. J., Graves, G. W., & Ronen, D. (1987). Real-time, wide area dispatch of mobil tank trucks. *Interfaces, 17*(1), 107–120.

Brown, G. G., & Graves, G. W. (1981). Real-time dispatch of petroleum tank trucks. *Management Science, 27*(1), 19–32.

Bruggen, L., Gruson, R., & Salomon, M. (1995). Reconsidering the distribution structure of gasoline products for a large oil company. *European Journal of Operational Research, 81*, 460–473.

Campbell, A., & Savelsbergh, M. (2004). A decomposition approach for the inventory-routing problem. *Transportation Science, 38*, 488–502.

Coehlo, L. C., Cordeau, J., & Laporte, G. (2012). The inventory-routing problem with transshipment. *Computers and Operations Research.* http://dx.doi.org/10.1016/j.cor.2011.12.020.

Cornillier, F., Boctor, F. F., Laporte, G., & Renaud, J. (2007). An exact algorithm for the petrol station replenishment problem. *Journal of the Operational Research Society, 59*, 607–615.

Cornillier, F., Boctor, F. F., Laporte, G., & Renaud, J. (2008). A heuristic for the multi-period petrol station replenishment problem. *European Journal of Operational Research, 202*, 295–305.

Derigs, U., Gottlieb, J., Kalkoff, J., Piesche, M., Rothlauf, F., & Vogel, U. (2010). Vehicle routing with compartments: Applications, modelling and heuristics. *OR Spectrum, 334*, 885–914.

Fallahi, A., Prins, C., & Calvo, R. W. (2008). A memetic algorithm and a tabu search for the multi-compartment vehicle routing problem. *Computers and Operations Research, 35*, 1725–1741.

Gillet, B. E., & Miller, L. R. (1974). A heuristic algorithm for the vehicle-dispatch problem. *Operations Research, 22*(2), 340–349.

Hansen, P., & Mladenovic, N. (2001). Variable Neighborhood Search: Principles and applications. *European Journal of Operational Research, 130*, 449–467.

Hansen, P., Mladenovic, N., & Perez, J. A. M. (2010). Variable neighbourhood search: Methods and applications. *Annals of Operation Research, 175*, 367–407.

Hemmelmayr, V., Doerner, K. F., Hartl, R. F., & Savelsbergh, M. W. P. (2009). Delivery strategies for blood products supplies. *OR Spectrum, 31*, 707–725.

Hemmelmayr, V., Doerner, K. F., Hartl, R. F., & Savelsbergh, M. W. P. (2010). Vendor managed inventory for environments with stochastic product usage. *European Journal of Operational Research, 202*, 686–695.

Li, J., Chen, H., & Chu, F. (2010). Performance evaluation of distribution strategies for the inventory routing problem. *European Journal of Operational Research, 202*, 412–419.

Li, J., Chu, F., & Chen, H. (2011). A solution approach to the inventory routing problem in a three-level distribution system. *European Journal of Operational Research, 210*, 736–744.

Liu, S., & Chen, J. (2011). A heuristic method for the inventory routing and pricing problem in a supply chain. *Expert Systems with Applications, 38*, 1447–1456.

Liu, S., & Chen, A. (2012). Variable neighborhood search for the inventory routing and scheduling problem in a supply chain. *Expert Systems with Applications, 39*, 4149–4159.

Liu, S., & Chung, C. (2009). A heuristic method for the vehicle routing problem with backhauls and inventory. *Journal of Intelligent Manufacturing, 20*, 29–42.

Liu, S., & Lee, W. (2011). A heuristic method for the inventory routing problem with time windows. *Expert Systems with Applications, 38*, 13223–13231.

Mendoza, J. E., Castanier, B., Gueret, C., Medaglia, A. L., & Velasco, N. (2010). A memetic algorithm for the multi-compartment vehicle routing problem with stochastic demands. *Computers and Operations Research, 37*, 1886–1898.

Mladenovic, N., & Hansen, P. (1997). Variable neighborhood search. *Computers and Operations Research., 24*(11), 1097–1100.

Moin, N. H., & Salhi, S. (2007). Inventory routing problem: A logistical overview. *Journal of the Operational Research Society, 58*, 1185–1194.

Moin, N. H., Salhi, S., & Aziz, N. A. B. (2011). An efficient hybrid genetic algorithm for the multi-product multi-period inventory routing problem. *International Journal of Production Economics, 133*, 334–343.

Ng, W. L., Leung, S. C. H., Lam, J. K. P., & Pan, S. W. (2008). Petrol delivery tanker assignment and routing: a case study in Hong Kong. *Journal of the Operational Research Society, 59*, 1191–1200.

Oppen, J., Lokketangen, A., & Desrosiers, J. (2010). Solving a rich vehicle routing and inventory problem using column generation. *Computers and Operations Research, 37*, 1308–1317.

Popovic, D., Bjelic, N., & Radivojevic, G. (2011). Simulation approach to analyse deterministic irp solution of the stochastic fuel delivery problem. 14th EWGT & 26th MEC & 1st RH, procedia social and behavioral sciences (Vol. 20, pp. 273–282).

Potvin, J., & Rousseau, J. (1995). An excange heuristic for routing problems with time windows. *Journal of the Operational Research Society, 46*, 1433–1446.

Shen, Q., Chu, F., & Chen, H. (2011). A Lagrangian relaxation approach for a multi-mode inventory routing problem with transshipment in crude oil transportation. *Computers and Chemical Engineering, 35*, 2113–2123.

Siswanto, N., Essam, D., & Sarker, R. (2011). Solving the ship inventory routing and scheduling problem with undedicated compartments. *Computers and Industrial Engineering, 61*, 289–299.

Stalhane, M., Rakke, J. G., Moe, C. R., Andersson, H., Christiansen, M., & Fagerholt, K. (2011). A construction and improvement heuristic for a liquefied natural gas inventory routing problem. *Computers and Industrial Engineering.* http://dx.doi.org/10.1016/j.cie.2011.09.011.

Uzar, M. F., & Catay, B. (2012). Distribution planning of bulk lubricants at BP Turkey. *Omega, 40*, 870–881.

Vidovic, M., Popovic, D., & Ratkovic, B. (submitted for publication). Mixed integer and heuristics model for the inventory routing problem in fuel delivery. International Journal of Production Economics, for the Special Issue: Interdisciplinary Research. Manuscript number IJPE-D-11-00007R1.

Yu, Y., Chen, H., & Chu, F. (2008). A new model and hybrid approach for large scale inventory routing problems. *European Journal of Operational Research, 1893*, 1022–1040.

Yu, Y., Chu, C., Chen, H., & Chu, F. (2010). Large scale stochastic inventory routing problems with split delivery and service level constraints. *Annals of Operation Research.* http://dx.doi.org/10.1007/s10479-010-0772-4.

Zachariadis, E. E., Tarantilis, C. D., & Kiranoudis, C. T. (2009). An integrated local search method for inventory routing decisions. *Experts Systems with Applications, 36*, 10239–10248.

Zao, Q., Chen, S., & Zang, C. (2008). Model and algorithm for inventory/routing decision in a three-echelon logistics system. *European Journal of Operational Research, 191*, 623–635.